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THE EFFECT OF DYNAMICAL SCREENING ON SELF-DIFFUSION IN A DENSE MAGNETIZED PLASMA

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The coefficient of self-diffusion along an external magnetic field in a dense plasma is evaluated with the use of a kinetic equation that takes dynamical screening effects into account. It is found that the self-diffusion coefficient diminishes monotonously as the magnetic field strength increases. Qualitatively the present calculation corroborates a result obtained recently in a cruder approximation in which only static screening effects are considered.

1. Introduction

In a previous paper[1]) we started the investigation of self-diffusion through a dense Coulomb plasma in a magnetic field. As a model we use the one-component plasma with a neutralizing background. The motion of a tagged particle is described by a kinetic equation for the time correlation function. We have derived this kinetic equation with the use of renormalized kinetic theory; it incorporates the plasma screening effects through a renormalization of the interparticle potential.

The collision term contains a Rostoker-type memory kernel which takes the dynamical screening effects into account. Owing to the presence of the frequency and wave vector dependent dielectric function the ensuing expression for the self-diffusion coefficient is somewhat unwieldy. A simplified (Landau) version that contains only static screening effects was found to lead to an abatement of the longitudinal self-diffusion coefficient as the magnetic field strength increases. This effect is corroborated by results from molecular dynamics[2]).

For an unmagnetized plasma it is known[3–5]) that the inclusion of the dynamical screening properties yields a more accurate result for the magnitude of the self-diffusion coefficient. In the present paper we shall use therefore the generalized Rostoker memory kernel to evaluate the self-diffusion coefficient for the magnetized plasma.

In the lowest Chapman–Cowling approximation the longitudinal self-diffusion coefficient is found to be represented by a three-dimensional integral.

For a weak magnetic field we shall obtain an expansion of this integral in the parameter $b = \omega_B/\omega_p$, the ratio of the Larmor frequency and the plasma frequency. For a very strong magnetic field, viz. for $b \to \infty$, the asymptotic value of the diffusion coefficient will be calculated.

For intermediate values of the magnetic field strength we shall evaluate the three-dimensional integral numerically. To that end the integrand must be calculated efficiently. We shall attain this goal by dividing the integration domain in various parts; for each of these regions a suitable representation of the integrand will be derived.

2. Evaluation of the longitudinal self-diffusion coefficient from the Rostoker kernel

The longitudinal self-diffusion coefficient follows from the time correlation function for a tagged particle by using a Green–Kubo relation, viz.

$$D_{\|} = \frac{\mathrm{i}}{m^2 n} \lim_{\eta \to 0} \int \mathrm{d}\boldsymbol{p}\, \mathrm{d}\boldsymbol{p}'\, p_{\|} C^s(\boldsymbol{k} = \boldsymbol{0}, \boldsymbol{p}\boldsymbol{p}'; \mathrm{i}\eta) p'_{\|}\,, \tag{2.1}$$

where $p_{\|}$ is the component of the momentum of the tagged particle in the direction of the magnetic field. The mass of the particles is denoted by m and the number of particles per unit volume by n. The Fourier-Laplace transform of the time correlation function is defined as:

$$C^s(\boldsymbol{k}, \boldsymbol{p}\boldsymbol{p}'; z) = -\mathrm{i} \int_0^\infty \mathrm{d}t\, \mathrm{e}^{\mathrm{i}zt} \frac{1}{V} \int \mathrm{d}\boldsymbol{r}\, \mathrm{d}\boldsymbol{r}'\, \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')} \langle \delta f^s(\boldsymbol{r}\boldsymbol{p}; t) \delta f^s(\boldsymbol{r}'\boldsymbol{p}'; 0) \rangle\,, \tag{2.2}$$

with the microscopic phase function:

$$f^s(\boldsymbol{r}\boldsymbol{p}; t) = \sqrt{N}\, \delta[\boldsymbol{r} - \boldsymbol{r}_s(t)] \delta[\boldsymbol{p} - \boldsymbol{p}_s(t)]\,. \tag{2.3}$$

The total number of particles is N and $\boldsymbol{r}_s(t)$, $\boldsymbol{p}_s(t)$ describe the trajectory in phase space of the tagged particle; furthermore δf^s is the deviation of f^s from its equilibrium ensemble average.

The self-diffusion coefficient can be related to the memory kernel φ^s, which describes the influence of the interaction between the particles on the evolution of the time correlation function. A particularly simple expression arises in the 1-Sonine approximation, namely

$$D_{\|}^{-1} = \mathrm{i}\beta^2 \lim_{\eta\to 0} \int \mathrm{d}\boldsymbol{p}\, \mathrm{d}\boldsymbol{p}'\, p_{\|} \varphi^{s}(\boldsymbol{k} = \boldsymbol{0}, \boldsymbol{pp}'; \mathrm{i}\eta) f_0(p') p'_{\|}\,, \tag{2.4}$$

with $f_0(p) = (\beta/2\pi m)^{3/2} \exp(-\beta p^2/2m)$ the normalized Maxwellian.

In a previous paper[1]) we derived approximate expressions for the memory kernel. By means of renormalized kinetic theory and the 'disconnected approximation' we obtained a kinetic equation that takes collective interactions in a dense plasma into account. The memory kernel that occurs in this equation will be referred to as the 'generalized Rostoker kernel'; explicitly we have:

$$\begin{aligned} &\varphi^{s}_{\mathrm{R}}(\boldsymbol{k}, \boldsymbol{pp}'; z) n f_0(p') \\ &\quad = -\frac{\mathrm{i}n^2}{\beta^2} \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3} c(q) h(q) \boldsymbol{q} \cdot \nabla_{\boldsymbol{p}} \boldsymbol{q} \cdot \nabla_{\boldsymbol{p}'} \\ &\quad \times \left[f_0(p) \int_0^\infty \mathrm{d}t \exp[\mathrm{i}zt - \mathrm{i}(\boldsymbol{k} - \boldsymbol{q}) \cdot \boldsymbol{\alpha}_t \cdot \boldsymbol{p}/m] \right. \\ &\quad \left. \times \frac{\mathrm{i}}{2\pi} \int_{-\infty+\mathrm{i}0}^{\infty+\mathrm{i}0} \mathrm{d}\zeta \frac{F_B(\boldsymbol{q}, \zeta)}{\varepsilon_B(\boldsymbol{q}, \zeta)} \mathrm{e}^{-\mathrm{i}\zeta t} \delta(\boldsymbol{\gamma}_t \cdot \boldsymbol{p} - \boldsymbol{p}') \right]. \end{aligned} \tag{2.5}$$

