Radiation pressure on the boundary of a fluid and a metal

I. Waves of infinite width

by L.G. Suttorp and S.R. de Groot

Institute of Theoretical Physics, University of Amsterdam, Valckenierstraat 65, 1018 XE Amsterdam, the Netherlands

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SYNOPSIS

The radiation pressure is evaluated for a plane electromagnetic wave that impinges obliquely through a polarized fluid on a metal surface. The starting point is the time-averaged momentum-balance equation in which both the pressure and the force density are specified. The results are in accordance with the recent experimental findings of Jones and Leslie.

1. INTRODUCTION

Recently Jones and Leslie [4] have measured to a very high accuracy the radiation pressure exerted by a wave that impinges obliquely through a polarized fluid on a strongly reflecting surface. They found that the radiation pressure is proportional to the refractive index of the fluid and independent of the plane of polarization. These facts are in contradiction to a prediction due to Peierls [7, 8]. His theory was based on a statistical consideration of the forces exerted on the particles of the system. In particular, he found that the force density in a polarized fluid would contain the divergence of an anisotropic tensor of the form $\frac{1}{5}\mathbf{PP}$, with $\mathbf{P}$ the polarization density. We showed in a recent paper [10] that such a term may be compensated by a similar contribution in the pressure tensor of the polarized fluid. In fact, both the force density and the divergence of the pressure tensor appear in the momentum-balance equation on an equal footing, so that terms may be shifted from the one to the other.

In this paper we shall derive the radiation pressure on the basis of a momentum-balance equation from which the anisotropic terms mentioned above have been eliminated. Since only the time-averaged form of the balance equation will be used, the precise form of the electromagnetic momentum density plays no role in the treatment. As a model for a strongly reflecting medium we choose a metal of high conductivity. It will turn out that, apart from the forces exerted on the charge carriers in the metal, pressure corrections in the fluid have to be taken into account. The expression for the radiation pressure obtained in this way is in accordance with the experimental results of Jones and Leslie.

2. FRESNEL RELATIONS

In this section we shall collect the Fresnel relations [1, 9] for the reflexion and transmission of a plane electromagnetic wave incident through an electrically polarized fluid of refractive index n on its (flat) boundary with a metal of conductivity σ. In the following these material properties will be assumed to be independent of the position. For oblique incidence one has to distinguish between transverse electric (E) and transverse magnetic (M) waves.

The electric and magnetic fields for an incident E-wave are given by the real parts of

$$\text{(1)} \quad \begin{cases} \mathbf{E}_i(\mathbf{R}, t) = \mathbf{n}_0 E_{i0} \exp(ik\mathbf{n}_i \cdot \mathbf{R} - i\omega t), \\ \mathbf{B}_i(\mathbf{R}, t) = (k/\omega)\mathbf{n}_i \wedge \mathbf{n}_0 E_{i0} \exp(ik\mathbf{n}_i \cdot \mathbf{R} - i\omega t). \end{cases}$$

Here $\mathbf{n}_i$ is the unit vector in the direction of the wave vector with length $k = n\omega$ (we have put $c = 1$). The unit vector $\mathbf{n}_0$ is perpendicular to the plane of incidence; the latter is determined by $\mathbf{n}_i$ and the normal $\mathbf{n}$ to the boundary. Choosing $\mathbf{n}$ as pointing into the metal, the angle of incidence θ_i is determined by $\cos\theta_i = \mathbf{n} \cdot \mathbf{n}_i$. The vector $\mathbf{n}_0$ is then equal to $\mathbf{n}_i \wedge \mathbf{n}/\sin\theta_i$.

