Some useful equivalences in **IPC**

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Let us write $\varphi \sim \psi$ for $\vdash_{\mathbf{IPC}} \varphi \leftrightarrow \psi$. If other logics are under discussion as well, we may write $\sim_{\mathbf{IPC}}$, etc.

Some useful equivalences.

1. $\varphi \to \psi \wedge \chi \ \sim \ (\varphi \to \psi) \wedge (\varphi \to \chi)$
2. $\varphi \vee \psi \to \chi \ \sim \ (\varphi \to \chi) \wedge (\psi \to \chi)$
3. $\varphi \wedge \psi \to \chi \ \sim \ \varphi \to (\psi \to \chi)$ and generalizations.
4. $\varphi \wedge (\psi \vee \chi) \ \sim \ (\varphi \wedge \psi) \vee (\varphi \wedge \chi)$
5. $\varphi \vee (\psi \wedge \chi) \ \sim \ (\varphi \vee \psi) \wedge (\varphi \vee \chi)$
6. More complex forms of the distributive laws.
7. $\neg\neg\neg\varphi \ \sim \ \neg\varphi$
8. $\neg\varphi \to \varphi \ \sim \ \neg\neg\varphi$
9. $\neg(\varphi \wedge \psi) \ \sim \ \varphi \to \neg\psi$
10. $\neg(\varphi \to \psi) \ \sim \ \neg\neg\varphi \wedge \neg\psi$
11. $\neg\neg(\varphi \wedge \psi) \ \sim \ \neg\neg\varphi \wedge \neg\neg\psi$
12. $\neg\neg(\varphi \to \psi) \ \sim \ \neg\neg\varphi \to \neg\neg\psi$