In this expression h and $c = h/(1 + nh)$ are the Fourier transforms of the pair correlation function and the direct correlation function, respectively. Furthermore

$$F_B(\boldsymbol{q}, z) = -\mathrm{i} \int_0^\infty \mathrm{d}t\, \mathrm{e}^{\mathrm{i}zt} \int \mathrm{d}\boldsymbol{p}\, f_0(p) \exp(-\mathrm{i}\boldsymbol{q} \cdot \boldsymbol{\alpha}_t \cdot \boldsymbol{p}/m) \tag{2.6a}$$

and

$$\varepsilon_B(\boldsymbol{q}, z) = 1 + nc(q)[zF_B(\boldsymbol{q}, z) - 1] \tag{2.6b}$$

describe the dynamical screening effects in the presence of the external magnetic field[1]). The motion of a particle with charge e in a magnetic field with strength B is given by $\boldsymbol{r}(t) = \boldsymbol{r} - \boldsymbol{\alpha}_{-t} \cdot \boldsymbol{p}/m$ and $\boldsymbol{p}(t) = \boldsymbol{\gamma}_{-t} \cdot \boldsymbol{p}$, where

$$\boldsymbol{\alpha}_t \cdot \boldsymbol{p} = \boldsymbol{p}_{\|} t + \frac{\boldsymbol{p}_{\perp}}{\omega_B} \sin \omega_B t - \frac{\boldsymbol{p}_{\perp} \wedge \hat{\boldsymbol{B}}}{\omega_B} (1 - \cos \omega_B t)\,, \tag{2.7a}$$

$$\boldsymbol{\gamma}_t \cdot \boldsymbol{p} = \boldsymbol{p}_{\|} + \boldsymbol{p}_{\perp} \cos \omega_B t - \boldsymbol{p}_{\perp} \wedge \hat{\boldsymbol{B}} \sin \omega_B t\,, \tag{2.7b}$$

with $\omega_B = eB/mc$ the Larmor frequency, $\hat{\boldsymbol{B}}$ the unit vector in the direction of the magnetic field and $\boldsymbol{p}_\perp$ the component of the momentum perpendicular to the magnetic field.

We get an approximate expression for the self-diffusion coefficient by inserting the generalized Rostoker kernel (2.5) into (2.4). The integrations over the momenta are readily performed; one finds:

$$D_{\parallel,\mathrm{R}}^{-1} = \lim_{\eta\to 0} n \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3} c(q)h(q)q_\parallel^2 \times \int_0^\infty \mathrm{d}t\, F_B(\boldsymbol{q}, t) \frac{\mathrm{i}}{2\pi} \int_{-\infty+\mathrm{i}0}^{\infty+\mathrm{i}0} \mathrm{d}\zeta \frac{F_B(\boldsymbol{q}, \zeta)}{\varepsilon_B(\boldsymbol{q}, \zeta)} \exp[-\mathrm{i}(\zeta - \mathrm{i}\eta t)] . \tag{2.8}$$

The (t, ζ)-integral can be replaced by

$$-\frac{1}{2\pi} \int_{-\infty+\mathrm{i}0}^{\infty+\mathrm{i}0} \mathrm{d}\zeta \frac{F_B(\boldsymbol{q}, \zeta)}{\varepsilon_B(\boldsymbol{q}, \zeta)} F_B(\boldsymbol{q}, -\zeta + \mathrm{i}\eta) . \tag{2.9}$$

To simplify this expression we note that $[F_B(\boldsymbol{q}, z)]^2/\varepsilon_B(\boldsymbol{q}, z)$ is analytic for $\mathrm{Im}\, z \geqslant 0$. Consequently

$$-\frac{1}{2\pi} \int_{-\infty+\mathrm{i}0}^{\infty+\mathrm{i}0} \mathrm{d}\zeta \frac{F_B(\boldsymbol{q}, \zeta)}{\varepsilon_B(\boldsymbol{q}, \zeta)} F_B(\boldsymbol{q}, \zeta + \mathrm{i}\eta) = 0 . \tag{2.10}$$

Adding this term to (2.9) and using the symmetry property $F_B(\boldsymbol{q}, z) = -F_B(\boldsymbol{q}, -z^*)^*$ we can write instead of (2.9):

$$-\frac{\mathrm{i}}{\pi} \int_{-\infty+\mathrm{i}0}^{\infty+\mathrm{i}0} \mathrm{d}\zeta \frac{F_B(\boldsymbol{q}, \zeta)}{\varepsilon_B(\boldsymbol{q}, \zeta)} \mathrm{Im}\, F_B(\boldsymbol{q}, \zeta) . \tag{2.11}$$

From (2.6b) it is seen that the dynamic screening function satisfies $\varepsilon_B(\boldsymbol{q}, z) = \varepsilon_B(\boldsymbol{q}, -z^*)^*$. By changing the integration variable in (2.11) to $-\zeta^*$ we may cast (2.11) into the form

$$\frac{1}{\pi} \int_{-\infty+\mathrm{i}0}^{\infty+\mathrm{i}0} \mathrm{d}\zeta\, \mathrm{Im} \left[\frac{F_B(\boldsymbol{q}, \zeta)}{\varepsilon_B(\boldsymbol{q}, \zeta)}\right] \mathrm{Im}\, F_B(\boldsymbol{q}, \zeta) . \tag{2.12}$$

As a consequence we obtain from (2.8) the following expression for the self-diffusion coefficient:

$$D_{\|,\mathrm{R}}^{-1} = \frac{n}{\pi} \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3} c(q)h(q)q_\|^2 \int_{-\infty+\mathrm{i}0}^{\infty+\mathrm{i}0} \mathrm{d}\zeta \, \mathrm{Im}\left[\frac{F_B(\boldsymbol{q},\zeta)}{\varepsilon_B(\boldsymbol{q},\zeta)}\right] \mathrm{Im}\, F_B(\boldsymbol{q},\zeta) \,. \tag{2.13}$$

From (2.6b) one easily concludes for real ω:

$$\mathrm{Im}\left[\frac{F_B(\boldsymbol{q},\omega)}{\varepsilon_B(\boldsymbol{q},\omega)}\right] = \frac{1-nc(q)}{|\varepsilon_B(\boldsymbol{q},\omega)|^2} \mathrm{Im}\, F_B(\boldsymbol{q},\omega) \,. \tag{2.14}$$

Hence (2.13) becomes:

$$D_{\|,\mathrm{R}}^{-1} = \frac{n}{\pi} \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3} c(q)^2 q_\|^2 \int_{-\infty+\mathrm{i}0}^{\infty+\mathrm{i}0} \mathrm{d}\zeta \, \frac{[\mathrm{Im}\, F_B(\boldsymbol{q},\zeta)]^2}{|\varepsilon_B(\boldsymbol{q},\zeta)|^2} \,. \tag{2.15}$$