The reflected and transmitted fields have the form

$$\text{(2)} \quad \begin{cases} \mathbf{E}_r = \mathbf{n}_0 E_{r0} \exp(ik\mathbf{n}_r \cdot \mathbf{R} - i\omega t - i\psi_r), \\ \mathbf{B}_r = (k/\omega)\mathbf{n}_r \wedge \mathbf{n}_0 E_{r0} \exp(ik\mathbf{n}_r \cdot \mathbf{R} - i\omega t - i\psi_r), \end{cases}$$

$$\text{(3)} \quad \begin{cases} \mathbf{E}_t = \mathbf{n}_0 E_{t0} \exp(ik'\mathbf{n}_t \cdot \mathbf{R} - k''\mathbf{n} \cdot \mathbf{R} - i\omega t - i\psi_t), \\ \mathbf{B}_t = [(k'\mathbf{n}_t + ik''\mathbf{n})/\omega] \wedge \mathbf{n}_0 E_{t0} \exp(ik'\mathbf{n}_t \cdot \mathbf{R} - k''\mathbf{n} \cdot \mathbf{R} - i\omega t - i\psi_t). \end{cases}$$

The unit vectors $\mathbf{n}_r$ and $\mathbf{n}_t$, with $\mathbf{n} \cdot \mathbf{n}_r = -\cos\theta_i$ and $\mathbf{n} \cdot \mathbf{n}_t = \cos\theta_t$, give the directions of the reflected and transmitted waves. The two wave numbers k' and k'' that characterize the propagation and damping in the metal fulfil the relations

$$\text{(4)} \quad \begin{cases} 2k'k'' \cos\theta_t = \sigma\omega, \\ k'^2 - k''^2 = \omega^2. \end{cases}$$

To keep the amplitudes E_{r0} and E_{t0} in (2) and (3) real (just as E_{i0} in (1)), we introduced phases ψ_r and ψ_t.

The boundary conditions yield Snell's law

$$\text{(5)} \quad k \sin\theta_i = k' \sin\theta_t$$

and the pair of relations for the amplitudes

$$(6)\qquad \begin{cases} E_{i0}+E_{r0}e^{-i\psi_r}=E_{t0}e^{-i\psi_t}, \\ k\cos\theta_i(E_{i0}-E_{r0}e^{-i\psi_r})=(k'\cos\theta_t+ik'')E_{t0}e^{-i\psi_t}. \end{cases}$$

(The origin of the co-ordinate system has been chosen to lie in the boundary.) The solutions of (6) are

$$(7)\qquad f_r^E\equiv\frac{E_{r0}}{E_{i0}}e^{-i\psi_r}=\frac{k\cos\theta_i-k'\cos\theta_t-ik''}{k\cos\theta_i+k'\cos\theta_t+ik''},$$

$$(8)\qquad f_t^E\equiv\frac{E_{t0}}{E_{i0}}e^{-i\psi_t}=\frac{2k\cos\theta_i}{k\cos\theta_i+k'\cos\theta_t+ik''}.$$

For fixed k and ψ_i the parameters k', k'' and θ_t follow from (4) and (5). In fact one finds

$$(9)\qquad \begin{cases} k'^2\cos^2\theta_t=\frac{1}{2}\omega^2(r+s), \\ k''^2=\frac{1}{2}\omega^2(r-s), \end{cases}$$

where

$$(10)\qquad \begin{cases} r\equiv\sqrt{(1-n^2\sin^2\theta_i)^2+\sigma_0^2}, \\ s\equiv 1-n^2\sin^2\theta_i, \end{cases}$$

with $\sigma_0\equiv\sigma/\omega$.

