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# INTERFERENCE EFFECTS IN THE LONG-TIME TAIL OF THE VELOCITY AUTOCORRELATION FUNCTION FOR A DENSE ONE-COMPONENT PLASMA IN A MAGNETIC FIELD

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The long-time behaviour of the velocity autocorrelation function that describes the motion of a tagged particle through a one-component plasma in a uniform magnetic field has been determined with the use of mode-coupling theory. The long-time tail depends on the orientation of the velocity with respect to the magnetic field. Owing to the anisotropy of the collective mode spectrum the mode-coupling integrals are afflicted with interference effects. As a consequence the long-time behaviour of the velocity autocorrelation function for a plasma in a magnetic field differs qualitatively from that found for an unmagnetized plasma.

## 1. Introduction

The velocity autocorrelation function of a tagged particle which moves through a fluid of neutral particles is, for large time $t$, proportional to $t^{-d/2}$, with $d$ the dimension of the system. A theoretical explanation of this well-known fact, which has first been observed in molecular dynamics calculations[1]), may be given by means of the theory of mode coupling[2,3]). In the framework of this theory the long-time tail of the velocity autocorrelation function is understood as a consequence of a persistent coupling of the tagged-particle density to the large scale viscous modes that are sustained by the fluid.

For a plasma the tail of the velocity autocorrelation function has a more complicated structure; it is the sum of a term proportional to $t^{-3/2}$ (for $d = 3$) and a term proportional to $t^{-3/2}\cos(\omega_p t + \theta)$, with $\omega_p$ the plasma frequency[4–8]). From the point of view of mode-coupling theory[8]) these terms are found to be the result of the coupling of the tagged-particle density to the viscous modes and to the plasmon modes, respectively.

In the presence of a magnetic field the velocity autocorrelation function depends on the direction of the velocity owing to the anisotropy of the system. As a consequence, a longitudinal and a transverse velocity autocorrelation function may be distinguished. The asymptotic form of these functions for large $t$ is expected to differ qualitatively from that found for an unmagnetized plasma. In fact, the collective modes, which dominate the asymptotic behaviour, are modified substantially if a magnetic field is turned on. In particular, the viscous modes and the plasmon modes merge in a set of four mixed 'gyro-plasmon' modes[9,10]). These modes, and the corresponding mode frequencies, depend on the orientation of the wave vector with respect to the magnetic field. As will be shown in the following by means of a mode-coupling analysis the gyro-plasmon modes determine the long-time tails of the velocity autocorrelation functions. The dependence of the mode spectrum on the direction of the wave vector gives rise to interference effects, which will be investigated in detail. As a model we shall adopt the classical one-component plasma, consisting of charged particles which are immersed in a neutralizing inert background and which interact through a Coulomb potential. The external magnetic field is assumed to be static and uniform in space.

## 2. Mode-coupling expression for the velocity autocorrelation function

The velocity autocorrelation function of a tagged particle with position $\boldsymbol{r}_s$, momentum $\boldsymbol{p}_s$ and mass $m$ in a magnetized plasma is defined as

$$F(\hat{\boldsymbol{k}}, t) = \lim_{\boldsymbol{k}\to \boldsymbol{0}} \frac{1}{k^2} \left\langle \boldsymbol{k} \cdot \frac{\boldsymbol{g}_s^*(\boldsymbol{k})}{m} \, e^{iLt} \boldsymbol{k} \cdot \frac{\boldsymbol{g}_s(\boldsymbol{k})}{m} \right\rangle . \tag{2.1}$$

The tagged-particle momentum density in Fourier space is given by

$$\boldsymbol{g}_s(\boldsymbol{k}) = \boldsymbol{p}_s \, e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_s} . \tag{2.2}$$

Furthermore, $L$ is the Liouville operator in phase space, which determines for an arbitrary function $F$ its time derivative as $\dot{F} = iLF$. The brackets in (2.1) denote a canonical ensemble average.

The velocity autocorrelation function (2.1) depends on the orientation of the wave vector, as determined by the unit vector $\hat{\boldsymbol{k}} = \boldsymbol{k}/k$. In fact, owing to the cylinder symmetry of the system, $F(\hat{\boldsymbol{k}}, t)$ will be a function of the angle between $\hat{\boldsymbol{k}}$ and the unit vector $\hat{\boldsymbol{B}}$ that points in the direction of the magnetic field.

The long-time behaviour of the velocity autocorrelation function can be determined if one assumes it to be adequately described by the theory of mode-coupling. According to mode-coupling theory the long-time tail of the velocity autocorrelation function is dominated by contributions originating from the coupling of the tagged-particle momentum density to the product of the tagged-particle density and one of the large-scale collective modes that may be sustained by the system. In a previous paper[10]) the complete set of collective modes of a one-component plasma in a magnetic field has been derived. The results of the paper will be briefly summarized in the following.