We introduce the dimensionless integration variables $\boldsymbol{x} = \boldsymbol{q}/k_\mathrm{D}$, $\nu = \zeta/\omega_\mathrm{p}$ and $z = \hat{\boldsymbol{x}} \cdot \hat{\boldsymbol{B}}$, with k_D the inverse Debye length and ω_p the plasma frequency; then we find:

$$D_{\|,\mathrm{R}}^{-1} = \frac{1}{\omega_\mathrm{p} a^2} \frac{12\sqrt{3}\,\Gamma^{5/2}}{\pi^2} \int_0^\infty \mathrm{d}x [nc(x)]^2 x^4 \int_0^1 \mathrm{d}z\, z^2 \int_0^\infty \mathrm{d}\nu h_\mathrm{R}(x,z,\nu) \,, \tag{2.16}$$

with the abbreviations:

$$h_\mathrm{R}(x,z,\nu) = \frac{[\mathrm{Im}\, I(x,z,\nu)]^2}{|1-nc(x)[1-\nu I(x,z,\nu)]|^2} \,, \tag{2.17}$$

$$I(x,z,\nu) = -\mathrm{i} \int_0^\infty \mathrm{d}t \exp\left[\mathrm{i}\nu t - \tfrac{1}{2}x^2z^2t^2 - \frac{x^2(1-z^2)}{b^2}(1-\cos bt)\right] . \tag{2.18}$$

Here ν is understood to have an infinitesimally small positive imaginary part. Futhermore $b = \omega_B/\omega_\mathrm{p}$ is the dimensionless magnetic field strength; we used the definitions $a = (3/4\pi n)^{1/3}$ for the ion radius and $\Gamma = \frac{1}{3}a^2k_\mathrm{D}^2$ for the plasma coupling constant.

An alternative approximation within the framework of the renormalized kinetic theory for plasmas leads to a modified Rostoker memory kernel[1]) that is obtained from (2.5) by replacing the direct correlation function by $-\beta v(q)$, with $v(q) = e^2/q^2$ the bare Coulomb interaction. The ensuing expression for the self-diffusion coefficient is furnished by the replacement in (2.16) of $[nc(x)^2]$ by $-nc(x)/x^2$.

In the static screening approximation the denominator in (2.17) is replaced by $[1 - nc(x)]^2$. Upon insertion into (2.16) one can perform the ν-integration, so that the expression for the self-diffusion coefficient becomes:

$$D_{\parallel,\mathrm{L}}^{-1} = \frac{1}{\omega_\mathrm{p} a^2} \frac{2\Gamma^{5/2} 3^{3/2}}{\pi} \int_0^\infty \mathrm{d}x [nh(x)]^2 x^4 \int_0^1 \mathrm{d}z\, z^2 \times \int_0^\infty \mathrm{d}t \exp\left[-x^2 z^2 t^2 - \frac{2x^2(1-z^2)}{b^2}(1 - \cos bt)\right]. \tag{2.19}$$

In a previous paper[1]) we derived this expression from the generalized Landau memory kernel for a dense magnetized plasma. If we start from the modified Rostoker kernel we arrive at an expression that is obtained from (2.19) by replacing $[nh(x)]^2$ by $-nh(x)[1 + nh(x)]/x^2$.

In the case of weak coupling, i.e. $\Gamma \ll 1$, we may use the Debye–Hückel approximation for the direct correlation function, $nc(x) \to -1/x^2$, in the right-hand side of (2.16). In this limit the x-integral diverges logarithmically; to make the integral convergent we shall impose a cut-off in (2.15) at wave vectors larger than the inverse Landau length. Correspondingly the upper bound for the x-integration in (2.16) becomes $X = 1/(\sqrt{3}\Gamma^{3/2})$. Then we can write the self-diffusion coefficient in a form that is similar to the expression we obtained for a weakly-coupled plasma[6]):

$$D_{\parallel,\mathrm{R}}^{-1} = \frac{1}{\omega_\mathrm{p} a^2} \Gamma^{5/2} \sqrt{\frac{3}{\pi}} [\tfrac{1}{2}\log(1 + X^2) + J_\mathrm{R}(b, \Gamma)]. \tag{2.20}$$

The difference between the Rostoker kernel and its modified version disappears in the Debye-Hückel limit.

The self-diffusion coefficient gets a simpler form in the extreme cases of zero and infinite magnetic field strength. If $b = 0$ the integral I becomes independent of z:

$$\lim_{b\to 0} I(x, z, \nu) = I^{(0)}(x, \nu) = -\mathrm{i} \int_0^\infty \mathrm{d}t\, \mathrm{e}^{\mathrm{i}\nu t - x^2 t^2/2} = \frac{1}{\nu}\left[1 - W\left(\frac{\nu}{x}\right)\right], \tag{2.21}$$

where W is the plasma dispersion function[7]) given by

$$W(\eta\sqrt{2}) = F_1(\eta) + \mathrm{i}F_2(\eta)\,, \tag{2.22}$$

$$F_1(\eta) = 1 - 2\eta\,\mathrm{e}^{-\eta^2}\psi(\eta)\,, \quad F_2(\eta) = \sqrt{\pi}\,\eta\,\mathrm{e}^{-\eta^2}\,, \tag{2.23}$$

with $\psi(\eta) = \int_0^\eta \mathrm{d}t\,\exp(t^2)$. The z-integration in (2.16) can be performed now; one obtains the Balescu–Guernsey–Lenard expression for the self-diffusion coefficient in a dense unmagnetized plasma[3,5]). In particular, we find upon employing a new integration variable $\eta = \nu/x\sqrt{2}$:

$$D^{-1}_{\parallel,\mathrm{R}}(b=0,\Gamma) = D^{-1}_{\parallel,\mathrm{BGL}}(\Gamma) = \frac{1}{\omega_\mathrm{p}a^2}\,\Gamma^{5/2}\sqrt{\frac{3}{\pi}}\,\Xi^{(0)}(\Gamma)\,, \tag{2.24}$$

with

$$\Xi^{(0)}(\Gamma) = 2\sqrt{\frac{2}{\pi}}\int_0^\infty \mathrm{d}x[nc(x)]^2x^3\int_0^\infty \mathrm{d}\eta\,\frac{\mathrm{e}^{-2\eta^2}}{[1-nc(x)F_1(\eta)]^2 + [nc(x)F_2(\eta)]^2}\,. \tag{2.25}$$

In the case $b\to\infty$ the integral I again reduces to a simple form:

$$\lim_{b\to\infty} I(x,z,\nu) = I^{(\infty)}(x,z,\nu) = -\mathrm{i}\int_0^\infty \mathrm{d}t\,\mathrm{e}^{\mathrm{i}\nu t - x^2z^2t^2/2}$$

$$= \frac{1}{\nu}\left[1 - W\!\left(\frac{\nu}{xz}\right)\right]. \tag{2.26}$$

In this case we use the integration variable $\eta = \nu/xz\sqrt{2}$ in (2.16), so that the z-integration can again be carried out:

$$D^{-1}_{\parallel,\mathrm{R}}(b\to\infty,\Gamma) = \frac{1}{\omega_\mathrm{p}a^2}\,\Gamma^{5/2}\sqrt{\frac{3}{\pi}}\,\frac{3}{2}\,\Xi^{(0)}(\Gamma)\,. \tag{2.27}$$

As a consequence we have the same proportionality relation,

$$\frac{D_{\parallel,\mathrm{R}}(b\to\infty,\Gamma)}{D_{\parallel,\mathrm{R}}(b=0,\Gamma)} = \frac{2}{3}\,, \tag{2.28}$$

which we have established earlier for the self-diffusion coefficient in the Landau approximation. Clearly also the modified Rostoker kernel leads to (2.28).

