

Assessment with Maple T.A.: Creation of Test Items

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In this report we give an overview of Maple T.A., a web-based system for generating questions and exercises, and automatically assessing a student's response via the computer algebra system Maple. We discuss its potential for mathematics education at Dutch secondary school level, both in the context of summative and formative assessment, by examples.

1. Summative vs Formative Assessment

In education one distinguishes between *summative* and *formative assessment*. Black and Wiliam (1998a,b) define assessment as all those activities undertaken by teachers – and by their students in assessing themselves – that provide information to be used as feedback to modify the teaching and learning activities in which they are engaged. Under this definition, assessment encompasses teacher observation, classroom discussion, and analysis of student work, including homework and tests. Such assessment becomes *formative assessment* when the evidence is actually used to adapt the teaching to meet student needs. *Summative assessment* summarizes the quality and characteristics of the student. It is often used to determine whether the student will pass or fail a course. Atkin et al (2001) describe the relationship between both forms of assessment in the context of science education. Boston (2002) addresses the purpose and benefits of formative assessment in education in general, and she provides examples and resources to support its implementation. The distinction between summative and formative assessment is summarized in Table 1, which is taken from the position paper on assessment of the Teaching Committee of the Mathematical Association in The United Kingdom, dated 7-9-2002 (Retrieved October 11, 2004 from www.m-a.org.uk).

Summative (assessment <u>of</u> learning)	Formative (assessment <u>for</u> learning)
To give a summary of current performance, typically through numerical data from the results of tests and examinations. This provides a measure of the attainment of individual students and is one source of evidence in monitoring both school and national performance.	To provide diagnostic evidence which teachers can use both to give feedback to students and to inform their future planning, and which students can use to help them learn more effectively by giving them ways to improve their knowledge, fluency and understanding.
Over-emphasis on tests, targets and comparisons with others encourages teaching that leads to superficial learning aimed at avoiding errors, rather than learning from them, and demoralises those who fail. Success with summative assessment is more likely when emphasis is given to enhancing thinking skills and the general classroom focus is on what students can do and how they can improve rather than on the marks, grades and levels that they have attained.	Such evidence is obtained in a wide variety of largely informal ways, involving oral and written classroom activities and tasks rather than exclusively through tests and examinations, although careful analysis of responses to these can be used formatively. Formative assessment can improve learning by focusing on misconceptions and the development of understanding and independent thinking.

Table 1. Summative vs Formative Assessment

It is noted in this position paper that both forms of assessment are necessary and that it is crucially important that the right balance is achieved between them. However, current systems and many traditional practices, particularly in mathematics, give a disproportionate emphasis to summative assessment with little attention being given to the extensive possibilities of formative assessment for raising standards.

Assessments for formative and summative purposes can actually be implemented in various ways: performance assessments (presentation, report, ...), portfolios (work plan, student notes, final product, reflection notes, ...), traditional tests used in a different setting (interview or peer-group assessment, 'think aloud' testing, ...), and so on. In this report we describe how Maple T.A., a web-based testing and assessment system using Maple in the background, can be used for both summative and formative purposes. First we describe what types of questions are available in Maple T.A. and how this relates to the IMS Question & Test Interoperability (QTI) Specifications, V2.0 Public Draft, available at www.imsglobal.org/question/. We provide illustrative examples of interaction types in the context of mathematics education at Dutch secondary school level and we describe how these question types can be specified in Maple T.A.

2. Types of Items

In the context of electronic testing and assessment one prefers to speak about items instead of questions because this has a broader meaning: an item is an object, usually a question and its associated information such as score, layout rendering, response checking, and feedback (including hints and solutions), contained within an on-line assessment or test. For easy transfer of items, tests, and results between electronic assessment systems and/or electronic learning environments an international standard, called the IMS QTI standard (see the web site www.imsglobal.org/question/), has been developed. It specifies how items in question banks can be specified using XML (extensible markup language). Many modern assessment systems like Respondus¹, Questionmark Perception², Diploma³, and the Dutch system Wintoets⁴ are QTI compliant, in that they have an IMS QTI import/export facility⁵, although most systems contain features that are not described in IMS QTI and for this reason are lost or cause trouble when they are imported or exported.

The information model, which is described in the Public Draft Version 2.0 of the IMS QTI Specification, distinguishes nineteen interaction types, each uniquely identified by its class name. In Table 2 we list these interactions, based on the organizing principle that an interaction is simple, text-based, graphical, or of miscellaneous type. Note that question types and layout are not identified as pairings of response type with cardinality and rendering type, which is a more flexible but ambiguous way of description used in earlier QTI versions. Instead, in version 2.0, the type of interaction is now the identifier of an item in the IMS QTI standard.

¹ www.respondus.com

² www.questionmark.com

³ <http://www.brownstone.net>

⁴ www.drp.nl/wintoets/

⁵ QTI compliance means in almost all case, import/export facility in accordance with QTI version 1.1 or 1.2

<i>Simple Types</i>	
<i>Class Name</i>	<i>Specified Interaction</i>
choiceInteraction	Selecting on or more items from a list
orderInteraction	Putting a list of items into a specific order
associateInteraction	Creating associations between items in a list
matchInteraction	Creating pairings between items in two lists, but not between choices in the same list
gapMatchInteraction	Selecting items (text or images) from lists to fill gaps in a text
<i>Text-Based Types</i>	
<i>Class Name</i>	<i>Specified Interaction</i>
inlineChoiceInteraction	Selecting one item from a set of choices, each of which is a simple piece of text, in order to fill a gap in a text
textEntryInteraction	Entering a simple piece of text at an entry point
extendedTextInteraction	Entering an extended amount of text, i.e., more lines of text
hottextInteraction	Making one or more selections of text within a surrounding text
<i>Graphical Types</i>	
<i>Class Name</i>	<i>Specified Interaction</i>
hotspotInteraction	Selecting one or more dedicated areas from a graphic image
selectPointInteraction	Selecting one or more points from a graphic image
graphicOrderInteraction	Imposing order on a set of defined (visible) areas of a graphic image
graphicAssociateInteraction	Creating associations between a set of defined (visible) areas of a graphic image
graphicGapMatchInteraction	Filling gaps in a graphic image with images
positionObjectInteraction	Positioning a single image on another graphic image ('the stage'); the response is the pair of coordinates of the centre of the object with respect to the stage
<i>Miscellaneous Types</i>	
<i>Class Name</i>	<i>Specified Interaction</i>
sliderInteraction	Selecting a numerical value between a lower and upper bound with a slider with a step size that can be set
drawingInteraction	Editing a graphic image with a common set of drawing tools
uploadInteraction	Uploading a pre-prepared file representing the response to a question
customInteraction	Interacting with an item in a way that is not yet documented in IMS QTI

Table 2. Interaction Types in IMS QTI

Item templates, i.e., templates that can be used for producing large numbers of similar items, have been introduced in the Public Draft version 2.0 of the IMS QTI specification. Such items are often called cloned items. They make it possible that students in a class see different versions of similar questions. The difference between clones is realized by means of so-called template variables that can be initialized to randomly selected values, and then the value of a template variable can be included within the text of a question. Note that earlier version of QTI did not support this randomisation within questions.

Actually, the main drawback of the IMS QTI standard is that it has not been developed with mathematics in mind, although it is fair to say that with the version 2.0 concrete steps are made in this direction. Mavrikis and González Palomo (2004) mention as biggest problems for authors of mathematical activities: “these activities are often complex with multiple parts, require multiple notations, must allow entry of mathematical expressions, and that answers often entail mathematical entities beyond simple numbers or strings yielding the need to handle accuracy and precision of numbers.” Further down in the introductory section of their paper, the authors state: “IMS QTI misses semantically tagged mathematics to support the creation of exercises and tests that adapt the presented parts and feedback according the students’ errors in different parts of the question.” All this has lead to systems that use different techniques and formats. The authors mention the following examples:

- WaLLiS⁶: Web-based Assistant for Learning on-Line Intelligent System (using JavaScript templates and JavaServer Pages);
- AIM⁷: Alice Interactive Mathematics (having its own programming language using AIM-specific flags and Maple-like code; see also (Hermans, 2003 and 2004) and (Sangwin, 2003a, 2004a, and 2004b));
- MapleTA⁸ (using Maple specific syntax to achieve randomisation of questions and response evaluation);
- StoMP⁹: Software Teaching of Modular Physics (a QTI 1.2 compliant version exists, but it uses own XML structures).