The collective modes of a one-component magnetized plasma consist of a purely dissipative thermal mode and four oscillating gyro-plasmon modes. The thermal mode is characterized by the frequency

$$z_{\mathrm{T}}(\boldsymbol{q}) = -\mathrm{i}q^2 D_{\mathrm{T}}(\hat{\boldsymbol{q}})\,, \tag{2.3}$$

with a (real) damping coefficient $D_{\mathrm{T}}$ that depends on the unit wave vector $\hat{\boldsymbol{q}} = \boldsymbol{q}/q$ and on the longitudinal and the transverse heat conductivities[10]). The mode amplitude is a linear combination of the particle density $n(\boldsymbol{q})$ and the energy density $\varepsilon(\boldsymbol{q})$ in Fourier space:

$$a_{\mathrm{T}}(\boldsymbol{q}) = C_{\mathrm{T}}[\varepsilon(\boldsymbol{q}) - hn(\boldsymbol{q})]\,. \tag{2.4}$$

Here $C_{\mathrm{T}}$ is a normalization constant and $h$ is the enthalpy per particle.

The four gyro-plasmon modes oscillate at frequencies $z_{\lambda\rho}(\boldsymbol{q})$ ($\lambda = \pm 1$, $\rho = \pm 1$), which are given by

$$z_{\lambda,1}(\boldsymbol{q}) = w_{\lambda}(\hat{\boldsymbol{q}}) - \mathrm{i}q^2 D_{\lambda}(\hat{\boldsymbol{q}})\,, \tag{2.5}$$

$$z_{\lambda,-1}(\boldsymbol{q}) = -[z_{\lambda,1}(\boldsymbol{q})]^*\,. \tag{2.6}$$

Here the frequencies for vanishing wave number are

$$w_{\lambda}(\hat{\boldsymbol{q}}) = \tfrac{1}{2}(\omega_{\mathrm{p}}^2 + \omega_B^2 + 2\omega_{\mathrm{p}}\omega_B\hat{q}_{\parallel})^{1/2} + \tfrac{1}{2}\lambda(\omega_{\mathrm{p}}^2 + \omega_B^2 - 2\omega_{\mathrm{p}}\omega_B\hat{q}_{\parallel})^{1/2}\,, \tag{2.7}$$

with $\omega_{\mathrm{p}}$ the plasma frequency, $\omega_B$ the Larmor frequency and $\hat{q}_{\parallel} = \hat{\boldsymbol{q}}\cdot\hat{\boldsymbol{B}}$. The coefficient $D_{\lambda}$ of damping and dispersion is a rather complicated function of the seven anisotropic viscosity coefficients[10]). The amplitudes of the gyro-plasmon modes read

$$a_{\lambda\rho}(\boldsymbol{q}) = C_{\lambda}(\hat{\boldsymbol{q}})\left[\frac{k_{\mathrm{D}}}{q}\,n(\boldsymbol{q}) + \frac{1}{k_{\mathrm{B}}Tc_V}\,(\tfrac{1}{3}c_V + \tfrac{1}{2}k_{\mathrm{B}})\,\frac{q}{k_{\mathrm{D}}}\,\varepsilon(\boldsymbol{q}) + \frac{1}{(mk_BT)^{1/2}}\,\boldsymbol{v}_{\lambda\rho}(\hat{\boldsymbol{q}})\cdot\boldsymbol{g}(\boldsymbol{q})\right]. \tag{2.8}$$

It contains, on a par with $n(\boldsymbol{q})$ and $\varepsilon(\boldsymbol{q})$, the momentum density $\boldsymbol{g}(\boldsymbol{q})$ in Fourier space. The vector $\boldsymbol{v}_{\lambda\rho}(\hat{\boldsymbol{q}})$ is defined as

$$\boldsymbol{v}_{\lambda\rho}(\hat{\boldsymbol{q}}) = \frac{\rho w_\lambda(\hat{\boldsymbol{q}})\omega_{\rm p}}{w_\lambda^2(\hat{\boldsymbol{q}}) - \omega_B^2}\,\hat{\boldsymbol{q}}_\perp + \frac{\rho\omega_{\rm p}}{w_\lambda(\hat{\boldsymbol{q}})}\,\hat{\boldsymbol{q}}_\parallel - \frac{{\rm i}\omega_{\rm p}\omega_B}{w_\lambda^2(\hat{\boldsymbol{q}}) - \omega_B^2}\,\hat{\boldsymbol{q}} \wedge \hat{\boldsymbol{B}}\,, \tag{2.9}$$

with $\hat{\boldsymbol{q}}_\parallel = \hat{q}_\parallel \hat{\boldsymbol{B}}$ and $\hat{\boldsymbol{q}}_\perp = \hat{\boldsymbol{q}} - \hat{\boldsymbol{q}}_\parallel$. The coefficients in (2.8) depend in addition on the temperature $T$, the Debye wave vector $k_{\rm D}$ and the specific heat per particle $c_V$. The normalization constant $C_\lambda(\hat{\boldsymbol{q}})$ is

$$C_\lambda(\hat{\boldsymbol{q}}) = \frac{\omega_{\rm p}}{(2n)^{1/2}}\left[\frac{w_\lambda^2(\hat{\boldsymbol{q}}) - \omega_B^2\hat{q}_\parallel^2}{w_\lambda^2(\hat{\boldsymbol{q}})(\omega_{\rm p}^2 + \omega_B^2) - 2\omega_{\rm p}^2\omega_B^2\hat{q}_\parallel^2}\right]^{1/2}. \tag{2.10}$$