3. Expansion of the self-diffusion coefficient for weak magnetic field

In this section we consider the case $b \ll 1$ in some detail. In the Appendix we shall show that the contribution from $|xz| \in [0, b]$ to the integral expression (2.16) is $\mathcal{O}(b^3)$ and hence negligible as $b \to 0$. For $|xz| \geq b$ the expression (2.18) for I contains an effective Gaussian cut-off so that we may employ the expansion:

$$\exp\left[-\frac{x^2(1-z^2)}{b^2}(1-\cos bt)\right]$$

$$= \exp[-\tfrac{1}{2}t^2x^2(1-z^2)]\left[1+\frac{1}{24}b^2t^4x^2(1-z^2)+\cdots\right]. \tag{3.1}$$

Hence we have:

$$I(x, z, \nu) \simeq I^{(0)}(x, \nu) + b^2 I^{(2)}(x, z, \nu), \tag{3.2}$$

with $I^{(0)}$ the contribution (2.21) that is independent of the magnetic field and

$$I^{(2)}(x, z, \nu) = -\frac{1-z^2}{24\nu x^2}\left\{\left[\left(\frac{\nu}{x}\right)^4 - 6\left(\frac{\nu}{x}\right)^2 + 3\right]W\left(\frac{\nu}{x}\right) + \left(\frac{\nu}{x}\right)^2 - 3\right\}. \tag{3.3}$$

To obtain an expansion for the self-diffusion coefficient we insert (3.2) into (2.16) with (2.17). Then we get up to second order in b:

$$D^{-1}_{\parallel,\mathrm{R}}(b, \Gamma) = \frac{1}{\omega_{\mathrm{p}}a^2}\Gamma^{5/2}\sqrt{\frac{3}{\pi}}\,[\Xi^{(0)}(\Gamma) + b^2\Xi^{(2)}(\Gamma)]. \tag{3.4}$$

The term that is independent of the magnetic field is, of course, the Balescu–Guernsey–Lenard expression given in (2.24) with (2.25). The correction for small magnetic field is determined by:

$$\Xi^{(2)}(\Gamma) = \frac{1}{5}\sqrt{\frac{2}{\pi}}\int_0^\infty \mathrm{d}x[nc(x)]^2x\int_0^\infty \mathrm{d}\eta\,\frac{\mathrm{e}^{-2\eta^2}}{\{[1-nc(x)F_1(\eta)]^2 + [nc(x)F_2(\eta)]^2\}^2}$$

$$\times[1-nc(x)F_1(\eta)][(\tfrac{4}{3}\eta^4 - 4\eta^2 + 1) + nc(x)(\tfrac{2}{3}\eta^2 - 1)]. \tag{3.5}$$

The corresponding expansion for the modified Rostoker expression of the self-diffusion coefficient is obtained by replacing $[nc(x)]^2$ by $-nc(x)/x^2$ in (2.25) and (3.5). For vanishing magnetic field one recovers the expression for the self-diffusion coefficient as determined from the Wallenborn–Baus memory kernel[4,5]).

The integrals $\Xi^{(0)}$ and $\Xi^{(2)}$ have been evaluated numerically. To that end we used the values for the direct correlation function $c(x)$ that are found by solving numerically the hypernetted-chain (HNC) equation. In this way a fair approximation for the static correlation functions of a dense plasma is obtained. The results for $\Xi^{(0)}$ and $\Xi^{(2)}$ are presented in table I, for both versions of the Rostoker kernel. From the table one may deduce that a weak magnetic field (with $b \leqslant 0.1$) gives rise to only a minor change in the self-diffusion coefficient. At a fixed value of b the influence of the magnetic field increases as the plasma becomes more strongly coupled.

In the case of weak coupling we can use the Debye–Hückel approximation for the direct correlation function. Then the x-integration can be performed. As a result we obtain an expression of the form (2.20) for the self-diffusion coefficient, with:

$$J_R(b,\Gamma) = J_R^{(0)}(\Gamma) + b^2 J_R^{(2)}(\Gamma)\,. \tag{3.6}$$

For the zeroth-order term one finds:

$$J_R^{(0)}(\Gamma) = \sqrt{\frac{2}{\pi}}\int_0^\infty \mathrm{d}\eta\; \mathrm{e}^{-2\eta^2}\left\{\tfrac{1}{2}\log\left[\frac{(F_1+X^2)^2+F_2^2}{(1+X^2)^2(F_1^2+F_2^2)}\right] - \frac{F_1}{F_2}\left[\arctan\left(\frac{F_1+X^2}{F_2}\right) - \arctan\left(\frac{F_1}{F_2}\right)\right]\right\}. \tag{3.7}$$

TABLE I

The coefficient $\Xi^{(0)}(\Gamma)$ and $\Xi^{(2)}(\Gamma)$ in the expansion (3.4) of the inverse self-diffusion coefficient, as obtained from the Rostoker kernel (R) and the modified Rostoker kernel (R′).

Γ		0.1	0.2	0.5	1	2	5	10
$\Xi^{(0)}$	R	2.05	1.32	0.629	0.318	0.145	0.0457	0.0180
	R′	2.66	1.83	0.979	0.540	0.266	0.0848	0.0281
$\Xi^{(2)}$	R	0.0225	0.0221	0.0202	0.0178	0.0149	0.0105	0.0074
	R′	0.0229	0.0229	0.0219	0.0205	0.0184	0.0148	0.0116

TABLE II
The coefficients $J_R^{(0)}(\Gamma)$ and $J_R^{(2)}(\Gamma)$ in the expansion (3.6) of the contribution $J_R(b, \Gamma)$ to the inverse self-diffusion coefficient (2.20) in the Debye–Hückel approximation.