The incident, reflected and transmitted fields of an M-wave read

$$(11)\qquad \begin{cases} \mathbf{E}_i=(\omega/k)\mathbf{n}_0\wedge\mathbf{n}_iB_{i0}\exp(ik\mathbf{n}_i\cdot\mathbf{R}-i\omega t), \\ \mathbf{B}_i=\mathbf{n}_0B_{i0}\exp(ik\mathbf{n}_i\cdot\mathbf{R}-i\omega t), \end{cases}$$

$$(12)\qquad \begin{cases} \mathbf{E}_r=(\omega/k)\mathbf{n}_0\wedge\mathbf{n}_rB_{r0}\exp(ik\mathbf{n}_r\cdot\mathbf{R}-i\omega t-i\psi'_r), \\ \mathbf{B}_r=\mathbf{n}_0B_{r0}\exp(ik\mathbf{n}_r\cdot\mathbf{R}-i\omega t-i\psi'_r), \end{cases}$$

$$(13)\qquad \begin{cases} \mathbf{E}_t=\mathbf{n}_0\wedge[(k'\mathbf{n}_t+ik''\mathbf{n})/\sqrt{\omega^2+\sigma^2}]B_{t0}\exp(ik'\mathbf{n}_t\cdot\mathbf{R}-k''\mathbf{n}\cdot\mathbf{R}-i\omega t-i\psi'_t), \\ \mathbf{B}_t=\mathbf{n}_0B_{t0}\exp(ik'\mathbf{n}_t\cdot\mathbf{R}-k''\mathbf{n}\cdot\mathbf{R}-i\omega t-i\psi''_t). \end{cases}$$

The phases ψ'_t and ψ''_t satisfy the relation

$$(14)\qquad \exp[i(\psi'_t-\psi''_t)]=(\omega+i\sigma)/\sqrt{\omega^2+\sigma^2}.$$

The boundary conditions are now (5) and

$$(15)\qquad \begin{cases} B_{i0}+B_{r0}e^{-i\psi'_r}=B_{t0}e^{-i\psi''_t}, \\ \omega\sqrt{\omega^2+\sigma^2}\cos\theta_i(B_{i0}-B_{r0}e^{-i\psi'_r})=k(k'\cos\theta_t+ik'')B_{t0}e^{-i\psi'_t}, \end{cases}$$

of which the solutions are

$$(16)\qquad f_r^M\equiv\frac{B_{r0}}{B_{i0}}e^{-i\psi'_r}=\frac{\omega(\omega+i\sigma)\cos\theta_i-k(k'\cos\theta_t+ik'')}{\omega(\omega+i\sigma)\cos\theta_i+k(k'\cos\theta_t+ik'')},$$

$$(17)\qquad f_t^M\equiv\frac{B_{t0}}{B_{i0}}e^{-i\psi'_t}=\frac{2\omega\sqrt{\omega^2+\sigma^2}\cos\theta_i}{\omega(\omega+i\sigma)\cos\theta_i+k(k'\cos\theta_t+ik'')}.$$

The Fresnel relations (7–8) and (16–17) will be needed for the derivation of the radiation pressure.

3. THE RADIATION PRESSURE

The momentum balance equation for a polarizable substance in an oscillating electromagnetic field gets a simple form upon time-averaging [3, 10]:

$$(18) \qquad -\nabla\cdot\bar{\boldsymbol{P}}+\bar{\mathbf{F}}=0,$$

if the macroscopic velocities can be neglected. Here $\bar{\mathbf{F}}$ is the averaged force density

$$(19) \qquad \bar{\mathbf{F}}=\overline{(\nabla\mathbf{E})\cdot\mathbf{P}}=\nabla\cdot[\bar{\mathbf{D}}\bar{\mathbf{E}}+\bar{\mathbf{B}}\bar{\mathbf{B}}-\tfrac{1}{2}(\overline{E^2}+\overline{B^2})\boldsymbol{U}]$$

and $\bar{\boldsymbol{P}}$ the averaged pressure tensor. For a fluid the latter is isotropic [10].