The rest of the paper of Mavrikis and González Palomo (2004) is about the information model that achieves in explicitly representing the mathematical entities in questions, as well as allowing the authoring of more complex structures and interactive exercises with multiple parts and adaptive feedback. This paper nicely illustrates the current state of affairs in developing web-based mathematical activities for educational purposes: there exist many ideas and projects at research level, significant progress has been made in electronic publishing of mathematics through mainstream web browsers and in support for delivering, re-using and exchanging mathematical expression via standards like Open-Math¹⁰, MathML¹¹ and OMDoc¹², but many things are not yet ready enough for easy deployment at practical level for daily use in the classroom or the study room at home.

⁶ www.maths.ed.ac.uk/wallis/

⁷ <http://aiminfo.net/>

⁸ www.maplesoft.com/mapleta/

⁹ www.ph.surrey.ac.uk/stomp/

¹⁰ www.openmath.org

¹¹ www.w3.org/Math/

Maple T.A. is a web-based system for generating questions and exercises, and automatically assessing a response. A response can be as simple as a multiple-choice selection, or as complex as a free-form entry of mathematical expressions and formulae, in which case Maple intelligently evaluates the response to decide if it is mathematically equivalent to the correct answer. Maple can also be used to generate from a question template a set of questions “clone” so that, for example, each student in a class would be unlikely to see the same version of the question as the next student. Items in a Maple T.A. question bank can be created in three ways:

- on-line via the Question Bank Editor;
- by editing questions, using the system plain-text script file syntax (both on-line and off-line);
- by editing questions using LaTeX and specific macros to generate via Internet plain-text script files that can be uploaded into the system.

The first and third methods are the most popular ones. Figure 1 shows as screen shot of the Question Bank Editor.

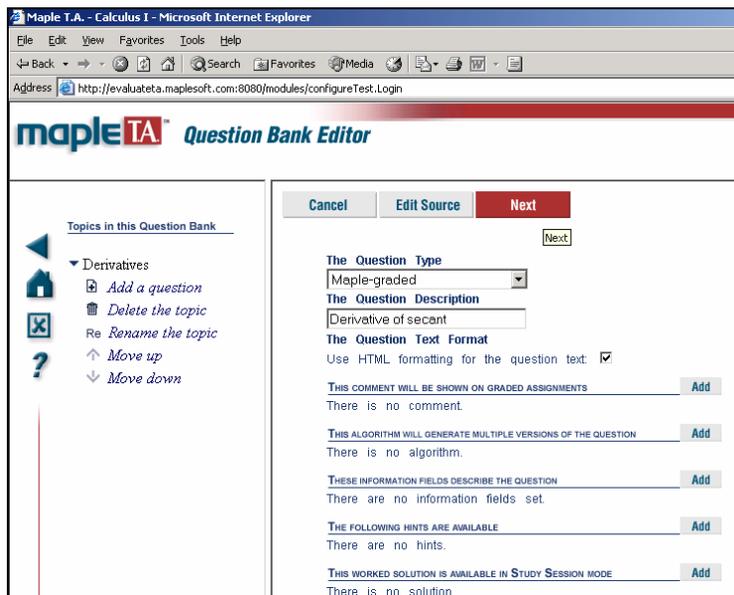


Figure 1. Screen shot of the Question Bank Editor

Authoring a test bank via LaTeX can be done through the following five steps:

1. Write your question bank in LaTeX using a text editor;
2. Run LaTeX on your text files. Preview the output to ensure it is correct. You may perform multiple iterations of the above steps;
3. Using the Web-based conversion service at <http://qa.brownstone.net/latex2edu/>, convert your LaTeX-format question bank to Maple T.A. format;
4. Upload the converted question bank;
5. Install the question bank into your class Web site.

¹² www.mathweb.org/omdoc/

A simple example of the LaTeX source of a multiple choice question, with choices in scrambled order, is the following:

```
\begin{question}{MultipleChoice}
\qutext{\$2\log{x}-\log{y}=\$}
\choice*{\$\log\left(\frac{x^2}{y}\right)\$}
\choice{\$\log\left(x^2-y\right)\$}
\choice{\$\log\left(2x-y\right)\$}
\end{question}
```

Algorithmic variables can randomize the above question. For example, the next code chooses random positive integer coefficients m and n , ranging from 2 and 6. The choices appear in the specific order listed.

```
\begin{question}{NonPermutingMultipleChoice}
\qutext{\$\var{m}\log{x}-\var{n}\log{y}=\$}
\choice*{\$\log\left(\frac{x^{\var{m}}}{y^{\var{n}}}\right)\$}
\choice{\$\log\left(x^{\var{m}}-y^{\var{n}}\right)\$}
\choice{\$\log\left(\var{m}x-\var{n}y\right)\$}
\choice{\$\log\left(x^{\var{m}}y^{\var{n}}\right)\$}
\code{\$m=range(2,6); \$n=range(2,6);}
\end{question}
```

Drawback is that Maple T.A. is not QTI compliant. But maybe this is just a matter of time because the underlying system of Brownstone is moving in this direction: Diploma already has an IMS QTI export facility. Anyway, the difficulty with QTI compliance is not too big a surprise: in (Gorissen, 2003) it is concluded that although it is possible to construct a basic set of QTI questions that can be imported by most of the computer-aided assessment application that support QTI in one way or the other, interoperability breaks down when test data become more complex. Maybe, in mathematics assessments we are still in this phase of low interoperability. We refer to (Milligan, 2003) and (Mavrikis, 2004) for further discussion of issues that relate to mathematical questions and the IMS QTI specification.

On the positive side, the Maple T.A. Building Block provides integration of the online testing and assessment system with the Blackboard Learning System¹³. Maple T.A. Building Block gives students and instructors access to all testing and assessment functionalities of Maple T.A., while requiring a single login point to access both systems. Instructors also can create Maple T.A. assignments and tests within their Blackboard courses. In addition, when students take a Maple T.A. test, both Maple T.A. and Blackboard grade books are automatically updated.

Although Maple T.A. is not IMS QTI compliant, you will recognize many of the QTI types of items – certainly when you look at QTI version 1.2 – that are also available in Maple T.A.. We list all possible Maple T.A. question types in Table 3 and the corresponding arguments of the LaTeX question environment together with available fields in the LaTeX questions.

¹³ www.blackboard.com

Table 3. Question Types in MapleTA

<i>Simple Types</i>	
<i>Question Type //</i> <i>LaTeX Environment Argument;</i> <i>Fields in Question (besides \qutext)</i>	<i>Meaning</i>
(Nonpermuting) Multiple Choice // MultipleChoice; \choice NonPermutingMultipleChoice; \choice	The student will be presented with a question and a list of choices that includes one correct answer. ‘Nonpermuting’ forces the choices to appear in the specific order listed. Algorithmic variables can be used in question and answers. These variables must be defined in the \code field.
(Nonpermuting) Multiple Selection // MultipleSelection; \choice NonPermutingMultipleSelection; \choice	The student will be presented with a question and a list of choices that includes possibly more than one correct answer. Use Nonpermuting if you want the choice to appear in the specific order listed. Algorithmic variables can be used in question and answers.
(Nonpermuting) True/False // TrueFalse; \true, \false	The student will be presented with a question and a list of two choices: true and false. Use Nonpermuting if you want the choice to appear in the specific order listed. Algorithmic variables can be used in the question.
Matching // Matching; \match	The student will be presented two lists and must match each entry on the first list with one entry from the second list. Matches of one element to many are not available in the system.
<i>Text-Based Types</i>	
<i>Question Type</i>	<i>Meaning</i>
<i>Fill-in-the-blank or</i> <i>Graded Free Response:</i>	There are four varieties, of which three have a textual question mode and one is based on mathematical formulas:
○ Menu-Style Response // Blanks; \blank[menu]	The student will be presented a question with a blank spot and a list of raw text choices in a drop-down menu with one correct answer.
○ Textbox Response // Blanks; \blank[text]	The student's response is searched for specified key words or phrases but other text is ignored.
○ Key Words or Phrase Question // Key Words; \answer	The student's response is searched for specified key words or phrases but other text is ignored.
○ Blanks Formula // Blanks; \blank[formula]	The student will be presented a question with a blank mathematical formula cell that appears in-line with the question statement (rather than on a separate line as in a formula question).
List // List Question;	A list question is similar to a Blanks Menu Style and Blanks Text Style question type in terms of their application, except the latter ones only