Γ	0.05	0.1	0.2
$J_R^{(0)}$	−0.301	−0.300	−0.297
$J_R^{(2)}$	0.0236	0.0235	0.0233

The quadratic correction for small magnetic field is found to be:

$$J_R^{(2)}(\Gamma) = \frac{1}{10\sqrt{2\pi}} \int_0^\infty d\eta\, e^{-2\eta^2} \left\{ \frac{1-\frac{2}{3}\eta^2}{F_2} \left[\arctan\left(\frac{F_1+X^2}{F_2}\right) - \arctan\left(\frac{F_1}{F_2}\right) \right] + \frac{4}{3} \frac{\eta^2 X^2}{(F_1+X^2)^2+F_2^2} \right\} . \tag{3.8}$$

The integrations in (3.7) and (3.8) have been performed numerically for various values of the coupling constant Γ. The results for $J_R^{(0)}$ and $J_R^{(2)}$ are presented in table II.

4. Numerical results

To be able to apply a numerical integration method to the expression (2.16) for the self-diffusion coefficient we first have to evaluate the function I, given by (2.18), for arbitrary values of its arguments. The dependence of I on the magnetic field is seen to be simple upon introducing the variables $y = x/b$ and $\omega = \nu/b$, namely:

$$I(y, z, \omega) = \frac{-i}{b} \int_0^\infty dt \exp[i\omega t - \tfrac{1}{2}y^2 z^2 t^2 - y^2(1-z^2)(1-\cos t)] . \tag{4.1}$$

We may express I in the plasma dispersion function W by using the identity

$$e^{-a(1-\cos t)} = \sum_{p=-\infty}^{\infty} \Lambda_p(a) e^{ipt} , \tag{4.2}$$

where $\Lambda_p(a) = \exp(-a) I_p(a)$ contains the modified Bessel function $I_p(a)$. One gets in this way:

$$I(y, z, \omega) = \frac{1}{b} \sum_{p=-\infty}^{\infty} \frac{1}{\omega - p} \left[1 - W\left(\frac{\omega - p}{yz}\right)\right] \Lambda_p[y^2(1-z^2)] . \tag{4.3}$$

This series converges slowly when $y^2(1-z^2) \gg 1$; in that case it becomes difficult to approximate the sum with sufficient accuracy. However, for $y^2(1-z^2) \gg 1$ the integrand in (4.1) is sharply peaked around $t = 2\pi n$, with n a non-negative integer. Hence we write:

$$I = \sum_{n=0}^{\infty} I_n , \tag{4.4a}$$

where

$$I_n = \frac{-\mathrm{i}}{b} \int_{a_n}^{(2n+1)\pi} \mathrm{d}t \exp[\mathrm{i}\omega t - \tfrac{1}{2} y^2 z^2 t^2 - y^2(1-z^2)(1-\cos t)] , \tag{4.4b}$$

with $a_0 = 0$ and $a_n = (2n-1)\pi$ for $n \geq 1$. In each term I_n we approximate the integrand by a Gaussian with a maximum at $t = 2n\pi$; explicitly we have

$$\begin{aligned} I_0 &\simeq \frac{-\mathrm{i}}{b} \int_0^{\infty} \mathrm{d}t \exp(\mathrm{i}\omega t - \tfrac{1}{2} y^2 t^2) \left[1 + \frac{1}{24} y^2 (1-z^2) t^4\right] \\ &= \frac{1}{b\omega} \left[1 - W\left(\frac{\omega}{y}\right)\right] + \frac{1-z^2}{24 b\omega y^2} \\ &\quad \times \left\{\left[\left(\frac{\omega}{y}\right)^4 - 6\left(\frac{\omega}{y}\right)^2 + 3\right]\left[1 - W\left(\frac{\omega}{y}\right)\right] - \left(\frac{\omega}{y}\right)^4 + 5\left(\frac{\omega}{y}\right)^2\right\} \end{aligned} \tag{4.5}$$

(cf. (3.2) with (2.21) and (3.3)) and:

$$\begin{aligned} I_n &\simeq \frac{-\mathrm{i}}{b} \int_{-\infty}^{\infty} \mathrm{d}t \exp[\mathrm{i}\omega t - \tfrac{1}{2} y^2 z^2 t^2 - \tfrac{1}{2} y^2 (1-z^2)(t - 2n\pi)^2] \\ &\quad \times \left[1 + \frac{1}{24} y^2 (1-z^2)(t-2n\pi)^4\right] \\ &= \frac{-\mathrm{i}(2\pi)^{1/2}}{by} \exp\left\{\frac{-\omega^2}{2y^2 z^2}[1 - (1-z^2)(1+\mathrm{i}\alpha_n)^2]\right\} \\ &\quad \times \left\{1 + \frac{1-z^2}{24 y^2}\left[\left(\frac{\omega}{y}\right)^4 (1+\mathrm{i}\alpha_n)^4 - 6\left(\frac{\omega}{y}\right)^2 (1+\mathrm{i}\alpha_n)^2 + 3\right]\right\} , \end{aligned} \tag{4.6}$$

with $\alpha_n = 2n\pi y^2z^2/\omega$. If we have $y^2z^2 \gg 1$ as well as $y^2(1-z^2) \gg 1$, the t-integration in (4.1) is effectively cut off at $t \simeq (yz)^{-1} \ll 1$; consequently we have to take into account only the term with $n = 0$ in (4.4).

On the other hand, if $y^2z^2 \ll 1$ the number of significant terms in (4.4) becomes very large. Owing to the presence of oscillating factors in (4.6) we are liable to lose accuracy. In this situation we may employ an alternative representation which has been suggested earlier[8]). The periodicity of the cosine in (4.1) can be used to write:

$$I = \frac{-\mathrm{i}}{b} \int_0^{2\pi} \mathrm{d}t \exp[\mathrm{i}\omega t - \tfrac{1}{2}y^2z^2t^2 - y^2(1-z^2)(1-\cos t)]$$
$$\times \sum_{n=0}^{\infty} \exp[2n\pi\mathrm{i}(\omega + \mathrm{i}y^2z^2t) - 2n^2\pi^2y^2z^2]\,. \tag{4.7}$$

Next we use the fact that the Fourier transform of a Gaussian is again a Gaussian, viz.:

$$\exp(-2n^2\pi^2y^2z^2) = \pi^{-1/2} \int_{-\infty}^{\infty} \mathrm{d}s \exp(-s^2 - 2^{3/2}\pi\mathrm{i}nyzs)\,. \tag{4.8}$$

As a consequence the summation in (4.7) gives rise to a geometric series, so that we obtain:

$$I = -\frac{\mathrm{i}}{b\pi^{1/2}} \int_0^{2\pi} \mathrm{d}t \exp[\mathrm{i}\omega t - \tfrac{1}{2}y^2z^2t^2 - y^2(1-z^2)(1-\cos t)]$$
$$\times \int_{-\infty}^{\infty} \mathrm{d}s \exp(-s^2)\{1 - \exp[2\pi\mathrm{i}(\omega + \mathrm{i}y^2z^2t - 2^{1/2}yzs)]\}^{-1}\,. \tag{4.9}$$

