Intuitionistic Logic
Spring 2012

Homework 4
(due Monday, 19th March)

1. (a) Show that **LC** characterizes the upwards linear frames ($\forall x, y, z(xRy \wedge xRz \rightarrow yRz \vee zRy)$), i.e., show that $\mathfrak{F} \models \mathbf{LC}$ iff R is upwards linear. [2 pts]
   (b) Prove, using the canonical model method, strong completeness of **LC** with respect to the linear frames. [2 pts]
   (c) Prove that **LC** has the FMP (finite model property w.r.t. linear frames) by means of filtration [2 pts]
2. (a) Let φ contain only $\wedge, \vee$ and $\rightarrow$ but no $\neg$ and no $\bot$. Let $\mathfrak{M}$ be any Kripke-model (for the language of φ). Extend the model $\mathfrak{M}$ to $\mathfrak{M}^+$ by adding one more node x at the top above all the nodes of $\mathfrak{M}$, and making all the propositional variables of φ true in x. Show that, for all the nodes w in $\mathfrak{M}$ we have:

   $$\mathfrak{M}, w \models \varphi \text{ iff } \mathfrak{M}^+, w \models \varphi$$

   (satisfaction in the old and new model is the same for φ). [4 pts]
   (b) Let φ contain only $\wedge, \vee$ and $\rightarrow$ but no $\neg$ and no $\bot$. Show that $\vdash_{\mathbf{IPC}} \varphi$ iff $\vdash_{\mathbf{KC}} \varphi$. (You may use what is claimed about completeness of **KC** in class.) [2 pts]
3. Give an example of a finite rooted frame $\mathfrak{F}$ with a rooted generated subframe $\mathfrak{G}$ that is not a p-morphic image of $\mathfrak{F}$. Sketch the proof. (Hint: there exists an example with 6 worlds.) [6pts]