<code>\answer, \comment, \grader</code>	allow you to set a single response object in the question. List questions offer more facilities such as multiple correct answers, credit control, and response-specific comments.
Ungraded Essay // Essay;	This is an open response question that is not automatically graded by the system. Student responses are sent to the instructor, who scores them traditionally, and assigns a grade in the system Gradebook. No answer or feedback is available to the student.
<i>Graphical Types</i>	
<i>Question Type</i>	<i>Meaning</i>
Clickable Image // Clickable Image; clickableimage environment	The student is presented an image, provided as a separate web-friendly file, together with some introductory text. As the student moves the mouse over the image, various "hotspots" light up. The goal is to click on the right hotspot.
<i>Questions with Applet Interaction:</i>	Maple T.A. supports 3 types of graphical applet question types. To use these questions, you must use LaTeX to author the questions or write raw question bank code.
○ Dynamic Figure Label // labelgraphic environment	The student is presented an image, provided as a separate jpg or gif file, together with some introductory text and with (dynamically generated) text labels on the image
○ Graph Sketching // Sketch; sketch environment, \check	The student is presented an introductory text accompanied by a set of axes. The student draws a graph on the axes by clicking points to be used as interpolation nodes.
○ Dynamic Plots in Questions \graph, \multigraph	The system allows the embedding of graphs from web-friendly files in questions. It also allows embedding Maple plots in any question type; however, this is not an Applet interaction type but a Maple type question.
<i>Mathematical & Scientific Free Response Question Types</i>	
<i>Question Type</i>	<i>Meaning</i>
Formula // Formula; \answer	The student is presented a question statement, followed on a separate line (rather than inline as in the Blanks Formula type) by a single free-response entry cell that accepts a mathematical formula, i.e, an expression consisting of numbers, variable names, and the standard arithmetic operators and functions. If the student response and the correct answer are algebraically or numerically equivalent, the response is graded correct. Algorithmic variables can be

	used in question and answers, as in all mathematical question types listed below; these variables must be defined in the <code>\code</code> field. The <code>\answer</code> field must be in calculator syntax, not in LaTeX. Enter the answer as you expect the student to enter it (although equivalent answers are graded correct)
Restricted Formula // Restricted Formula; <code>\answer</code>	Similar to the Formula type, but answers may only use the standard arithmetic operations +, -, *, /, and ^, and the <code>sqrt()</code> function. Functions such as <code>log()</code> , <code>sin()</code> , and so on, are not allowed. Answers may include variables. The <code>\answer</code> field must be in calculator syntax, not in LaTeX.
Formula mod C // Formula mod C; <code>\answer</code>	Similar to the Formula type. Answers may be any kind of formula, entered in calculator syntax, but responses are graded as correct if they differ from the given answer by a constant value. Answers may include variables and standard functions. This question type is intended for use in questions asking the student to evaluate indefinite integrals. Do not include +C in the answer.
Numeric // Numeric; <code>\answer</code> , <code>\digits</code> , <code>\err</code> , <code>\perc</code>	Then Numeric question type accepts either a number in decimal or scientific notation, or a number with units for an answer. If the system has an answer with units, it will provide separate entry cells for numbers and units. If the system has an answer with no units specified, it displays a single entry cell. Grading can be done in exact mode, in any precision, and in any requested tolerance. The <code>\answer</code> field must be in calculator syntax, not in LaTeX.
Equation // Equation; <code>\answer</code>	This question type accepts an equation (two formulas separated by an '=' sign) as an answer.
Ntuple // Ntuple; <code>\answer</code>	This question type accepts an <u>ordered</u> sequence of numbers or formulas separated by <u>commas</u> and embraced by round brackets. It is intended for coordinates of points and vectors.
Matrix // Matrix; <code>\answer</code> , <code>\size</code>	The student is presented a question about a matrix and is offered a layout of a matrix with text fields, each of which accepts a number or a formula.
Formula List // Formula List; <code>\answer</code>	This question type accepts an <u>ordered</u> list of numbers or formulas separated by <u>commas</u> . It is intended for multiple number or formulas in the student's response, but the entries in the

	response must appear in the same order as in the correct answer.
MultiFormula // Multi Formula; \answer	This question type accepts an <u>unordered</u> list of numbers or formulas separated by <u>semicolons</u> . The entries in the student's response do <u>not</u> need to appear in the same order as in the correct answer. It is intended for mathematical questions with possible more than one correct answer (e.g., finding roots of polynomials).
Maple // Maple; \maple, \type{formula}, \plot, \drawMaplePlot, \drawmaplePlot3d, \maple*, \plot*, \drawMaplePlot*, \drawmaplePlot3d*	The 'Maple' question type allows the creation of questions that are graded using Maple 9 code. Maple questions use the \maple field instead of \answer to specify an answer. The argument to \maple is the correct answer, expressed in Maple syntax. The response (stored in \$RESPONSE) and the <i>expression</i> specified in the \maple field are compared by the Maple statement $\text{evalb}(\$RESPONSE=expression);$ The expression may be a Maple command. To allow students to use less stringent syntax for formulas in their responses, while at the same time disallowing advanced Maple syntax answers, you must include \type{formula} in the question body (e.g., the response $2x$ would be accepted as well as $2*x$, but advanced Maple commands like $\text{diff}(x^2,x)$ will be rejected). With the starred field \maple* an author can provide own Maple code for production of the correct answer. You may also insert Maple-driven plots in the question part with the \drawMaplePlot and \drawMaplePlot3d field. You can allow students to view a plot of their response by including the \plot field in the question body. Starred versions of the plotting fields are provided in case you want to specify your own raw Maple code to produce the intended result. Algorithmic variables can be used in question and answers; they are defined in the \code field. You can use the MathML export facilities of Maple to display the generated mathematical expressions in traditional notation.
<i>Miscellaneous Question Types</i>	
Multipart Formula // Multipart;	To make a question that consists of several parts, declare a question of type Multipart and include other questions (of any type) in its body.

	<p>Multipart questions can be nested and their numbering style can be changed via the \numbering field. The Question Bank Editor is often the recommended method for authoring multipart questions, because it allows incorporation of existing questions and easy specification of partial credit.</p>
<p>Inline //</p> <p>Inline; \blank and blank environment</p>	<p>Inline questions are a more flexible and extensible form of free response question that can include multiple response cells. In this way, they are similar to multipart questions in that they provide a shell structure in which to embed other questions. Inline questions support the following features:</p> <ul style="list-style-type: none"> ○ Questions can contain multiple response cells, for example, Fill-in-the-Blanks questions; ○ The widest variety of types of response cells (linked to specific question grader types) anywhere in the body of a question, including HTML tables; ○ Question Types: all Formula Question types, Numeric Question types, Multiple Choice and Multiple Selection Question types, and List Question types. <p>As in a multipart question, each subpart of the Inline question must itself be a well-formed question. The only exception is that, when used to define a blank inside an inline question, any of the above question types may omit the question field (because this is actually defined <i>inline</i> in the body of the larger question statement).</p> <p>In the Question Bank Editor, the Visual Editor is provided for easy question creation, either in WYSIWYG or HTML source mode. See the screen shot below.</p>

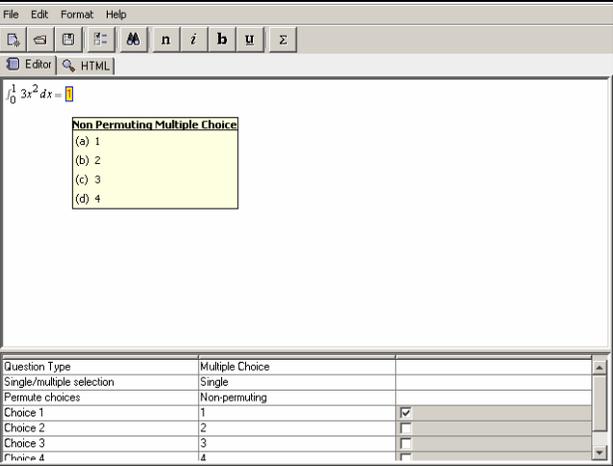
	 <table border="1" data-bbox="760 548 1373 657"> <tr> <td>Question Type</td> <td>Multiple Choice</td> <td></td> </tr> <tr> <td>Single/multiple selection</td> <td>Single</td> <td></td> </tr> <tr> <td>Permute choices</td> <td>Non-permuting</td> <td></td> </tr> <tr> <td>Choice 1</td> <td>1</td> <td><input checked="" type="checkbox"/></td> </tr> <tr> <td>Choice 2</td> <td>2</td> <td><input type="checkbox"/></td> </tr> <tr> <td>Choice 3</td> <td>3</td> <td><input type="checkbox"/></td> </tr> <tr> <td>Choice 4</td> <td>4</td> <td><input type="checkbox"/></td> </tr> </table>	Question Type	Multiple Choice		Single/multiple selection	Single		Permute choices	Non-permuting		Choice 1	1	<input checked="" type="checkbox"/>	Choice 2	2	<input type="checkbox"/>	Choice 3	3	<input type="checkbox"/>	Choice 4	4	<input type="checkbox"/>
Question Type	Multiple Choice																					
Single/multiple selection	Single																					
Permute choices	Non-permuting																					
Choice 1	1	<input checked="" type="checkbox"/>																				
Choice 2	2	<input type="checkbox"/>																				
Choice 3	3	<input type="checkbox"/>																				
Choice 4	4	<input type="checkbox"/>																				
<p>Palette Question</p>	<p>Palette questions enable you to use the WebEQ programmable editor¹⁴ interface to create your own customized palette for student response entries. This kind of questions is preferably created with the Question Bank Editor, and cannot be defined via LaTeX authoring. Predefined toolbars can be taken from the WebEQ Developer's Suite</p>																					

Table 3. Question Types in MapleTA

3. Authoring of Items in Maple T.A.

Items in a Maple T.A. question bank can be created in three ways:

- on-line via the Question Bank Editor;
- by editing questions, using the system plain-text script file syntax (both on-line and off-line);
- by editing questions using LaTeX and specific macros to generate via Internet plain-text script files that can be uploaded into the system.