The mode-coupling expression for the long-time tail of the velocity autocorrelation function (2.1) reads

$$F(\hat{\boldsymbol{k}}, t) \simeq \lim_{\boldsymbol{k}\to 0} \frac{1}{k^2 V} \sum_i \sum_{\boldsymbol{q}} |A_i(\boldsymbol{k}, \boldsymbol{q})|^2\, {\rm e}^{-{\rm i}[z_i(\boldsymbol{q}) + z_{\rm s}(\boldsymbol{k}-\boldsymbol{q})]t}\,. \tag{2.11}$$

The sums are extended over the five collective modes $i = {\rm T}, \lambda\rho$ and over all values of the wave vector $\boldsymbol{q}$ of these modes; furthermore $V$ is the volume of the system. The tagged-particle density mode

$$a_{\rm s}(\boldsymbol{q}) = {\rm e}^{-{\rm i}\boldsymbol{q}\cdot\boldsymbol{r}_{\rm s}} \tag{2.12}$$

has a frequency

$$z_{\rm s}(\boldsymbol{q}) = -{\rm i}q^2 D_{\rm s}(\hat{\boldsymbol{q}}) = -{\rm i}q^2(\hat{q}_\parallel^2 D_\parallel + \hat{q}_\perp^2 D_\perp)\,, \tag{2.13}$$

with $D_\parallel$ and $D_\perp$ the longitudinal and the transverse self-diffusion coefficients. The amplitudes in (2.11) are given by

$$A_i(\boldsymbol{k}, \boldsymbol{q}) = \left\langle a_i^*(\boldsymbol{q}) a_{\rm s}^*(\boldsymbol{k}-\boldsymbol{q})\boldsymbol{k}\cdot\frac{\boldsymbol{g}_{\rm s}(\boldsymbol{k})}{m}\right\rangle. \tag{2.14}$$

As the momentum integration, which is implied in the canonical average, is isotropic, only the gyro-plasmon modes, with $i = \lambda\rho$, lead to nonvanishing amplitudes $A_{\lambda\rho}$. Inserting (2.2), (2.8) and (2.12) we obtain for these amplitudes

$$A_{\lambda\rho}(\boldsymbol{k}, \boldsymbol{q}) = C_\lambda(\hat{\boldsymbol{q}})(k_{\rm B}T/m)^{1/2}\boldsymbol{v}^*_{\lambda\rho}(\hat{\boldsymbol{q}})\cdot\boldsymbol{k}\,. \tag{2.15}$$

Substituting these into (2.11) and replacing the sum over the wave vectors $\boldsymbol{q}$ by a continuous integral we finally get

$$F(\hat{\boldsymbol{k}}, t) \simeq \frac{k_B T}{m} \sum_{\lambda\rho} \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3}\, C_\lambda^2(\hat{\boldsymbol{q}}) \hat{\boldsymbol{k}} \cdot \boldsymbol{v}_{\lambda\rho}^*(\hat{\boldsymbol{q}}) \boldsymbol{v}_{\lambda\rho}(\hat{\boldsymbol{q}}) \cdot \hat{\boldsymbol{k}}\, \mathrm{e}^{-\mathrm{i}[z_{\lambda\rho}(\boldsymbol{q}) + z_s(\boldsymbol{q})]t} . \tag{2.16}$$

In the next section this expression will be used to extract the dominant terms of the velocity autocorrelation function for large $t$.

## 3. Evaluation of the long-time tails

To evaluate the integral in (2.16) we choose a spherical coordinate system for $\boldsymbol{q}$, with a polar axis in the direction of the magnetic field. Since both the frequencies $z_{\lambda\rho}$, $z_s$ and the normalization constant $C_\lambda$ depend on $\hat{\boldsymbol{q}}$ only through $\hat{q}_\parallel$ (and $\hat{q}_\perp = (1 - \hat{q}_\parallel^2)^{1/2}$), the integration over the azimuthal angle is easily carried out after insertion of (2.9). As a result the right-hand side of (2.16) gets the form

$$\hat{k}_\parallel^2 F_\parallel(t) + \hat{k}_\perp^2 F_\perp(t) , \tag{3.1}$$

which defines the asymptotic longitudinal and transverse autocorrelation functions $F_i(t)$, with $i = \parallel, \perp$. Substituting (2.5), (2.6) and (2.13) into (2.16) we may perform the integration over $|\boldsymbol{q}|$ as well, with the result

