Logic, Agency, and Games, Day Four

Topic flow:

1 L ⇔ G: Logic *of* games: use logic as it is to analyze reasoning in, or about, games. Logic *as* games: use games as they are to analyze logical notions. Today the latter in focus.

2 Evaluation games for first-order logic (pull things apart into two roles): logical constants as game operations of choice, switch, etc. Adequacy Lemma: φ is true in $\boldsymbol{M}$, s iff Verifier has a winning strategy in the *game*(φ, $\boldsymbol{M}$, s), same for falsity and a w.s. for Falsifier. Games for other languages can be much more complex, e.g., infinite 'parity games' needed for fixed-point logics.

3 Game-theoretic background: Zermelo's Theorem.

4 Kiss of Death: 'games bring nothing new'? Rethink logical constants: choice, switch, quantifiers are atomic games of object picking with the real game operation the ; of sequential composition. (Issue: for which quantifiers does this work? E.g., drawing objects with or without replacement, drawing larger samples in the probabilistic manner, etc.) New game algebra inside first-order logic. Even further natural game operations\logical constants, e.g., parallel play of two games.

5 Ehrenfeucht-Fraïssé games. Comparing models for 'analogy' by Duplicator and Spoiler. Adequacy theorem. More precisely, Spoiler's winning strategies are tightly correlated with first-order formulas true in one of the models and false in the other, Duplicator's winning strategies: finite back-and-forth morphisms, or for the infinite game: potential isomorphisms.

6 Game-theoretic background: Gale-Stewart Theorem.

7 Temporal forcing logic of powers provides compact logical forms for the Gale-Stewart setting: e.g., Weak Determinacy is $\{i\}\varphi \vee \{j\}G\neg\{i\}\varphi$. Decidable logic, but explicit axiomatization unknown.

8 Weak thesis (majority view among logicians): logic games are useful didactical tools, just that. Strong thesis (minority view among logicians): the meanings of the logical constants are given by games, and the resulting logics may well deviate from classical logic. 'In between' position: the games do suggest interesting new things. E.g., predicate logic gets deconstructed into a decidable algebra of the basic game constructions (this can be proved by a reduction of evaluation games into the decidable 'Game Logic' of Parikh 1985) with on top of that a theory of object picking (just one special mathematical operation) that leads to undecidability.

9 Combinations/hybrids. Design new games that are not quite logic games for new scenarios. We did the sabotage game, originally proposed for analyzing algorithmic tasks under disturbances, now with an interpretation in terms of learning. Game solution complexity goes up, the game suggests a new modal logic that contains, in addition to the usual modalities over edges, an edge-deletion modality interpreting formulas by reference to a changed model. Surprise: Sabotage Modal Logic is undecidable, one of the simplest modal logics showing this feature. The sabotage game has also been used to study the development of strategic reasoning with young children, and in giving logic lectures to children from age 6 onward. General current trend: design and study of logical network games, 'gamification' is a general trend also in society.

10 Comments in class: * evaluation games when we do not know the truth value (i.e., combinations with knowledge), * restricting attention to 'effective strategies', * evaluation games with more than two truth values, * games for constructing models (also in my book, e.g., the Henkin completeness proof has a game-like character), * spectrum of game layers in FOL.

For further details on all this: see the course materials.