The first and third methods are the most popular ones. In this section we shall show by examples how questions in a question bank can be created in these two ways. This guides you through the first steps of creating questions.

3.1. Question Bank Editor

Example 1: a basic question on transformations

In this first example, we build a multiple-choice question that gives a function $f(x)$ and asks to find $f(x+5)$. It offers some choices for the answers. The question we have in mind is:

Given $f(x) = (x+3)^2$, find $f(x+5)$

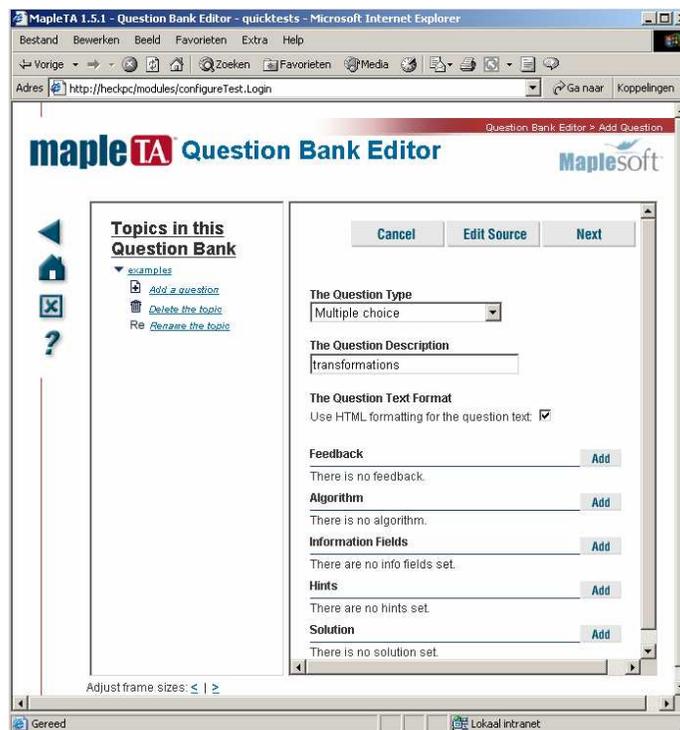
¹⁴ WebEQ is a tool for authoring mathematical expression for display on web pages. It is a product of Design Science. See www.dessci.com/en/products/webeq/ for more information

- $(x+8)^2$
- $x^2 + 6x + 14$
- $x^2 + 7x + 14$
- none of these

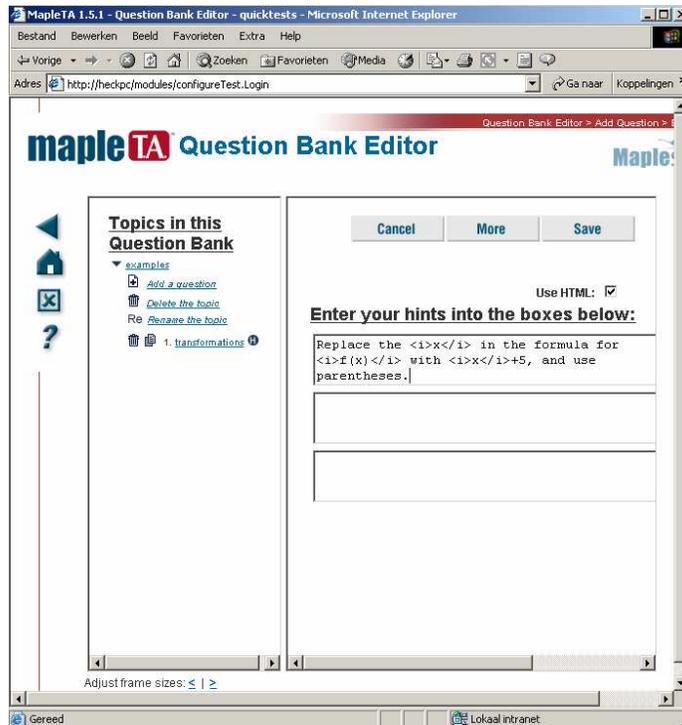
Assuming that you already have a question bank opened with the editor and that you are editing the topic 'examples'.

1. Click **Add a new question** in the left half of the Question Bank Editor screen.
2. Choose "Multiple choice" from the **Question Type** menu and type a description in the **Question Description** text box.
3. If you want to include math expressions or plots in the question text, select the check box next to **Use HTML formatting for the question text**.

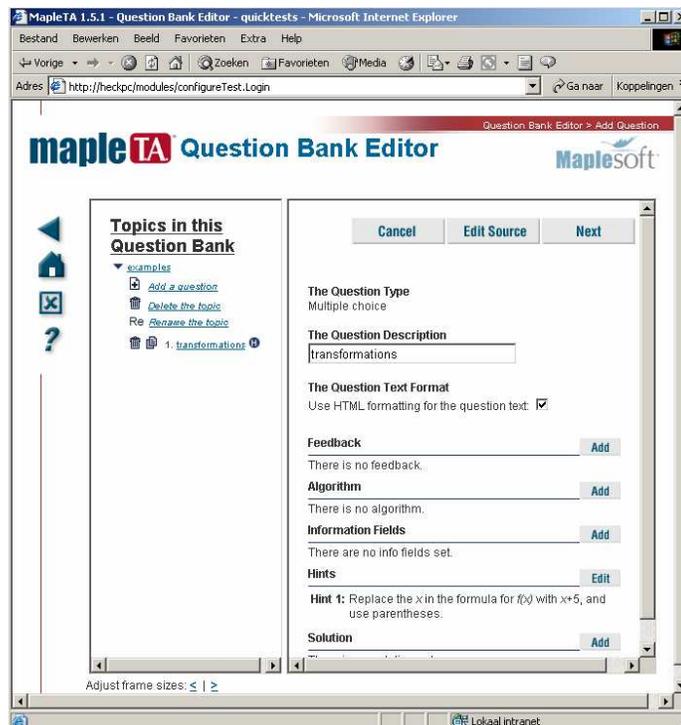
The screen now looks like this.



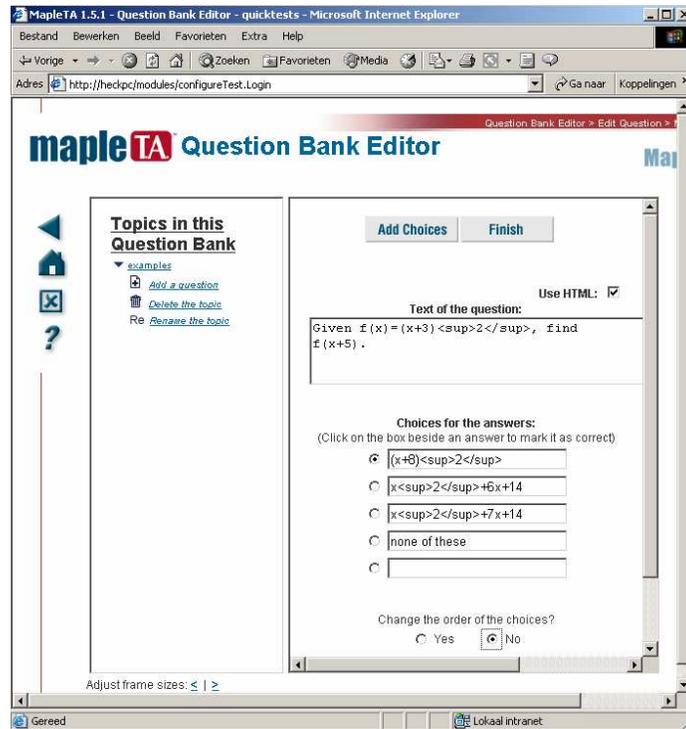
4. Click the **Add** button if you want to give Hints. This will take you to a screen in which you type the hint text, if required in HTML. For example you may enter