$$F_i(t) = \frac{k_B T}{m(4\pi t)^{3/2}} \sum_{\lambda = \pm 1} \mathrm{Re} \int_{-1}^{1} \mathrm{d}\hat{q}_\parallel\, C_\lambda^2(\hat{\boldsymbol{q}}) \Phi_i(\hat{\boldsymbol{q}}) \frac{\mathrm{e}^{-\mathrm{i}w_\lambda(\hat{\boldsymbol{q}})t}}{[D_s(\hat{\boldsymbol{q}}) + D_\lambda(\hat{\boldsymbol{q}})]^{3/2}} . \tag{3.2}$$

Here we introduced the abbreviations

$$\Phi_\parallel(\hat{\boldsymbol{q}}) = \frac{\omega_p^2}{w^2}\, \hat{q}_\parallel^2 , \tag{3.3}$$

$$\Phi_\perp(\hat{\boldsymbol{q}}) = \tfrac{1}{2}\, \frac{(w^2 + \omega_B^2)\omega_p^2}{(w^2 - \omega_B^2)^2}\, \hat{q}_\perp^2 , \tag{3.4}$$

with $w = w_\lambda(\hat{\boldsymbol{q}})$.

Let us consider separately the contributions $F_i^\pm(t)$, for $\lambda = \pm 1$, to (3.2). It will be convenient to introduce instead of $\hat{q}_\parallel$ the integration variable $w =$

$w_\lambda(\hat{\boldsymbol{q}})$. Since $w_\lambda$ is a solution of the equation

$$w^4 - (\omega_p^2 + \omega_B^2)w^2 + \omega_p^2\omega_B^2\hat{q}_\parallel^2 = 0 , \tag{3.5}$$

one has, for $\hat{q}_\parallel > 0$, and $w > 0$,

$$\hat{q}_\parallel = \frac{w}{\omega_p \omega_B} (\omega_p^2 + \omega_B^2 - w^2)^{1/2} . \tag{3.6}$$

The contribution $F_i^\lambda(t)$ to (3.2) may hence be written as

$$F_i^\lambda(t) = \frac{k_B T}{mn(4\pi t)^{3/2}} \operatorname{Re} \int_{w_1^\lambda}^{w_2^\lambda} dw\, \Psi_i(w) \frac{e^{-iwt}}{(D_s + D_\lambda)^{3/2}} , \tag{3.7}$$

with

$$\Psi_\parallel(w) = \frac{|w^2 - \omega_B^2|(\omega_p^2 + \omega_B^2 - w^2)^{1/2}}{\omega_p \omega_B^3} , \tag{3.8}$$

$$\Psi_\perp(w) = \frac{|w^2 - \omega_p^2|(w^2 + \omega_B^2)}{2\omega_p \omega_B^3(\omega_p^2 + \omega_B^2 - w^2)^{1/2}} . \tag{3.9}$$

The integration boundaries in (3.7) are

$$w_1^+ = \omega_M \equiv \max(\omega_p, \omega_B) , \quad w_1^- = 0 , \tag{3.10}$$

$$w_2^+ = \omega_0 \equiv (\omega_p^2 + \omega_B^2)^{1/2} , \quad w_2^- = \omega_m \equiv \min(\omega_p, \omega_B) . \tag{3.11}$$

The integrand in (3.7) is a regular nonvanishing function for all $w$ in the open interval $(w_1, w_2)$. At the boundaries the factor $\Psi_i(w)$ may introduce a branch cut or a zero of the integrand. In fact, at the lower boundary $w_1^\pm$ the functions $\Psi_i(w)$ behave as follows:

$$\Psi_\parallel(w) \simeq \frac{w^2 - \omega_B^2}{\omega_B^2 \omega_M} , \qquad w \downarrow w_1^+ , \tag{3.12}$$

$$\Psi_\parallel(w) \simeq \frac{\omega_0}{\omega_p \omega_B} , \qquad w \downarrow w_1^- , \tag{3.13}$$

$$\Psi_\perp(w) \simeq \frac{(\omega_M^2 + \omega_B^2)(w^2 - \omega_p^2)}{2\omega_p \omega_B^3 \omega_m} , \qquad w \downarrow w_1^+ , \tag{3.14}$$

$$\Psi_\perp(w) \simeq \frac{\omega_p}{2\omega_B\omega_0}\,, \qquad w\downarrow w_1^- \,. \tag{3.15}$$