To obtain the radiation pressure we integrate (18–19) over a thin circular-cylindrical slab lying symmetrically on both sides of the boundary. With Gauss's theorem one obtains then

$$(20) \qquad \begin{cases} \mathbf{n}\cdot[\overline{\boldsymbol{P}_f}-\overline{\mathbf{D}_f\mathbf{E}_f}-\overline{\mathbf{B}_f\mathbf{B}_f}+\tfrac{1}{2}(\overline{E_f^2}+\overline{B_f^2})\boldsymbol{U}]= \\ =\mathbf{n}\cdot[\overline{\boldsymbol{P}_m}-\overline{\mathbf{E}_m\mathbf{E}_m}-\overline{\mathbf{B}_m\mathbf{B}_m}+\tfrac{1}{2}(\overline{E_m^2}+\overline{B_m^2})\boldsymbol{U}], \end{cases}$$

where the subscripts f and m indicate fluid and metal, respectively. The averaged pressure tensor $\overline{\boldsymbol{P}_f}$, being isotropic, may be written as $\overline{\boldsymbol{P}_f}=\overline{p_f}\boldsymbol{U}$. The scalar pressure satisfies the time-averaged momentum balance in the fluid, *viz.* $\nabla\overline{p_f}=\overline{(\nabla\mathbf{E})\cdot\mathbf{P}}$, which is equal to $\tfrac{1}{2}\nabla(\overline{\mathbf{P}\cdot\mathbf{E}})-\tfrac{1}{2}\overline{E^2}\nabla n^2$, since $\mathbf{P}=(n^2-1)\mathbf{E}$. We wish to use this relation to connect the pressure $\overline{p_f}$ to the pressure p_{f0} at a position in the fluid, where the field strength is negligible. Strictly spoken such a position exists only if the incoming and the reflected waves have finite extension [11], while we considered infinite plane waves up to now. For wide beams, however, the Fresnel relations of § 2 will remain valid in the centre of the beam. If the electrostriction effects near the edge of the beam may be neglected, as is the case for fluids with low compressibility, the term proportional to ∇n^2 may be neglected. We then have

$$(21) \qquad \overline{p_f}=p_{f0}+\tfrac{1}{2}\overline{\mathbf{P}_f\cdot\mathbf{E}_f}.$$

The fields in the metal have a finite penetration depth. The pressure tensor $\boldsymbol{P}_{m0}$ at a field-free position may therefore be related to $\overline{\boldsymbol{P}_m}$ by considering the momentum balance for a cylindrical volume with axis perpendicular to the surface, with one flat side, of unit area, near the boundary and with the other deep in the metal. Then the right-hand side of (20) is found to be equal to $\mathbf{n}\cdot\boldsymbol{P}_{m0}$, since the contributions of the cylinder mantle vanish on grounds of symmetry.

The radiation pressure is defined as

$$(22) \qquad p^{\mathrm{rad}}=\mathbf{nn}:\boldsymbol{P}_{m0}-p_{f0}.$$

According to (20) and (21) it is equal to

(23) $$p^{\mathrm{rad}} = -\mathbf{nn}: [\overline{\mathbf{D}_f \mathbf{E}_f} + \overline{\mathbf{B}_f \mathbf{B}_f} - \tfrac{1}{2}(\overline{\mathbf{D}_f \cdot \mathbf{E}_f} + \overline{\mathbf{B}_f \cdot \mathbf{B}_f}) \boldsymbol{U}].$$

Besides this perpendicular component of the radiation force density per unit surface, one may introduce a "parallel radiation pressure"

(24) $$\mathbf{p}_{//}^{\mathrm{rad}} = \mathbf{n} \cdot \boldsymbol{P}_{m0} \cdot (\boldsymbol{U} - \mathbf{nn}),$$

which is a vector parallel to the boundary. Introducing the fields in the fluid we obtain from (20)

(25) $$\mathbf{p}_{//}^{\mathrm{rad}} = -\mathbf{n} \cdot (\overline{\mathbf{D}_f \mathbf{E}_f} + \overline{\mathbf{B}_f \mathbf{B}_f}) \cdot (\boldsymbol{U} - \mathbf{nn}).$$

For the transverse electric and the transverse magnetic waves of section 2 the parallel radiation pressure is a vector in the direction $\mathbf{n} \wedge \mathbf{n}_0$, so that one may write $\mathbf{p}_{//}^{\mathrm{rad}} = p_{//}^{\mathrm{rad}} \mathbf{n} \wedge \mathbf{n}_0$.