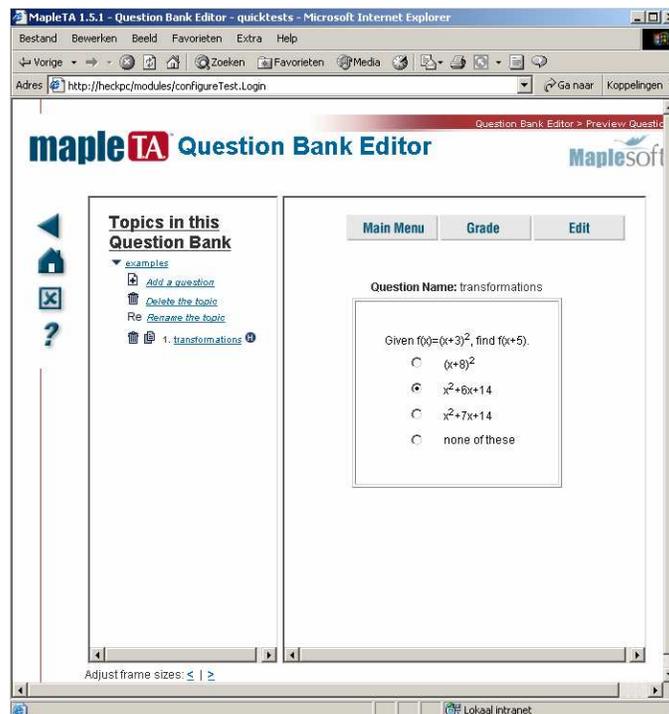
5. Press **Save**. The hint that you just entered is now visible in the screen.



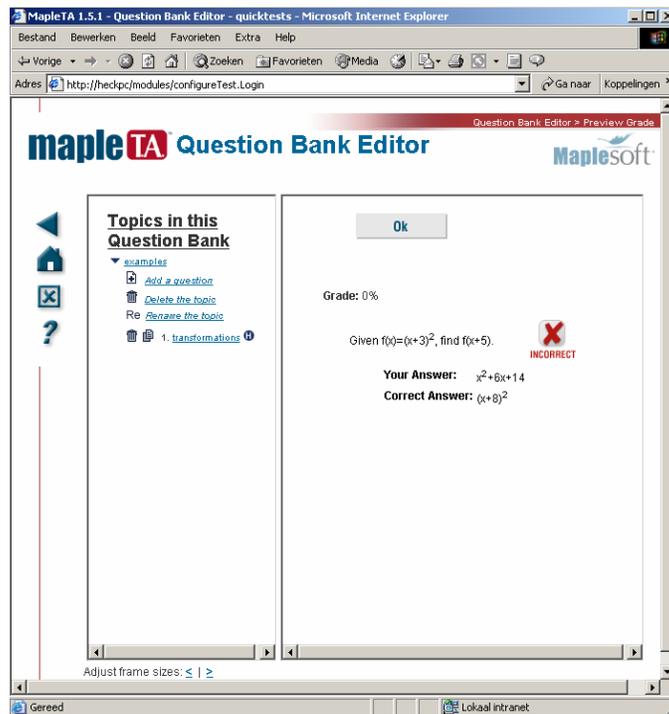
6. Click the **Next** button. This will take you to a screen in which you type the question text and the choices. In the screen shot below we use the HTML tags `^{...}` to specify powers. We prefer not to change the order of choices.



7. **Finish** editing. This will take you to a screen in which you can test your question, edit again, or return to the main menu.
8. Now pretend you are a student doing the assignment and enter a response by checking some box. For example



9. Click the **Grade** button to check that Maple is correctly assessing the response.



Click the **Ok** button to take you back to the previous screen. Try a variety of student responses to convince yourself that the grading the question is adequate. If the question needs revision, click the **Edit** button.

We will now discuss some variations on the theme

Example 2: including formatted mathematical expression in a question

Maple T.A. renders the question text as HTML/MathML if the **Use HTML** check box has been selected. This allows you to include formatted math expressions in questions.

In this section, we enhance Example 1 by making the question text appear as

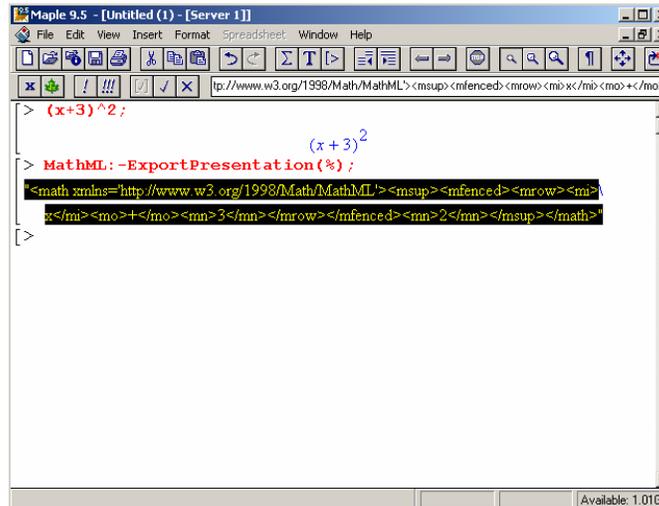
Given $f(x) = (x+3)^2$, find $f(x+5)$

- $(x+8)^2$
- $x^2 + 6x + 14$
- $x^2 + 7x + 14$
- none of these

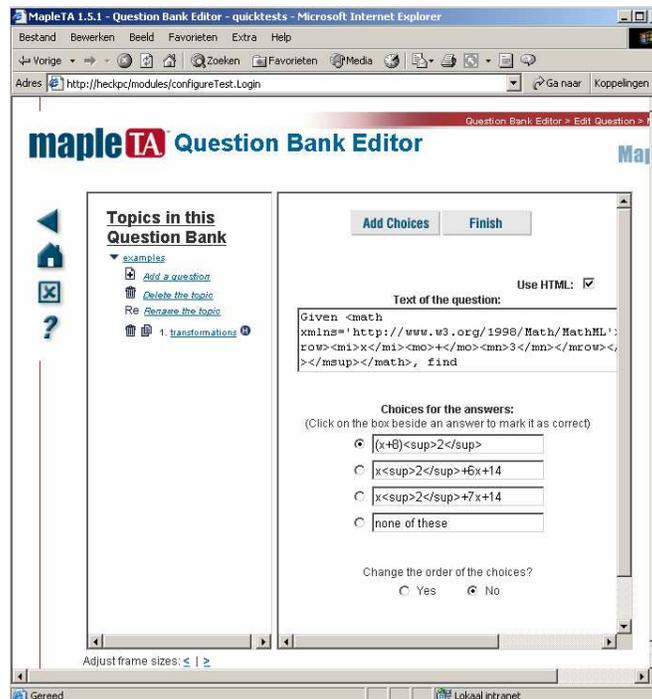
and at the same time avoiding the cumbersome entering of HTML -code

The easiest way to include a formatted math expression in a question is to use a formula editor like *Mathtype* and to copy/paste a formula in MathML presentation mode to the question. Alternatively you can use Maple's **MathML:-ExportPresentation** command as follows.

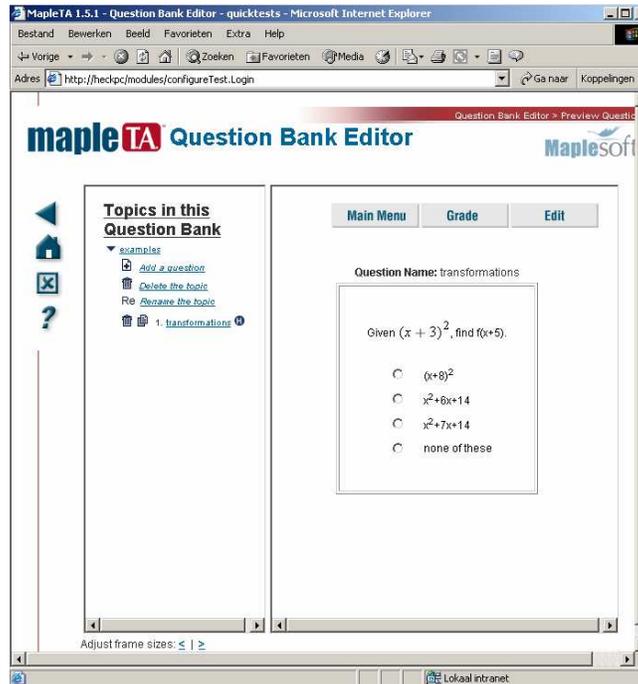
1. Navigate to the question text area (Step 6 in the previous section) and type the text that precedes the math expression.
2. Open a Maple worksheet and type the Maple code for the math expression you want to display.
3. Call **MathML:-ExportPresentation** on the Maple output. The Maple worksheet might look like this:



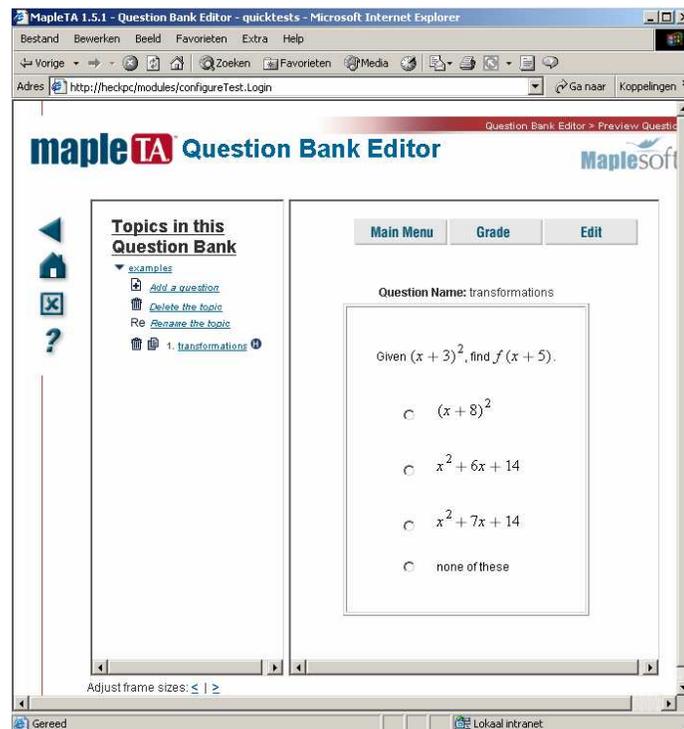
4. Copy the resulting MathML string to the clipboard and paste it into the Maple T.A. question text area.
5. Remove the quotation marks from the MathML string. **This step is important!** The screen now looks like this.



6. Click **Finish**. The question will now be displayed with pretty-print mathematics, as shown in the following picture (at least for the first part that we changed).



In this style of copying/pasting between Maple and Maple T.A. you can adjust the rest of the question to get all mathematics in proper display through MathML. The result will look like

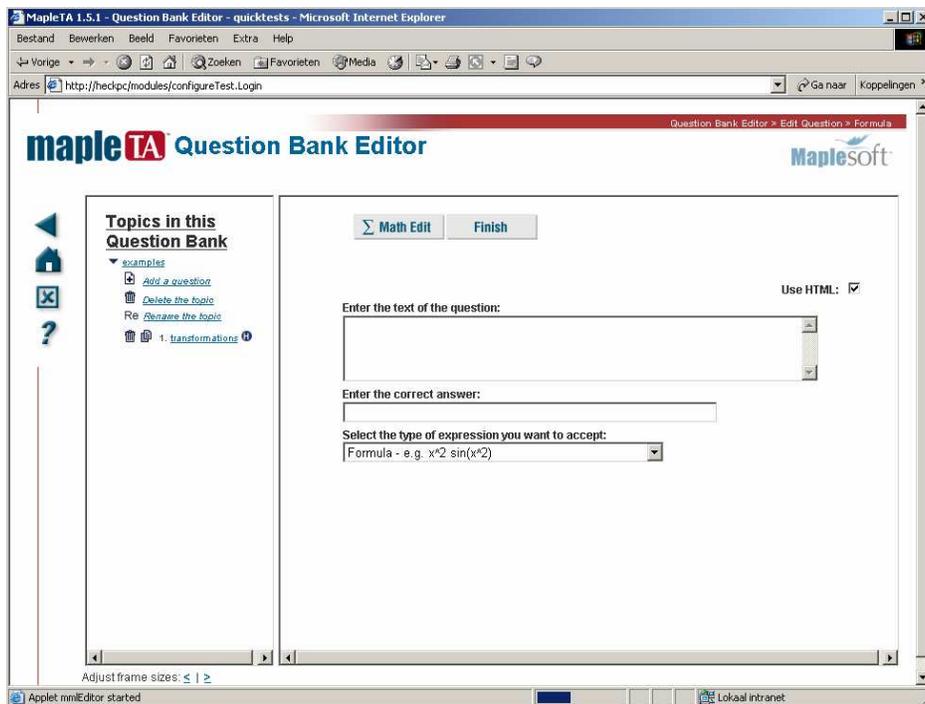


Of course this above method is a cumbersome way of getting nicely formatted mathematical expressions in a particular question. There are better ways to do this, viz., use LaTeX authoring for questions that involve many mathematical expressions. For the “Mathematical formula” question type, Maple T.A. provides a graphical MathML editor. In the next example we show how this can be used to ask students a similar question as before, but instead of checking one of predefined choices, they must enter a mathematical formula. The answer is checked automatically

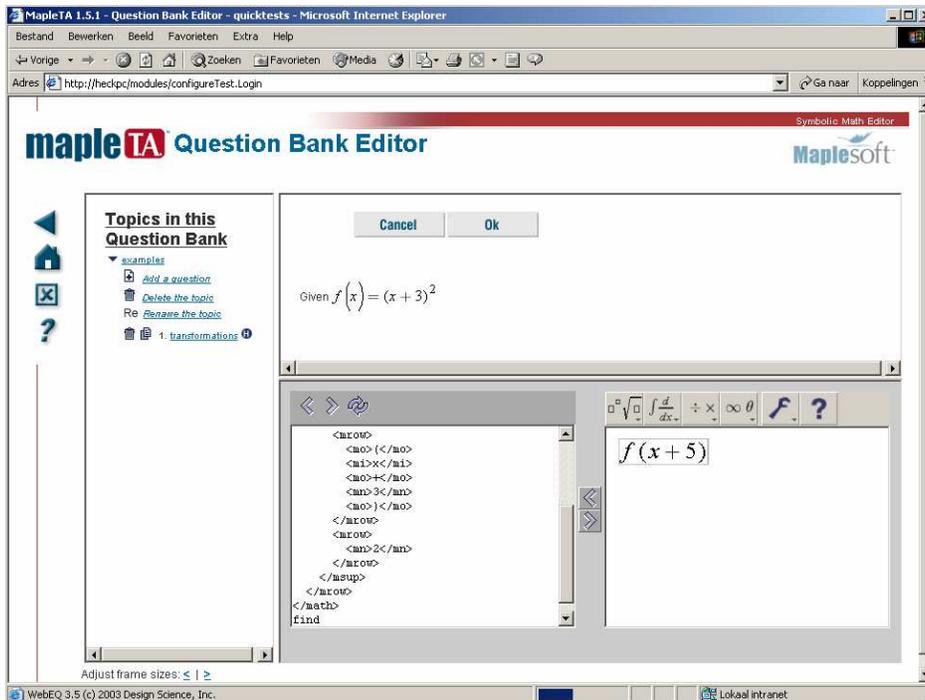
Example 3: Formatted math expressions in a “Mathematical formula” question type

Assuming that you already have a question bank opened with the editor and that you are editing the topic ‘examples’.

1. Click **Add a new question** in the left half of the Question Bank Editor screen.
2. Choose “Mathematical formula” from the **Question Type** menu and type a description in the **Question Description** text box.
3. Select the check box next to **Use HTML formatting for the question text**.
4. Advance to the screen in which you enter the question text. This brings you to a MathML editor screen. It looks as follows:



5. Put the cursor in the question text and press the **Math Edit** button. This will open the graphical MathML editor to create your expression.
6. Start entering the question and when you get to the point that you need a mathematical expression, build it in the right-hand panel, and move it into the right-hand panel by clicking the left-arrow button.
7. Continue editing in this style. See the screen shot below.

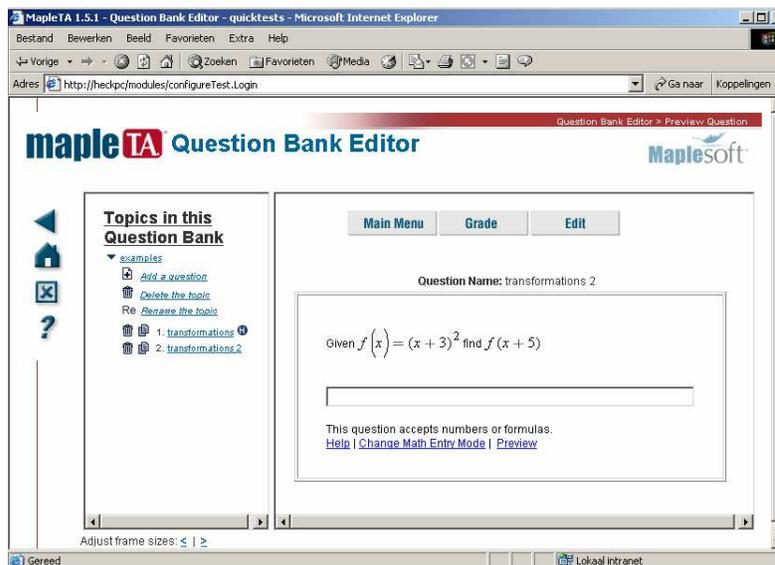


8. When you are finished with the formula, move the MathML expression into the left-hand panel by clicking the left-arrow button.
9. Click the **Ok** button when you are done with the question.