Likewise, at the upper boundary $w_2^\pm$ the functions $\Psi_i(w)$ have the form

$$\Psi_\parallel(w) \simeq \frac{\omega_p(\omega_0^2 - w^2)^{1/2}}{\omega_B^3}\,, \qquad w\uparrow w_2^+ \,, \tag{3.16}$$

$$\Psi_\parallel(w) \simeq \frac{\omega_B^2 - w^2}{\omega_B^2\omega_m}\,, \qquad w\uparrow w_2^- \,, \tag{3.17}$$

$$\Psi_\perp(w) \simeq \frac{\omega_p^2 + 2\omega_B^2}{2\omega_p\omega_B(\omega_0^2 - w^2)^{1/2}}\,, \qquad w\uparrow w_2^+ \,, \tag{3.18}$$

$$\Psi_\perp(w) \simeq \frac{(\omega_m^2 + \omega_B^2)(\omega_p^2 - w^2)}{2\omega_p\omega_B^3\omega_M}\,, \qquad w\uparrow w_2^- \,. \tag{3.19}$$

For large values of $t$ the contribution of the interior of the integration domain in (3.7) may be disregarded. As a consequence of the phase factor $\exp(-\mathrm{i}wt)$, destructive interference damps all contributions from the interior region. The main contributions to the asymptotic expression for the integral originate from the boundaries of the integration domain, since there the interference is not completely destructive. A derivation of the asymptotic form of integrals of the form (3.7) for large $t$ is given in the appendix. From (A.1), with (A.2) and (A.13)–(A.16), it follows that the asymptotic form of $F_i^+(t)$ is

$$F_i^+(t) \simeq A_{2,i}^+ t^{-\nu_{2,i}^+}\cos(\omega_0 t + \theta_{2,i}^+) - A_{1,i}^+ t^{-\nu_{1,i}^+}\cos(\omega_M t + \theta_{1,i}^+)\,. \tag{3.20}$$

The indices 1,2 indicate contributions from the boundaries at $w_1^+$ and $w_2^+$, respectively. The exponent $\nu_{2,i}^+$ equals 3 for $i = \parallel$ and 2 for $i = \perp$. The other exponent $\nu_{1,i}^+$ depends on the relative magnitude of $\omega_p$ and $\omega_B$; for $\omega_p > \omega_B$ one has $\nu_{1,\parallel}^+ = \frac{5}{2}$, $\nu_{1,\perp}^+ = \frac{7}{2}$, while for $\omega_p < \omega_B$ these values are interchanged.

The asymptotic form of $F_i^-$ is determined by the upper boundary of the integral in (3.7) only, since the contribution from the lower boundary is purely imaginary. The result is

$$F_i^-(t) \simeq A_{2,i}^- t^{-\nu_{2,i}^-}\cos(\omega_m t + \theta_{2,i}^-)\,. \tag{3.21}$$

The exponents $\nu_{2,i}^-$ are related to $\nu_{1,i}^+$ through

$$\nu_{2,i}^- = 6 - \nu_{1,i}^+\,. \tag{3.22}$$

TABLE I

Frequencies $w_k^\lambda$ and exponents $\nu_{k,i}^\lambda$ emerging from the lower ($k = 1$) and upper ($k = 2$) boundaries in the integral expression (3.7) for $F_i^\lambda(t)$, with $\lambda = \pm$, $i = \|, \perp$.

| $\lambda, k$ | $\omega_p > \omega_B$ | | | $\omega_p < \omega_B$ | | |
|---|---|---|---|---|---|---|
| | $w_k^\lambda$ | $\nu_{k,\|}^\lambda$ | $\nu_{k,\perp}^\lambda$ | $w_k^\lambda$ | $\nu_{k,\|}^\lambda$ | $\nu_{k,\perp}^\lambda$ |
| $+, 1$ | $\omega_p$ | $\frac{5}{2}$ | $\frac{7}{2}$ | $\omega_B$ | $\frac{7}{2}$ | $\frac{5}{2}$ |
| $+, 2$ | $\omega_0$ | 3 | 2 | $\omega_0$ | 3 | 2 |
| $-, 1$ | 0 | – | – | 0 | – | – |
| $-, 2$ | $\omega_B$ | $\frac{7}{2}$ | $\frac{5}{2}$ | $\omega_p$ | $\frac{5}{2}$ | $\frac{7}{2}$ |

A summary of the frequencies $w_k^\lambda$ and the exponents $\nu_{k,i}^\lambda$ is given in table I.

