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Tentamen *Chaotic Dynamical Systems*

1.[20%]

Let T_2 be the tent map on $[0,1]$,

$$T_2(x) = \begin{cases} 2x, & 0 \le x \le 1/2, \\ 2-2x, & 1/2 \le x \le 1. \end{cases}$$

(a) Give the definition, from Devaney, of the statement "T_2 is chaotic on $[0,1]$";

(b) Prove that T_2 is chaotic on $[0,1]$.

2.[30%]

Consider the family of maps

$$A_\lambda(x) = \lambda \arctan(x)$$

on the real line.

(a) Identify the bifurcation that occurs at $x = 0$, $\lambda = -1$;

(b) Examine existence and stability of fixed and period points near $x = 0$, for λ near -1;

(c) Sketch the bifurcation diagram.

3.[20%]

Let $\Sigma_2 = \{0,1\}^{\mathbb{Z}}$ be the set of two-sided sequences of 0's and 1's.

(a) Denote $\mathbf{s} = (s_i)_{i\in\mathbb{Z}}$, $\mathbf{t} = (t_i)_{i\in\mathbb{Z}}$. Prove that

$$d(\mathbf{s},\mathbf{t}) = \sum_{i=-\infty}^{\infty} \frac{|s_i - t_i|}{2^{|i|}}$$

is a metric on Σ_2;

(b) Let Σ_2 be endowed with this metric. Prove that periodic points are dense for the shift operator σ on Σ_2.

4.[30%]

Let L_A be a hyperbolic torus automorphism on the two-torus $\mathbb{T}^2$, corresponding to the matrix $A \in GL(2,\mathbb{Z})$.

(a) Prove that periodic points are dense in $\mathbb{T}^2$;

(b) Let $(x,y) \in \mathbb{T}^2$. Prove that both the stable and unstable manifold of (x,y) is dense in $\mathbb{T}^2$;

(c) Prove that L_A is topologically transitive.