The perpendicular and the parallel radiation pressures will be evaluated consecutively for the transverse electric and the transverse magnetic waves, the fields of which have been given in the previous section. The fields in the fluid are the sums of incident and reflected contributions. For the transverse electric waves we find upon inserting (1) and (2) into (23)

(26) $$p^{\mathrm{rad},E} = \tfrac{1}{2} n^2 \cos^2 \theta_i (E_{i0}^2 + E_{r0}^2).$$

The time-averaged incident energy current S is equal to $\frac{1}{2} n E_{i0}^2$, so that the reduced radiation pressure $\hat{p} = p/S$ becomes

(27) $$\hat{p}^{\mathrm{rad},E} = n \cos^2 \theta_i (1 + |f_r^E|^2),$$

with f_r^E given in (7). Likewise one finds for the transverse magnetic fields upon insertion of (11) and (12) into (23):

(28) $$p^{\mathrm{rad},M} = \tfrac{1}{2} \cos^2 \theta_i (B_{i0}^2 + B_{r0}^2)$$

or with $S = \frac{1}{2} n^{-1} B_{i0}^2$:

(29) $$\hat{p}^{\mathrm{rad},M} = n \cos^2 \theta_i (1 + |f_r^M|^2),$$

where f_r^M has been defined in (16). For $\theta_i = 0$ both (27) and (29) become

(30) $$\hat{p}^{\mathrm{rad},\perp} = n(1 + |f_r^\perp|^2),$$

where $f_r^\perp$ equals f_r^E or f_r^M for normal incidence.

The quantities $1 + |f_r^\lambda|^2$, with λ standing for E, M or $\perp$, follow from (7) and (16) with (9). In fact, one may write

(31) $$1 + |f_r^\lambda|^2 = \frac{2}{1 + A^\lambda}$$

with

(32) $$A^E = \frac{\sqrt{2(r+s)}\, n \cos \theta_i}{r + n^2 \cos^2 \theta_i},$$

$$(33)\qquad A^M=\frac{[\sqrt{2(r+s)}+\sigma_0\sqrt{2(r-s)}]n\cos\theta_i}{n^2r+(1+\sigma_0^2)\cos^2\theta_i},$$

$$(34)\qquad A^\perp=\frac{n\sqrt{2(1+\sqrt{1+\sigma_0^2})}}{\sqrt{1+\sigma_0^2}+n^2}.$$

The radiation pressure (27), (29) and (30) then become

$$(35)\qquad p^{\mathrm{rad},\lambda}=2n\cos^2\theta_i\,\frac{1}{1+A^\lambda}.$$

As a consequence the ratio of the transverse electric and the transverse magnetic radiation pressures is

$$(36)\qquad \mathscr{R}\equiv\frac{p^{\mathrm{rad},E}}{p^{\mathrm{rad},M}}=\frac{1+A^M}{1+A^E}.$$

For small angles θ_i the expressions (32) and (33) with (10) can be expanded into powers of $\sin\theta_i$:

$$(37)\qquad A^{E,M}=A^\perp\left[1\mp\frac{(n^2-1)^2+\sigma_0^2}{2\sqrt{1+\sigma_0^2}(n^2+\sqrt{1+\sigma_0^2})}\sin^2\theta_i+\dots\right],$$

where the upper and lower signs refer to E and M, respectively.