Comparing the exponents we conclude that the dominant terms in $F_i(t)$ have the form

$$F_\|(t) \propto t^{-5/2} \cos(\omega_p t + \theta_\|) , \tag{3.23}$$

$$F_\perp(t) \propto t^{-2} \cos(\omega_0 t + \theta_\perp) . \tag{3.24}$$

A detailed calculation, with the use of (3.12)–(3.19) and of the formulae from the appendix, leads to explicit expressions for the proportionality factors and for the phase angles. The result for the longitudinal velocity autocorrelation function is

$$F_\|(t) \simeq \frac{k_B T(\omega_B^2 - \omega_p^2)}{8\pi^{3/2} n m \omega_p \omega_B^2 t^{5/2}} \operatorname{Re}\left[\frac{\mathrm{i}e^{-\mathrm{i}\omega_p t}}{(D_\| + D_{1,\|})^{3/2}}\right] , \tag{3.25}$$

with $D_\| = D_s(\hat{q})$ and $D_{\lambda,\|} = D_\lambda(\hat{q})$ for $\hat{q} = \hat{q}_\|$. Explicitly one has

$$D_\| + D_{1,\|} = D_\| + \frac{\mathrm{i}}{2}\frac{c_s^2}{\omega_p} + \frac{1}{2nm}\left(\tfrac{4}{3}\eta_1 + \eta_V + 4\zeta\right) , \tag{3.26}$$

with $D_\|$ the (real) static longitudinal self-diffusion coefficient and $\eta_i$, $\zeta$ (complex) dynamical viscosity coefficients at the frequency $z = \omega_p$. Furthermore, $c_s$ is the velocity of sound, defined through

$$\frac{mc_s^2}{k_B T} = \frac{1}{nk_B T\kappa_T} + \frac{1}{k_B c_V}\left(\tfrac{1}{3}c_V + \tfrac{1}{2}k_B\right)^2 , \tag{3.27}$$

with $\kappa_T$ the isothermal compressibility.

The transverse velocity autocorrelation function is found as

$$F_\perp(t) \simeq \frac{k_B T(\omega_p^2 + 2\omega_B^2)}{16\sqrt{2}\pi nm\omega_p \omega_B \omega_0^{1/2} t^2} \operatorname{Re}\left[\frac{e^{i\pi/4 - i\omega_0 t}}{(D_\perp + D_{1,\perp})^{3/2}}\right]. \tag{3.28}$$

Here $D_\perp = D_s(\hat{q})$ and $D_{\lambda,\perp} = D_\lambda(\hat{q})$ for $\hat{q} = \hat{q}_\perp$; in fact one has:

$$D_\perp + D_{1,\perp} = D_\perp + \frac{i}{2}\frac{c_s^2}{\omega_0} - \frac{i\eta_4 \omega_B}{nm\omega_0} - \frac{1}{2nm}\left[\frac{2}{3}\eta_1 - 2\eta_2 - \eta_V + 2\zeta + \frac{\omega_B^2}{\omega_0^2}(\eta_1 - 2\eta_2)\right], \tag{3.29}$$

with $\eta_i$, $\zeta$ dynamical viscosities at the frequency $z = \omega_0$.

The asymptotic expression (3.25) for $F_\parallel(t)$ is clearly useless for the case of resonance, with $\omega_B = \omega_p$. In that case, the dominant term in $F_\parallel(t)$ stems from the upper boundary of the integral in (3.7) for $F_\parallel^+(t)$. It leads to the asymptotic result

$$F_\parallel(t) \simeq \frac{k_B T}{8\pi 2^{1/4} nm\omega_p^{3/2} t^3} \operatorname{Re}\left[\frac{i\, e^{i\pi/4 - i\sqrt{2}\omega_p t}}{(D_\perp + D_{1,\perp})^{3/2}}\right]. \tag{3.30}$$

The denominator is given by (3.29), with viscosities to be evaluated at the frequency $z = \omega_0 = \sqrt{2}\omega_p$.

The results (3.25), (3.28) and (3.30) show that as a consequence of the interference effects that are brought about by the anisotropy of the mode spectrum the tails of the velocity autocorrelation functions drop off more rapidly than those of the correlation functions for an unmagnetized plasma. Moreover, a second frequency, viz. $\omega_0 = (\omega_p^2 + \omega_B^2)^{1/2}$ shows up on a par with the plasma frequency. In the general off-resonant case this frequency determines the oscillations of the tail of the transverse velocity autocorrelation function, whereas the tail of the longitudinal function still oscillates at the plasma frequency, as in the unmagnetized case. In the particular case of resonance the frequency $\omega_0 = \sqrt{2}\omega_p$ determines the oscillations of the tails of both the transverse and the longitudinal autocorrelation functions.

The oscillating tails in the velocity autocorrelation functions may lead to a peak structure at the corresponding frequencies in the Fourier transforms of these functions. These peaks are expected to be particularly pronounced in the transverse autocorrelation function, since it has got the slowest decaying tail. Indeed, molecular dynamics computations[11]) for the velocity autocorrelation functions of a one-component plasma in a magnetic field have demonstrated the presence of such a peak structure. It was found that the power spectrum of the transverse velocity autocorrelation function for strongly coupled plasmas

($\Gamma = 10$ or 100) in a magnetic field with a resonant Larmor frequency ($\omega_B = \omega_p$) shows a peak at a frequency $\omega \simeq 1.3\omega_p$, which is rather near to $\sqrt{2}\omega_p$. The plasmon peak, which is present for vanishing magnetic fields, turned out to be suppressed completely in the resonant case.