As a function of s the integrand possesses simple poles at:

$$\sigma_n(t) = \frac{1}{2^{1/2}yz}(\omega - n + \mathrm{i}y^2z^2t)\,, \tag{4.10}$$

for integer n. We can apply the Mittag–Leffler theorem to write the factor in brackets in (4.9) as a partial fractions expansion:

$$\{1-\exp[2\pi i(\omega+iy^2z^2t-2^{1/2}yzs)]\}^{-1}$$
$$=\frac{1}{2^{3/2}\pi i yz}\sum_{n=-\infty}^{\infty}\frac{1}{s-\sigma_n(t)}+G(s)\,, \tag{4.11}$$

where $G(s)$ is an entire bounded function. From the first Liouville theorem we conclude that G must be a constant, so that $G(s)=G(0)$. Inserting (4.11) into (4.9) we can now perform the s-integration. Thus we obtain the following expression for I:

$$I=\frac{-i}{b}\int_0^{2\pi}dt\,\exp[i\omega t-\tfrac{1}{2}y^2z^2t^2-y^2(1-z^2)(1-\cos t)]$$
$$\times\left[\{1-\exp[2\pi i(\omega+iy^2z^2t)]\}^{-1}+\frac{1}{2^{3/2}\pi i yz}\sum_{n=-\infty}^{\infty}\frac{W[2^{1/2}\sigma_n(t)]}{\sigma_n(t)}\right], \tag{4.12}$$

which is equivalent to the representation given earlier[8]).

If $yz \ll 1$ we have $|\sigma_n(t)| \gg 1$ for all $n \neq n_\omega$ with n_ω the integer that is closest to ω. Therefore we may use the asymptotic expansion[7]) for W in all but one term of the sum. Subsequently the summation can be performed easily; indeed one has

$$\sum_{n=-\infty}^{\infty}\frac{1}{[\sigma_n(t)]^3}=2^{3/2}(\pi yz)^3\frac{\cos[\pi(\omega+iy^2z^2t)]}{\sin^3[\pi(\omega+iy^2z^2t)]}\,. \tag{4.13}$$

An approximate expression for I, which is valid if both yz and $|\omega-n_\omega|$ are small, is obtained now by expanding the last factor in the integrand of (4.12) in powers of yz and $\delta\omega=\omega-n_\omega$. The leading term turns out to be independent of t, viz. $i[1-W(\delta\omega/yz)]/(2\pi\delta\omega)$. The first few higher-order terms can be calculated likewise in a straightforward manner.

For $y^2(1-z^2)\gg 1$ the integral in (4.12) is dominated by the contributions from t in the vicinities of 0 and of 2π. We may use an expansion analogous to (4.5) and (4.6) to derive an asymptotic expression for I of the form:

$$I\simeq\frac{\exp(-\omega^2/2y^2)}{(2\pi)^{1/2}by\delta\omega}\left[1-W\left(\frac{\delta\omega}{yz}\right)\right]. \tag{4.14}$$

For simplicity we have written only the leading term explicitly here; however, in the actual calculations contributions have been taken into account that are of first order in $(1-z^2)/y^2$ and of second order in yz and $\delta\omega$ (as compared to (4.14)).

Although for small values of $y^2(1-z^2)$ the representation (4.3) can be employed as such, it is more convenient to make use of an alternative form for I that is obtained by inserting the expression

$$\exp[-y^2(1-z^2)(1-\cos t)] = 1 - y^2(1-z^2)(1-\cos t) + \dots, \tag{4.15}$$

valid for $y^2(1-z^2) \ll 1$, into (4.1); in this way one gets:

$$I \simeq \frac{1}{b\omega}\left[1 - W\left(\frac{\omega}{y}\right)\right] + \frac{y^2(1-z^2)}{b}\left\{\frac{1}{2(\omega+1)}\left[1 - W\left(\frac{\omega+1}{y}\right)\right] + \frac{1}{2(\omega-1)}\left[1 - W\left(\frac{\omega-1}{y}\right)\right] - \frac{1}{\omega}\left[1 - W\left(\frac{\omega}{y}\right)\right]\right\}. \tag{4.16}$$

In evaluating the expression (2.16) for the inverse self-diffusion coefficient we have used the various representations for I, viz. (4.3), (4.4)–(4.6), (4.14) and (4.16), in appropriate regions of the $[y^2z^2, y^2(1-z^2)]$-plane as illustrated in fig. 1.

The direct correlation function $c(x)$ in (2.16) is obtained by solving numerically the HNC equation, as in the preceding section. For $\Gamma \leq 0.2$ we have also used the Debye–Hückel expression $nc(x) = -x^{-2}$.

Upon introducing the integration variables y and ω instead of x and ν in the integral (2.16), its integrand is seen to have a structure in the variable ω on a scale of order 1, as is apparent from (4.3). Therefore it is appropriate to add

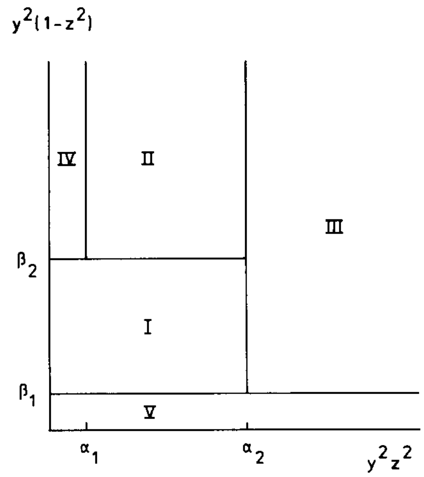

Fig. 1. The domains for the various representations of the function I. We used eq. (4.3) in I, eqs. (4.4–4.6) in II, eq. (4.5) in III, eq. (4.14) in IV and eq. (4.16) in V. To obtain I with a relative accuracy of 10^{-4} we employed the values $\alpha_1 = 5 \times 10^{-4}$, $\alpha_2 = 10$, $\beta_1 = 0.05$ and $\beta_2 = 50$.

within the integrand the contributions from all points $\omega = \bar{\omega} + n$, with $\bar{\omega} \in [0, 1)$ and n a non-negative integer. For small yz the integrand has sharp peaks with a width of order yz at $\bar{\omega} = 0$ and $\bar{\omega} = 1$. If in addition the variable $\bar{\omega}$ is rescaled to $\bar{\omega}/yz$ in this case, these peaks are properly taken into account by the numerical integration procedure.

As remarked in section 2 the expression (2.16) reduces to the Landau version (2.19) in the static screening approximation. As we have calculated (2.19) quite accurately before[1]) we shall concentrate here on the evaluation of the difference $D^{-1}_{\parallel,R} - D^{-1}_{\parallel,L}$ rather than of $D^{-1}_{\parallel,R}$. In this way a considerable improvement in the numerical accuracy is obtained.