If the conductivity of the metal is high the quantities A^λ (32–34) become with (10):

$$(38)\qquad A^{E,M}\simeq A^\perp(\cos\theta_i)^{\pm1}$$

with

$$(39)\qquad A^\perp\simeq n\sqrt{2\sigma_0^{-1}}.$$

Insertion of these expressions into (35) brings the radiation pressure for the various cases in the form

$$(40)\qquad p^{\mathrm{rad},E,M}=2n\cos^2\theta_i[1-n\sqrt{2\sigma_0^{-1}}(\cos\theta_i)^{\pm1}+\dots],$$

$$(41)\qquad p^{\mathrm{rad},\perp}=2n(1-n\sqrt{2\sigma_0^{-1}}+\dots).$$

The ratio (36) of the E- and M-radiation pressures is then:

$$(42)\qquad \mathscr{R}=1-n\sqrt{2\sigma_0^{-1}}\,[\cos\theta_i-(\cos\theta_i)^{-1}]+\dots.$$

Another form for this ratio makes use of the reflectivity for normal incidence, which according to (31) and (39) reads

$$(43)\qquad R^\perp=|f_r^\perp|^2=1-2n\sqrt{2\sigma_0^{-1}}+\dots.$$

In terms of this reflectivity the ratio (42) gets the form

$$(44)\qquad \mathscr{R}=1-\tfrac{1}{2}(1-R^\perp)[\cos\theta_i-(\cos\theta_i)^{-1}]+\dots.$$

If the normal reflectivity is close to 1, as is the case for high conductivity, the transverse electric and the transverse magnetic radiation pressures become almost indistinguishable.

The parallel radiation pressure follows from (25). For E-waves one finds with (1) and (2)

$$p_{//}^{\text{rad},E} = \tfrac{1}{2} n^2 \sin\theta_i \cos\theta_i (E_{i0}^2 - E_{r0}^2), \tag{45}$$

so that the reduced parallel radiation pressure $\hat{p} = p/S$ becomes

$$\hat{p}_{//}^{\text{rad},E} = n \sin\theta_i \cos\theta_i (1 - |f_r^E|^2). \tag{46}$$

The M-waves give rise to the parallel radiation pressure

$$p_{//}^{\text{rad},M} = \tfrac{1}{2} \sin\theta_i \cos\theta_i (B_{i0}^2 - B_{r0}^2) \tag{47}$$

or

$$\hat{p}_{//}^{\text{rad},M} = n \sin\theta_i \cos\theta_i (1 - |f_r^M|^2). \tag{48}$$

Introducing the quantities A^λ through (31) we obtain for (46) and (48):

$$\hat{p}_{//}^{\text{rad},\lambda} = 2n \sin\theta_i \cos\theta_i \frac{A_\lambda}{1 + A_\lambda} \tag{49}$$

with $\lambda = E, M$. For small angles θ_i one can employ for A^λ the approximate expression (37), while for high conductivity (38) may be used.

The expressions (41) for the (perpendicular) radiation pressure at normal incidence and (42) or (44) for the ratio of the pressures due to E- and M-radiation at oblique incidence are in agreement with the measurements of Jones-Richards [5] and Jones-Leslie [4]. In the former experiment the radiation pressure at normal incidence on a highly reflecting surface was found to be proportional to the refractive index n of the fluid in conformity with the leading term of (41). In the second experiment, of a much higher accuracy, the radiation pressure has been measured also for oblique incidence. The ratio $\mathscr{R}$ was found to be equal to unity in conformity with the leading term of (42) or (44). The second term of (44) gives a correction of the order of 2 ‰ for the maximum angle θ_i considered by Jones and Leslie. A correction of this size is just below the level of accuracy of their experiment. (It should be noted, however, that they use a multi-layer mirror with a high reflectivity, while (44) is in fact valid only for a metal surface.)

In some recent papers Peierls [7, 8] predicted a dependence of the radiation pressure on the refractive index n, which is more complicated than (40) for M-radiation. Moreover in his theory the ratio $\mathscr{R}$ should differ from unity even if the metal has an infinite conductivity. Neither of these predictions was borne out by the experiments. The reason for the discrepancy is that he argued that the averaged pressure tensor $\overline{\boldsymbol{P}_f}$ should contain anisotropic terms. In a preceding paper [10] we showed that this is not the case.