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**Appendix**

*Asymptotic behaviour of integral expressions*

The mode-coupling formula for the velocity autocorrelation function leads to integral expressions of the form

$$F(t) = \int_a^b \mathrm{d}w\, \mathrm{e}^{-\mathrm{i}wt} f(w)\,, \tag{A.1}$$

with some complex function $f(w)$. To determine the long-time tail of the autocorrelation function we need the asymptotic form of $F(t)$ valid for large $t$. This asymptotic expression is easily obtained if the function $f(w)$ is regular in the interval $[a, b]$, with the inclusion of the boundaries. In fact, by repeated partial integration one derives for large $t$

$$F(t) \simeq G_b(t) - G_a(t)\,, \tag{A.2}$$

with $G_a$ and $G_b$ given by the asymptotic series

$$G_w(t) \simeq -\mathrm{e}^{-\mathrm{i}wt} \sum_{n=0}^{\infty} \left(\frac{-\mathrm{i}}{t}\right)^{n+1} f^{(n)}(w)\,. \tag{A.3}$$

Here $f^{(n)}(w)$ denotes the $n$th order derivative of $f(w)$. As (A.2) with (A.3) shows, the asymptotic form of $F(t)$ for large $t$ is determined by the properties of $f(w)$ at the boundaries. The phase factor $\mathrm{e}^{-\mathrm{i}wt}$ leads to destructive interference of the contributions from the interior points of the integration domain. At the boundaries of the integration interval this interference effect does not lead to a complete cancellation of the contributions of neighbouring points. As a result the asymptotic series (A.3) is obtained.

If $f(w)$ has a branch point at one or both of the end points of the integration interval, while it is still regular in the interior of the interval, the derivation of the asymptotic form is more involved. Let us assume that a branch cut is present at the lower boundary. Correspondingly the integral (A.1) may be written as

$$F(t) = \int_a^b \mathrm{d}w\, \mathrm{e}^{-\mathrm{i}wt}(w-a)^{\alpha-1}\bar{f}(w)\,, \tag{A.4}$$

with $0<\alpha<1$ and a function $\bar{f}(w)$ that is regular in the closed interval $[a, b]$. To determine the asymptotic form of (A.4) we split the interval into two parts $[a, a+\varepsilon]$ and $[a+\varepsilon, b]$, with a small positive $\varepsilon$ chosen such that $\bar{f}(w)$ may be represented by its Taylor expansion around $w=a$ for all $w \in [a, a+\varepsilon]$. The contribution $F_1(t)$ of the interval $[a, a+\varepsilon]$ may then be written as

$$F_1(t) = \mathrm{e}^{-\mathrm{i}at} \sum_{n=0}^{\infty} \frac{1}{n!}\, I_n^{\varepsilon}(t)\bar{f}^{(n)}(a)\,, \tag{A.5}$$

with $I_n^{\varepsilon}(t)$ defined as

$$I_n^{\varepsilon}(t) = \int_0^{\varepsilon} \mathrm{d}w\, \mathrm{e}^{-\mathrm{i}wt} w^{\alpha+n-1}\,, \tag{A.6}$$

for $n=0, 1, \ldots$. By partial integration one obtains

$$I_n^{\varepsilon}(t) = -\mathrm{e}^{-\mathrm{i}\varepsilon t} \sum_{m=0}^{n-1} \left(\frac{-\mathrm{i}}{t}\right)^{m+1} \frac{\Gamma(\alpha+n)}{\Gamma(\alpha+n-m)}\, \varepsilon^{\alpha+n-m-1} + \left(\frac{-\mathrm{i}}{t}\right)^{n} \frac{\Gamma(\alpha+n)}{\Gamma(\alpha)}\, I_0^{\varepsilon}(t)\,. \tag{A.7}$$

For $n=0$ one may write (A.6) as

$$I_0^{\varepsilon}(t) = \int_0^{\infty} \mathrm{d}w\, \mathrm{e}^{-\mathrm{i}wt} w^{\alpha-1} - \int_{\varepsilon}^{\infty} \mathrm{d}w\, \mathrm{e}^{-\mathrm{i}wt} w^{\alpha-1}\,. \tag{A.8}$$

The first integral can be evaluated by rotating the integration contour in the complex plane so as to obtain the well-known integral representation of $\Gamma(\alpha)$.