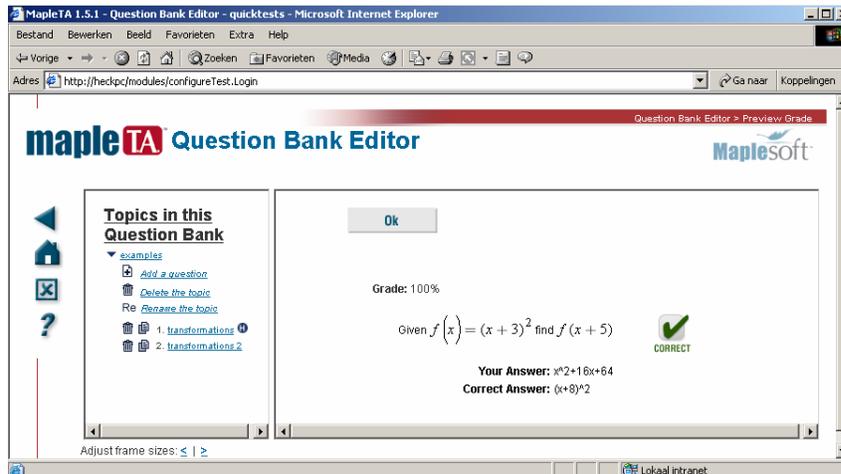
The editor uses WebEQ technology to translate the graphical form of your expression to MathML code.

We continue with building up the question.

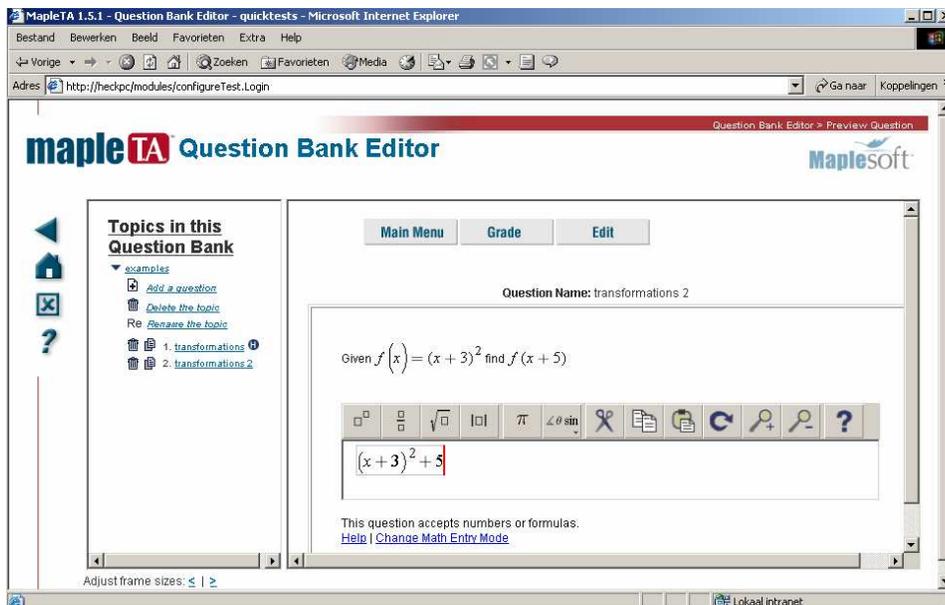
10. Select the type of question that you want to accept – in this case “Formula - e.g. $x^2 \sin(x^2)$ ” – and enter the correct answer $(x+8)^2$ in the answer box.
11. Click **Finish**. The question will now be displayed with pretty-print mathematics, as shown in the following picture



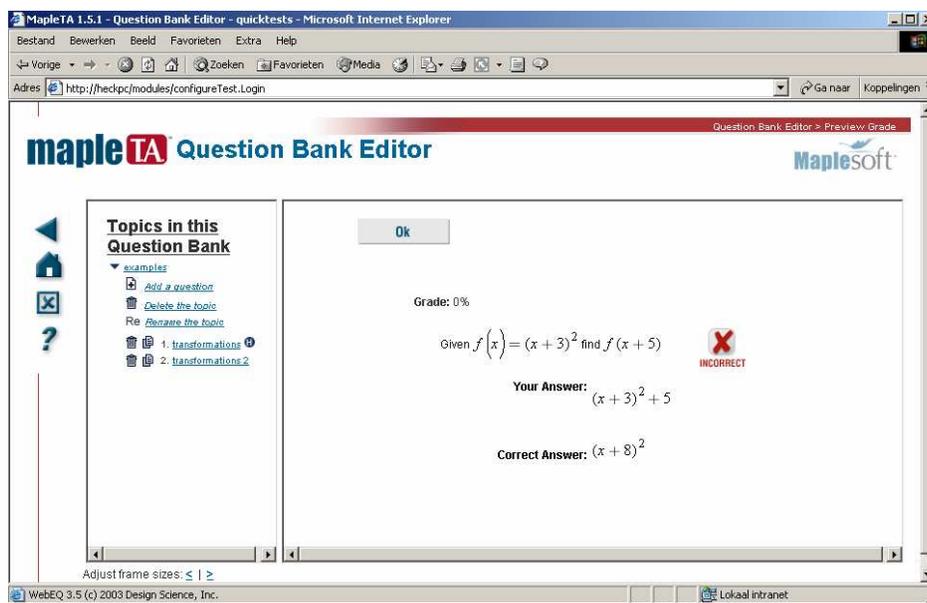
12. Try some responses to see if everything works as expected. If you enter $(x+8)^2$ you will get the reply that the answer is correct, but Maple T.A. will do some simple algebraic testing to verify that $x^2+16x+64$ is also correct. See the screen shot below.



- It is possible to specify in the question that the student response is required not only to be mathematically equal to the correct answer, but also to be expressed in the same form. But this is not supported in the Question Bank Editor; only in LaTeX authoring or editing source scripts.
13. Change the **Math Entry Mode** from “Text mode” into “Symbol Mode”, Click OK, and reload the question by clicking on the question-hyperlink in the left-hand panel. The screen will show a palette for entering a response. In the screen shot below we entered a wrong answer with the palette.



The given feedback to this response is



14. Go back to the **main menu** and click **Save/Install**. You will be prompted for the name of the question bank where you want to store the questions. This step of saving/installing question is a crucial step if you do not want to loose work. Your questions are only stored permanently into the Maple T.A. system when you explicitly ask for it. Until now, our questions were only store in some work cache.
15. To go really save, you can download the question bank to you local disk via the **Download** button.

For more advanced grading of mathematical/scientific free-response question you will need the full capacity of the computer algebra system Maple. We will come back to this later, but first we introduce *algorithmic variables* to generate random questions of the given type.

Example 4: randomising a question via algorithmic variables

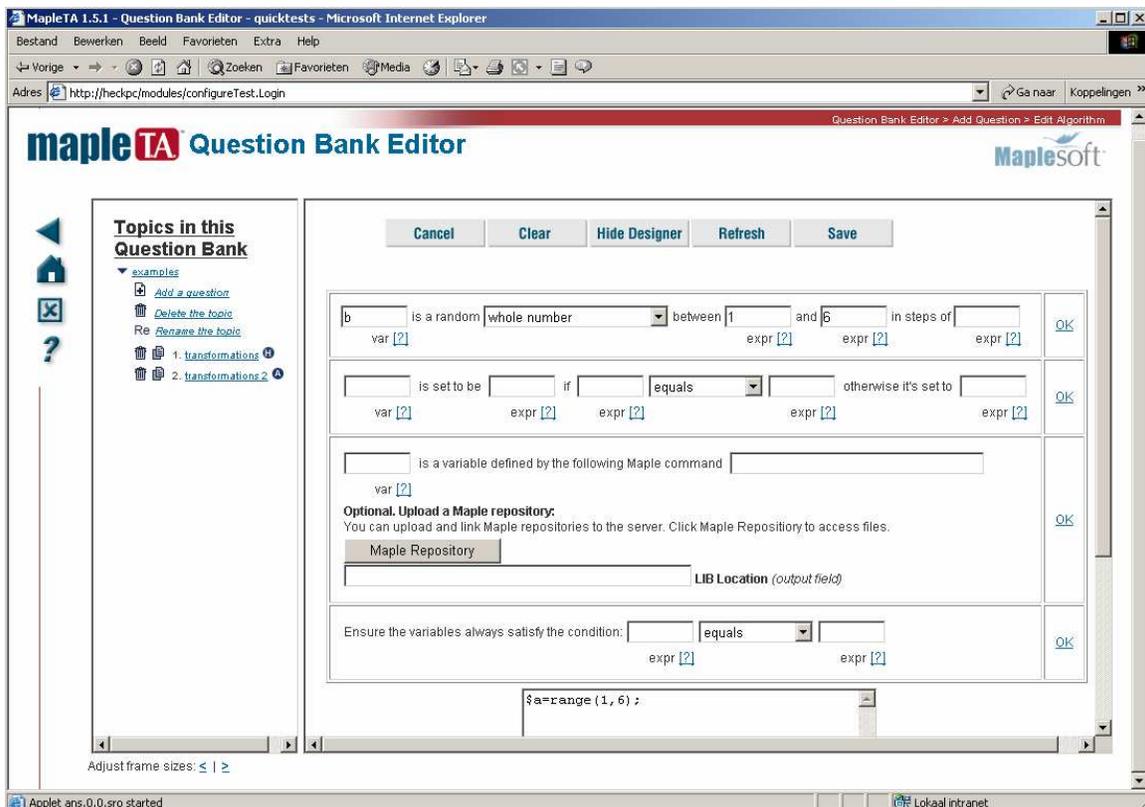
It is clear that the multiple-choice question