The expressions for the perpendicular radiation pressure as found here followed by inserting into (23) the expressions for the fields in the fluid. The right-hand side of (23) contains a tensor that is in fact the spatial section of the field energy-momentum tensor as put forward by Minkowski and Abraham [2, 3]. However, in confining one's attention to the energy-momentum tensor of these authors one naively considers only the field part of the total energy-momentum tensor, and ignores the material part completely. In the approach followed here one starts from the complete momentum-balance equation and takes due account of the changes in the material pressure brought about by the electromagnetic field.

4. CONNEXION OF THE RADIATION PRESSURE AND THE LORENTZ FORCES ON THE METAL

An alternative method to evaluate the radiation pressure starts from the total time-averaged Lorentz force on the charged particles in the metal:

$$\sigma \int^{V} d\mathbf{R}\ \overline{\mathbf{E}\wedge\mathbf{B}}; \tag{50}$$

here V is the volume of a circular cylinder lying in the metal with its axis orthogonal to the boundary, with one flat side, of unit area, near the boundary, and with the other deep in the metal (just as considered below (21)). In writing (50) we made use of the fact that the charge density in the metal vanishes both for E- and M-waves. By applying Gauss's theorem, using symmetry arguments to dismiss the contribution of the mantle and taking account of the finite penetration depth of the metal fields, one may write (50) as

$$-\mathbf{n}\cdot[\overline{\mathbf{E}_m\mathbf{E}_m}+\overline{\mathbf{B}_m\mathbf{B}_m}-\tfrac{1}{2}(\overline{\mathbf{E}_m^2}+\overline{\mathbf{B}_m^2})\boldsymbol{U}], \tag{51}$$

where $\mathbf{E}_m$ and $\mathbf{B}_m$ indicate the metal fields near the boundary. The components of (51) parallel and orthogonal to $\mathbf{n}$ will be denoted as p^{Lor} and $p_{//}^{\mathrm{Lor}}$, respectively.

For an E-wave the radiation pressure follows on comparing (51) with (23) and using the boundary conditions as:

$$p^{\mathrm{rad},E}=p^{\mathrm{Lor},E}+\Delta p^{f,E}, \tag{52}$$

where

$$\Delta p^{f,E}=\tfrac{1}{2}\overline{\mathbf{P}_f\cdot\mathbf{E}_f} \tag{53}$$

is the pressure correction in the fluid, *v.* (21). The formula (52) shows that the radiation pressure for an E-wave is the sum of the force per unit surface, associated to the Lorentz forces, and the fluid-pressure correction. The ratio of these two contributions is found to be

$$\frac{\Delta p^{f,E}}{p^{\mathrm{Lor},E}}=\frac{n^2-1}{r+s} \tag{54}$$

with r and s defined in (10). Both for $n\simeq 1$ and for $\sigma\to\infty$ the fluid-pressure correction can be neglected as compared to the Lorentz-force contribution.

On a par with (52) one finds for the parallel radiation pressure of an E-wave

$$p_{//}^{\mathrm{rad},E}=p_{//}^{\mathrm{Lor},E}. \tag{55}$$

The fluid-pressure correction plays no role in this case.

For the transverse magnetic wave an analogous connexion between the radiation pressure and the Lorentz forces on the metal can be established. It should be borne in mind, however, that an M-wave generates surface charges on the metal (with density ϱ_s), the Lorentz forces of which should be accounted for. As a matter of fact one gets

$$(56) \qquad p^{\mathrm{rad},M} = p^{\mathrm{Lor},M} + p^{\mathrm{Lor}',M} + \Delta p^{f,M}.$$