In the second integral of (A.8) we perform a repeated integration, which leads to an asymptotic series valid for large $t$:

$$I_0^\varepsilon(t) \simeq \frac{\Gamma(\alpha)\,\mathrm{e}^{-\mathrm{i}\alpha\pi/2}}{t^\alpha} + \frac{\mathrm{i}\mathrm{e}^{-\mathrm{i}\varepsilon t}}{t^\alpha} \sum_{m=0}^{\infty} \frac{(-\mathrm{i})^m \Gamma(\alpha)}{\Gamma(\alpha - m)} (\varepsilon t)^{\alpha - m - 1} . \qquad (A.9)$$

The contribution $F_2(t)$ of the interval $[a + \varepsilon, b]$ to $F(t)$ follows directly by employing (A.1)–(A.3):

$$\begin{aligned} F_2(t) &\simeq G_b(t) - G_{a+\varepsilon}(t) \\ &\simeq G_b(t) + \mathrm{e}^{-\mathrm{i}(a+\varepsilon)t} \sum_{n=0}^{\infty} \left(\frac{-\mathrm{i}}{t}\right)^n \frac{\mathrm{d}^n}{\mathrm{d}\varepsilon^n} [\varepsilon^{\alpha-1} \bar{f}(a + \varepsilon)] . \end{aligned} \qquad (A.10)$$

A Taylor expansion of $\bar{f}(a + \varepsilon)$ around $\bar{f}(a)$ then leads to

$$\begin{aligned} F_2(t) \simeq G_b(t) + \mathrm{e}^{-\mathrm{i}(a+\varepsilon)t} \sum_{n=0}^{\infty} \frac{1}{n!} \bar{f}^{(n)}(a) \sum_{m=0}^{\infty} \left(\frac{-\mathrm{i}}{t}\right)^{m+1} \\ \times \frac{\Gamma(\alpha + n)}{\Gamma(\alpha + n - m)} \varepsilon^{\alpha + n - m - 1} . \end{aligned} \qquad (A.11)$$

Adding this result to (A.5), with (A.7) and (A.9) inserted, we arrive at an expression of the form (A.2), with $G_a(t)$ given by

$$G_a(t) \simeq -\mathrm{e}^{-\mathrm{i}at} \frac{\mathrm{e}^{-\mathrm{i}\alpha\pi/2}}{t^\alpha} \sum_{n=0}^{\infty} \left(\frac{-\mathrm{i}}{t}\right)^n \frac{\Gamma(\alpha + n)}{n!} \bar{f}^{(n)}(a) . \qquad (A.12)$$

For $\alpha \to 1$ this expression reduces to (A.3), with $\bar{f}$ instead of $f$.

The results obtained above may be summarized as follows. The asymptotic form, for large $t$, of an integral (A.1), with an integrand function $f(w)$ that is regular in the interior of the integration interval, but that may possess branch points at the integration boundaries, is given by the general form (A.2). The functions $G_w(t)$ (for $w = a, b$) depend on the behaviour of $f(w)$ near the boundaries. If one has

$$f(w) \simeq c(b - w)^\mu , \quad w \uparrow b , \qquad (A.13)$$

with some complex constant $c \neq 0$ and some real exponent $\mu > -1$, the leading term of the asymptotic expansion of $G_b(t)$ for large $t$ is

$$G_b(t) \simeq c\, \frac{\mathrm{e}^{\mathrm{i}(\mu+1)\pi/2} \Gamma(\mu + 1)\, \mathrm{e}^{-\mathrm{i}bt}}{t^{\mu+1}} . \qquad (A.14)$$

Likewise, if at the lower boundary one has

$$f(w) \simeq c'(w-a)^{\nu}\,, \quad w\downarrow a\,, \tag{A.15}$$

with a constant $c' \neq 0$ and an exponent $\nu > -1$, the leading term of $G_a(t)$ is

$$G_a(t) \simeq -c'\,\frac{e^{-i(\nu+1)\pi/2}\Gamma(\nu+1)\,e^{-iat}}{t^{\nu+1}}\,. \tag{A.16}$$

In the main text the formulae (A.1), (A.2), with (A.13)–(A.16), have been used for the cases $\mu = -\frac{1}{2}, 0, \frac{1}{2}, 1$ and $\nu = 0, 1$.

## References

1) B.J. Alder and T.E. Wainwright, Phys. Rev. A **1** (1970) 18.
2) K. Kawasaki, Phys. Lett. A **32** (1970) 379.
3) M.H. Ernst, E.H. Hauge and J.M.J. van Leeuwen, Phys. Lett. A **34** (1971) 419; J. Stat. Phys. **15** (1976) 7.
4) H. Gould and G.F. Mazenko, Phys. Rev. Lett. **35** (1975) 1455.
5) P.V. Giaquinta, M. Parrinello, M.P. Tosi and N.H. March, Phys. Chem. Liquids **5** (1976) 197.
6) R.L. Varley, Phys. Lett. A **62** (1977) 340.
7) T. Gaskell, J. Phys. C **15** (1982) 1601.
8) M.C. Marchetti and T.R. Kirkpatrick, Phys. Rev. A **32** (1985) 2981.
9) L.G. Suttorp and J.S. Cohen, Physica **133A** (1985) 370.
10) L.G. Suttorp and A.J. Schoolderman, Physica **141A** (1987) 1.
11) B. Bernu, J. de Phys. Lett. **42** (1981) L253.