

Foundations of Electrodynamics

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Preface

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Electrodynamics may be said to consist of two parts, at different levels: microscopic and macroscopic theory. The first contains the laws that govern the interaction of fields and point particles – often grouped into stable sets such as atoms and molecules – and the second those that describe the interaction of fields and continuous media. The two theories are linked together, since the phenomena at the macroscopic level may be looked upon as being the result of the interplay of many particles. Therefore one should be able to obtain the electromagnetic laws for continuous media from those for point particles. Such a derivation, together with a discussion of the microscopic starting points, forms the subject of this monograph.

The programme will be carried out in the framework of both classical and quantum theory. The classical theory is given in the non-relativistic approximation and then in covariant formulation. In the latter various topics will receive special attention: among these figure the covariant description of composite particles, the obtention of statistical averages in a relativistically invariant way and a discussion of the energy-momentum tensor for continuous media. The quantum-mechanical theory will be formulated in such a fashion that the analogy with classical theory can be exploited as far as possible. This is achieved by representing the physical quantities by ordinary functions rather than by operators. Again the non-relativistic approximation will be studied first. Subsequently magnetic effects are discussed in a 'semi-relativistic' theory, which goes one step beyond the non-relativistic treatment. The completely covariant extension of quantum theory will be confined to the discussion of the motion of single particles with and without spin in slowly varying external fields. The covariant generalization to statistical assemblies of particles moving in each other's fields would require quantization of the electromagnetic field together with its sources: this forms the subject of quantum electrodynamics not dealt with here.

Part is meant especially to serve as textbook material for graduate students who take courses in electromagnetic theory. By reading the first two chapters they will get acquainted with the way in which the macroscopic laws of electrodynamics are obtained from a microscopic basis, albeit in the framework of classical, non-relativistic theory. In the relativistic part the third chapter may be useful as an exposé of the covariant equations for fields and particles with the inclusion of the effects of radiation damping, while the final results of the fourth and fifth chapters give an idea of the way in which the non-relativistic laws may be generalized. Similarly the results of chapters VI and VII show the consequences of the use of quantum mechanics. The special formulation of quantum mechanics in terms of Weyl transforms and Wigner functions can be studied independently from the appendix of chapter VI.

More advanced students will be interested in the covariant formulation of the equations of motion for composite particles in chapter IV, relativistic statistics as discussed in chapter V, the covariant quantum-mechanical equations of motion for particles with spin 0 and $\frac{1}{2}$ in chapter VIII, and in the semi-relativistic treatment of magnetic effects given in chapters IX and X.

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L.G.S.

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