Here $p^{\mathrm{Lor}',M}$ is the component parallel to $\mathbf{n}$ of the Lorentz force on the surface charge:

$$(57) \qquad p^{\mathrm{Lor}',M} = \tfrac{1}{2}\overline{\varrho_s(\mathbf{E}_m + \mathbf{D}_f)}\cdot\mathbf{n};$$

it contains the effective field $\frac{1}{2}(\mathbf{E}_m + \mathbf{D}_f)$ in the direction orthogonal to the boundary. The fluid-pressure correction in (56) is now

$$(58) \qquad \Delta p^{f,M} = \tfrac{1}{2}\overline{\mathbf{P}_f\cdot\mathbf{E}_f} + \tfrac{1}{2}\overline{(\mathbf{P}_f\cdot\mathbf{n})^2}.$$

The last term is the Liénard pressure [3, 6]; it does not contribute for an E-wave, since in that case the polarization $\mathbf{P}$ is parallel to the boundary. The ratio of the fluid-pressure correction and the Lorentz-force contributions to the radiation pressure turns out to be

$$(59) \qquad \frac{\Delta p^{f,M}}{p^{\mathrm{Lor},M} + p^{\mathrm{Lor}',M}} = \frac{(n^2-1)[r + (1+\sigma_0^2)\sin^2\theta_i]}{r + (1+\sigma_0^2)s}.$$

For $n \simeq 1$ the Lorentz-force contributions dominate the fluid-pressure correction. In contrast to what was the case for the E-wave the fluid-pressure correction does not become negligible in the limit $\sigma \to \infty$.

The parallel radiation pressure is not afflicted with fluid-pressure corrections, just as was the case for the E-wave:

$$(60) \qquad p_{//}^{\mathrm{rad},M} = p_{//}^{\mathrm{Lor},M} + p_{//}^{\mathrm{Lor}',M};$$

here the last term is the component parallel to the boundary of the Lorentz force on the surface charge:

$$(61) \qquad p_{//}^{\mathrm{Lor}',M} = \tfrac{1}{2}\overline{\varrho_s(\mathbf{E}_m + \mathbf{E}_f)}\cdot(\mathbf{n}\wedge\mathbf{n}_0)$$

with the effective field $\frac{1}{2}(\mathbf{E}_m + \mathbf{E}_f)$ in the direction parallel to the boundary. In conclusion we have found that in general the radiation pressure is only partly due to the Lorentz forces on the metal; the pressure correction in the fluid is equally important.

REFERENCES

1. Born, M. and E. Wolf – Principles of Optics, Pergamon, London (1959).
2. Brevik, I. – Experiments in phenomenological electrodynamics and the electromagnetic energy-momentum tensor, Phys. Reports **52**, 133 (1979).
3. Groot, S.R. de and L.G. Suttorp – Foundations of Electrodynamics, North-Holland, Amsterdam (1972).

4. Jones, R.V. and B. Leslie – The measurement of optical radiation pressure in dispersive media, Proc. Roy. Soc. London **A360,** 347 (1978).
5. Jones, R.V. and J.C.S. Richards – The pressure of radiation in a refracting medium, Proc. Roy. Soc. London **A221,** 480 (1954).
6. Liénard, A. – Equilibre et déformation de systèmes de conducteurs traversés par des courants et de corps magnétiques sans hystérésis, Ann. Physique **20,** 249 (1923).
7. Peierls, R. – The momentum of light in a refracting medium, Proc. Roy. Soc. London **A347,** 475 (1976).
8. Peierls, R. – The momentum of light in a refracting medium II. Generalization. Application to oblique reflexion, Proc. Roy. Soc. London **A355,** 141 (1977).
9. Stratton, J.A. – Electromagnetic Theory, McGraw-Hill, New York (1941).
10. Suttorp, L.G. and S.R. de Groot – On anisotropy of the pressure tensor for a polarizable fluid, Physica (1981), to be published.
11. Suttorp, L.G. and S.R. de Groot – Radiation pressure on the boundary of a fluid and a metal II. Waves of finite width, Proc. Kon. Ned. Akad. Wet. **B84** (3), 325 (1981).