The final form of the integral in (2.16) that has been employed in the numerical integration procedure is:

$$\int_0^Y \mathrm{d}y [nc(by)]^2 y^4 \int_0^1 \mathrm{d}z\, z^2 \int_0^1 \mathrm{d}\bar{\omega} \sum_{n=0}^{N} h(y, z, n + \bar{\omega}) \,, \tag{4.17}$$

with

$$h(y, z, \omega) = [\operatorname{Im} I(y, z, \omega)]^2 \left\{ \frac{1}{|1 - nc(by)[1 - \omega I(y, z, \omega)]|^2} - \frac{1}{[1 - nc(by)]^2} \right\} . \tag{4.18}$$

The upper bounds Y and N have been chosen such that the omitted contributions are less than 0.1%.

The numerical calculation of (4.17) has been performed with an adaptive Monte Carlo integration method[9]). For each pair of values (Γ, b) we used a

TABLE III

The reduced self-diffusion coefficient $R_{\parallel}(b, \Gamma)$, as found from the Rostoker memory kernel in the 1-Sonine approximation.

b \ Γ	0.1	0.2	0.5	1	2	5	10
0.1	1.00	1.00	1.00	0.999	0.999	0.998	0.997
0.2	1.00	1.00	0.998	0.998	0.994	0.990	0.982
0.5	0.997	0.996	0.991	0.984	0.973	0.937	0.894
1	0.986	0.981	0.963	0.942	0.903	0.830	0.780
2	0.956	0.937	0.892	0.842	0.784	0.729	0.704
5	0.865	0.828	0.756	0.714	0.690	0.678	0.674
10	0.790	0.745	0.696	0.679	0.673	0.669	0.668
20	0.732	0.697	0.675	0.670	0.668	0.668	0.667
50	0.688	0.673	0.668	0.668	0.668	0.668	0.667

dozen iterations; in each iterative step the integrand was evaluated at a few thousands of points in order to obtain results for the reduced self-diffusion coefficient $R_{\|}(b, \Gamma) = D_{\|}(b, \Gamma)/D_{\|}(0, \Gamma)$ with a relative accuracy better than 0.5%.

The HNC results for $R_{\|}$ are presented in tables III and IV. For a few small values of Γ we have also evaluated the contribution $J_{\mathrm{R}}(b, \Gamma)$ to the Debye–Hückel approximation of the inverse self-diffusion coefficient (2.20). In table V we give values both for $J_{\mathrm{R}}(b, \Gamma)$ and for the ensuing ratios $R_{\|}(b, \Gamma)$. As the tables show, the diffusion process is gradually impeded when the magnetic field strength increases. The dependence of the reduced self-diffusion coefficient $R_{\|}$

TABLE IV
The reduced self-diffusion coefficient $R_{\|}(b, \Gamma)$, as found from the modified Rostoker memory kernel in the 1-Sonine approximation.

b \ Γ	0.1	0.2	0.5	1	2	5	10
0.1	1.00	1.00	1.00	0.999	0.999	0.999	0.995
0.2	1.00	0.999	0.999	0.998	0.997	0.992	0.983
0.5	0.997	0.996	0.994	0.989	0.980	0.949	0.882
1	0.990	0.986	0.973	0.958	0.926	0.837	0.726
2	0.965	0.951	0.919	0.873	0.808	0.712	0.669
5	0.889	0.853	0.787	0.733	0.693	0.673	0.668
10	0.819	0.776	0.715	0.684	0.672	0.669	0.668
20	0.759	0.719	0.681	0.670	0.669	0.668	0.667
50	0.705	0.681	0.668	0.668	0.667	0.667	0.667

TABLE V
The contribution $J_{\mathrm{R}}(b, \Gamma)$ to the inverse self-diffusion coefficient in the Debye–Hückel approximation and the ensuing values for the reduced self-diffusion coefficient $R_{\|}(b, \Gamma)$.

b \ Γ	0.05		0.1		0.2	
	J_{R}	$R_{\|}$	J_{R}	$R_{\|}$	J_{R}	$R_{\|}$
0.1	−0.301	1.00	−0.300	1.00	−0.297	1.00
0.2	−0.300	1.00	−0.299	1.00	−0.296	0.999
0.5	−0.294	0.998	−0.293	0.997	−0.290	0.996
1	−0.272	0.992	−0.273	0.990	−0.270	0.983
2	−0.203	0.974	−0.202	0.964	−0.200	0.942
5	0.043	0.914	0.044	0.883	0.039	0.825
10	0.326	0.853	0.324	0.807	0.279	0.733
20	0.648	0.793	0.627	0.737	0.426	0.686
50	1.08	0.725	0.910	0.683	0.482	0.670

TABLE VI
Comparison of self-diffusion coefficients ($D_\parallel$, in units $\omega_p a^2$) and reduced self-diffusion coefficients ($R_\parallel$) as obtained from the Rostoker kernel (which includes dynamical screening effects) and from the Landau kernel (which contains only static screening effects).

	Rostoker	Landau
$D_\parallel(b=0, \Gamma=1)$	3.22	4.18
$D_\parallel(b=2, \Gamma=1)$	2.71	3.53
$R_\parallel(b=2, \Gamma=1)$	0.842	0.844

on b and Γ is found to be quite similar to the result that we obtained in the static screening approximation[1]). It should be emphasized, however, that the inclusion of the dynamical screening effects in the memory kernel does influence the numerical values of the diffusion coefficient $D_\parallel$ itself, as is obvious from table VI. This fact has been established before for an unmagnetized dense plasma[5]). The ratio $R_\parallel$ appears to be much less sensitive to the precise type of the screening effects that are taken into account.

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Appendix

Estimate of the contribution from $|xz| < b$ to the inverse self-diffusion coefficient for small b

Using the cylindrical coordinates $x_\parallel = xz$, $x_\perp = x(1-z^2)^{1/2}$ we write the integral in (2.16) as:

$$\int_0^\infty \mathrm{d}x_\parallel x_\parallel^2 K(x_\parallel), \tag{A.1}$$

where

$$K(x_\|) = \int_0^\infty \mathrm{d}x_\perp\, x_\perp \int_0^\infty \frac{\mathrm{d}\nu}{\nu^2} \frac{G_2^2(x_\|, x_\perp, \nu)}{G_1^2(x_\|, x_\perp, \nu) + G_2^2(x_\|, x_\perp, \nu)}\,, \tag{A.2}$$

with

$$G_1(x_\|, x_\perp, \nu) = 1 - [nc(x)]^{-1} - \nu \operatorname{Re} I(x_\|, x_\perp, \nu)\,, \tag{A.3a}$$

$$G_2(x_\|, x_\perp, \nu) = -\,\nu \operatorname{Im} I(x_\|, x_\perp, \nu)\,, \tag{A.3b}$$

$$I = -\mathrm{i} \int_0^\infty \mathrm{d}t \exp\left[\mathrm{i}\nu t - \tfrac{1}{2}x_\|^2 t^2 - \frac{x_\perp^2}{b^2}(1 - \cos bt)\right]. \tag{A.3c}$$

We shall study the function $K(x_\|)$ for $0 \leqslant x_\| \leqslant b \ll 1$.

