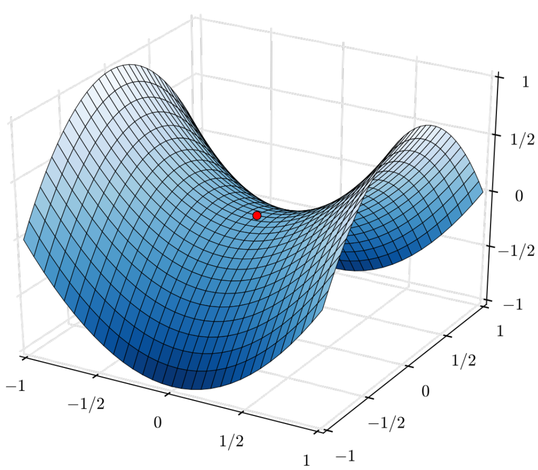
Werkcollege 6

Stationary points

Suppose that $f(x, y)$ is a function with 2 variables.

- a point (a, b) is *stationary* if $\nabla f(a, b)$ is null (that is, $f_x(a, b) = f_y(a, b) = 0$)
- local maximum and local minimum are defined as in the case with one-variable functions. If a stationary point is neither a local maximum nor a local minimum, it is a saddle point
- example of a saddle point:



- how to determine if a stationary point is a local maximum, a local minimum or a saddle point?
- we use the determinant of the Hessian matrix. The Hessian matrix is the matrix

$$\begin{bmatrix} f_{xx}(x, y) & f_{xy}(x, y) \\ f_{yx}(x, y) & f_{yy}(x, y) \end{bmatrix}$$

  We denote by $H(x, y)$ its determinant.
- for a "smooth" function $f(x, y)$ and a stationary point (a, b), we have the following criteria
  - if $H(a, b) < 0$, then (a, b) is a saddle point
  - if $H(a, b) > 0$ and $f_{xx}(a, b) > 0$, then (a, b) is a local minimum
  - if $H(a, b) > 0$ and $f_{xx}(a, b) < 0$, then (a, b) is a local maximum

  Note: if $H(a, b) = 0$, then the test is not conclusive

Polar coordinates

Instead of describing a point of the 2D plane by a pair (x, y) (which corresponds to the projections of the point on the X and Y axis), we can use its polar coordinates (r, θ)

- r is the distance to the origin
- θ is the angle between 0 and 2π $(0 \leq \theta < 2\pi)$, as pictured below