Since the behaviour of the integrand in (A.2) is rather complicated it is useful to decompose the $(x_\perp, \nu)$-integration domain such that alternative estimates for the integrand can be employed. Let us first consider $x_\perp \in [0, \sqrt{b}]$ and $v \in [0, \frac{1}{4}x_\|]$. In this region we have

$$\nu \operatorname{Re} I(x_\|, x_\perp, \nu) \leqslant \nu \int_0^\infty \mathrm{d}t \exp(-\tfrac{1}{2}x_\|^2 t^2) = \frac{\nu\sqrt{2\pi}}{x_\|} \leqslant \tfrac{1}{2}\sqrt{\frac{\pi}{2}}\,. \tag{A.4}$$

Since $b \ll 1$ implies $(x_\|^2 + x_\perp^2)^{1/2} \ll 1$ and hence $|nc(x)|^{-1} \ll 1$, it follows that $G_1 \geqslant c_1$ for some positive constant c_1. As a consequence a lower bound for the denominator in (A.2) is obtained. Furthermore we derive from (A.3b) in a way similar to (A.4) that $G_2 \leqslant \nu\sqrt{2\pi}/x_\|$, so that the following estimate is obtained:

$$K_{11}(x_\|) \equiv \int_0^{\sqrt{b}} \mathrm{d}x_\perp\, x_\perp \int_0^{x_\|/4} \frac{\mathrm{d}\nu}{\nu^2} \frac{G_2^2}{G_1^2 + G_2^2} \leqslant c_2 \int_0^{\sqrt{b}} \mathrm{d}x_\perp\, x_\perp \int_0^{x_\|/4} \frac{\mathrm{d}\nu}{x_\|^2} = \mathcal{O}\left(\frac{b}{x_\|}\right) \tag{A.5}$$

with some constant $c_2 > 0$. For $\nu \in [\frac{1}{4}x_\|, \infty)$ we use the coarse estimate $G_2^2/(G_1^2 + G_2^2) \leqslant 1$ which yields:

$$K_{12}(x_\|) \equiv \int_0^{\sqrt{b}} \mathrm{d}x_\perp\, x_\perp \int_{x_\|/4}^\infty \frac{\mathrm{d}\nu}{\nu^2} \frac{G_2^2}{G_1^2 + G_2^2} \leqslant \int_0^{\sqrt{b}} \mathrm{d}x_\perp\, x_\perp \int_{x_\|/4}^\infty \frac{\mathrm{d}\nu}{\nu^2} = \mathcal{O}\left(\frac{b}{x_\|}\right). \tag{A.6}$$

Next we study the case $x_\perp \in [\sqrt{b}, \infty)$. In this region we have $x_\perp/b \gg 1$ as $b \ll 1$, so that I can be approximated quite accurately by a sum of Gaussians (cf. (4.4)–(4.6)). Explicitly we get:

$$G_1(x_\parallel, x_\perp, \nu) \simeq -[nc(x)]^{-1} + \mathrm{Re}\, W\left(\frac{\nu}{x}\right) - \sqrt{2\pi}\,\frac{\nu}{x}\,\mathrm{Im}\,\Phi(x_\parallel, x_\perp, \nu)\,, \tag{A.7a}$$

$$G_2(x_\parallel, x_\perp, \nu) \simeq \sqrt{\frac{\pi}{2}}\,\frac{\nu}{x}\exp\left(-\frac{\nu^2}{2x^2}\right) + \sqrt{2\pi}\,\frac{\nu}{x}\,\mathrm{Re}\,\Phi(x_\parallel, x_\perp, \nu)\,, \tag{A.7b}$$

with

$$\Phi(x_\parallel, x_\perp, \nu) = \sum_{n=1}^{\infty} \exp\left\{-\frac{\nu^2}{2x_\parallel^2}\left[1 - \frac{x_\parallel^2}{x^2}(1 + \mathrm{i}\alpha_n)^2\right]\right\} \tag{A.7c}$$

and $\alpha_n = 2\pi n x_\parallel^2/(\nu b)$. Replacing the summation by an integration renders an upper bound for the real and imaginary parts of the function Φ:

$$\left.\begin{array}{l}|\mathrm{Re}\,\Phi| \\ |\mathrm{Im}\,\Phi|\end{array}\right\} \leq \sum_{n=1}^{\infty} \exp\left(-\frac{\nu^2}{2x^2} - 2\pi^2 n^2 \frac{x_\parallel^2 x_\perp^2}{b^2 x^2}\right) \leq \frac{b}{2x_\parallel}\,. \tag{A.8}$$

We now split the ν-integration domain into $[0, \nu_0]$ and $[\nu_0, \infty)$ with ν_0 depending on the value of $nc(x)$, viz.:

$$\nu_0 = \frac{x_\parallel x}{2b\sqrt{2\pi}}\,\bar{\nu}_0(x)\,, \tag{A.9}$$

with

$$\bar{\nu}_0(x) = \begin{cases} 1 & \text{if } nc(x) \leq -1\,, \\ -[nc(x)]^{-1} & \text{if } -1 \leq nc(x) < 0\,, \\ \infty & \text{if } nc(x) = 0\,, \\ [nc(x)]^{-1} - 1 & \text{if } 0 < nc(x) < 1\,. \end{cases} \tag{A.10}$$

Then for $\nu \in [0, \nu_0]$ we find

$$\frac{G_2^2}{G_1^2 + G_2^2} \leq c_3\,\frac{\nu^2}{\nu_0^2}\,, \tag{A.11}$$

for a positive constant c_3, so that

$$K_{21}(x_\parallel) \equiv \int_{\sqrt{b}}^{\infty} \mathrm{d}x_\perp\, x_\perp \int_0^{\nu_0} \frac{\mathrm{d}\nu}{\nu^2}\,\frac{G_2^2}{G_1^2 + G_2^2} = \mathcal{O}\left(\frac{b}{x_\parallel}\right). \tag{A.12}$$

For $\nu > \nu_0$ we use again $G_2^2/(G_1^2 + G_2^2) \leq 1$, which yields for the corresponding contribution K_{22} to K the estimate $K_{22}(x_\parallel) = \mathcal{O}(b/x_\parallel)$ as well. Now we conclude from (A.5), (A.6), and (A.12) that:

$$K(x_\parallel) = \mathcal{O}\left(\frac{b}{x_\parallel}\right) \tag{A.13}$$

and hence, with (A.1) and (A.2), that the contribution from $x_\parallel \in [0, b]$ to (A.1) is $\mathcal{O}(b^3)$.

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