Given $f(x) = (x + 3)^2$, find $f(x + 5)$

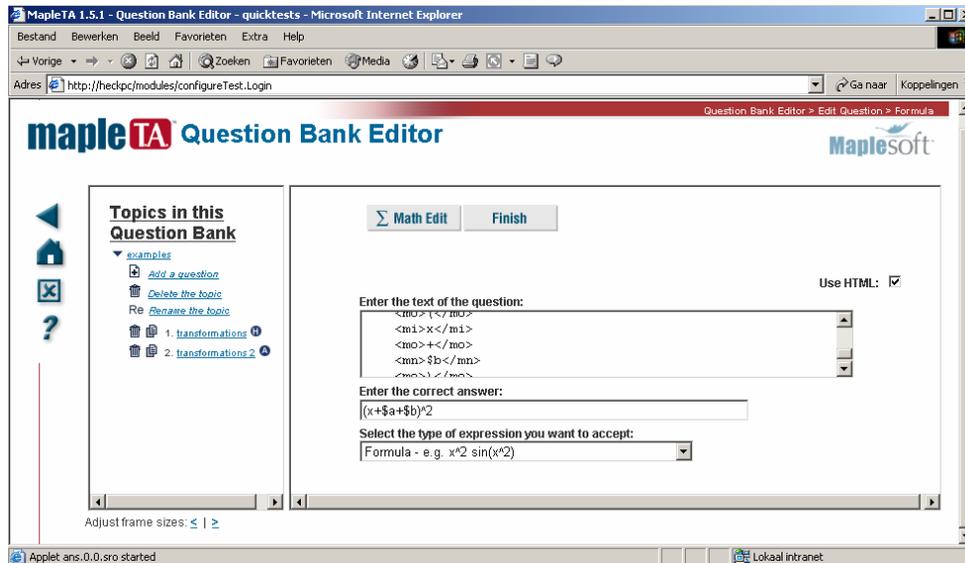
- $(x + 8)^2$
- $x^2 + 6x + 14$
- $x^2 + 10x + 24$
- none of these

is more or less a template for questions of similar type but with integers 3 and 5 replaced by others, and the choices adapted accordingly. This can be arranged in Maple T.A. by introducing algorithmic variables in the question and answers. We will show how this is done in the “Mathematical formula” question type that we created in the previous example..

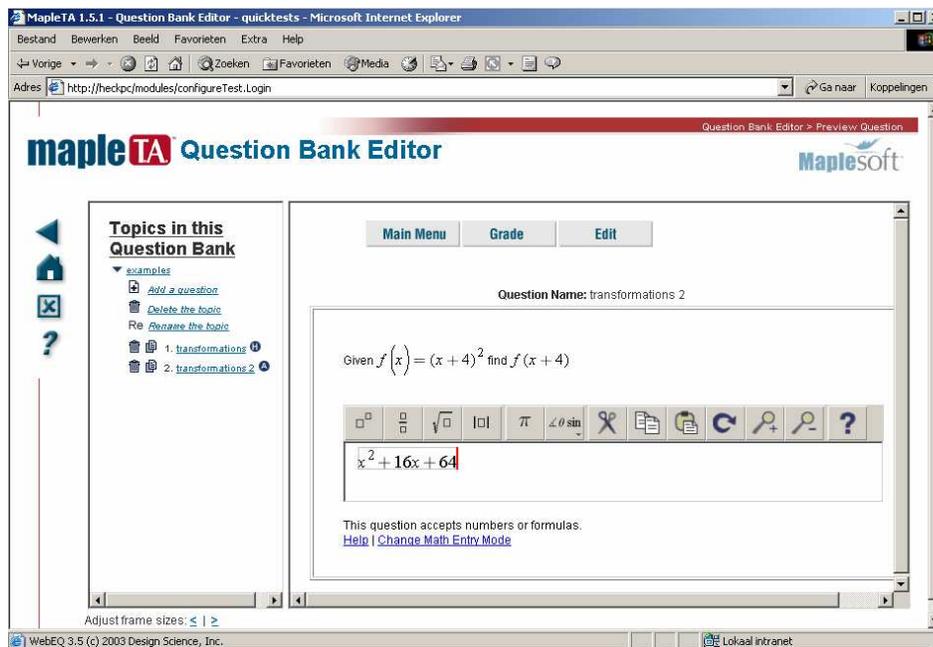
1. Navigate to the question that you created in example 3 and click the **Edit** button.
2. Click the **Add** button next to the heading **Algorithm**. This will take you to a screen in which you define the random variables.
3. Click the **Show Designer** button. This will take you to a point-and-click interface for designing random variables. (Once you become familiar with Maple T.A.'s syntax for designing random variables, you can simply type the randomization code into the text area and skip the **Show Designer** step.)
4. Here we will define two random variables (that replace the integers 3 and 5 in the existing question. In this example, we call them **a** and **b**. Enter the name of the variable in the first text box. Select the data type (in this case whole number) from the adjacent menu. Enter the lower and upper bounds of the variable in the next two text boxes (we choose ranges from 1 to 6). Click the **OK** label to the right. Do this for **a** and **b**. The screen will look like this.



5. When you are finished defining variables, click the **Save** button at the top of the screen.
6. Click **Next** in the following screen to get to the question text and correct answer area. Replace in the question text the number 3 by $\$a$ and the number 5 by $\$b$. Also adjust the correct answer to $(x + \$a + \$b)^2$. You can use the Math Editor or type manually in the boxes as shown in the screen shot below.



7. When you **Finish**, you are presented a randomly generated question with pretty-print mathematics and you can enter possible responses, as shown in the following sequence of pictures, to verify that everything works as expected.



2. Choose “Maple-graded” from the **Question Type** menu and type a description in the **Question Description** text box.
3. If you want to include math expressions or plots in the question text, select the check box next to **Use HTML formatting for the question text**.
4. Click the **Add** button next to the heading **Algorithm**. This will take you to a screen in which you define the random variables. Enter the following three lines of code:

```
$p = maple("randomize():RandomTools:-Generate(
           polynom(integer(range=-5..5),x,degree=3))");
$poly = maple("($p)+x^4");
$polydisplay = maple("printf(MathML:-ExportPresentation(
                       $poly))");
```

This cries for explanation:

You can create random Maple variables by statements of type

```
$<var_name> = maple("<maple command>");
```

For example, in the first line of code we create a random polynomial in x of degree 3 and with integral coefficients between -5 and 5 , using the Maple procedure `Generate` from the `RandomTools` package, and assign it to the random variable `$p`. The prefix `randomize():` is just required for a different instance of the polynomial to be generated upon each instance of the question.

In the next line of code we create a new random variable by adding the term x^4 .

Since we want a nice mathematical display of our randomly generated polynomial in the question part, we must introduce a random variable to store the MathML-formatted polynomial. As we have seen before, Maple generate the MathML-string with surrounding quotes. We use the formatted I/O routine `printf` form Maple to remove the superfluous quotes.

5. Click the **Next** button. This will take you to a screen in which you type the question text and the Maple code for grading the student response (see the screen shot below).
6. Enter the question by typing in the corresponding text box
 Compute the derivative of `$polydisplay` .
7. We let Maple compute the correct answer, as it will be shown in the report when you view the grading of the question. In order to avoid unforeseen male-syntax errors you are advised to put random variables in a Maple statement between brackets. So we enter the following line for computing the derivative as correct answer:

```
diff(($poly),x);
```

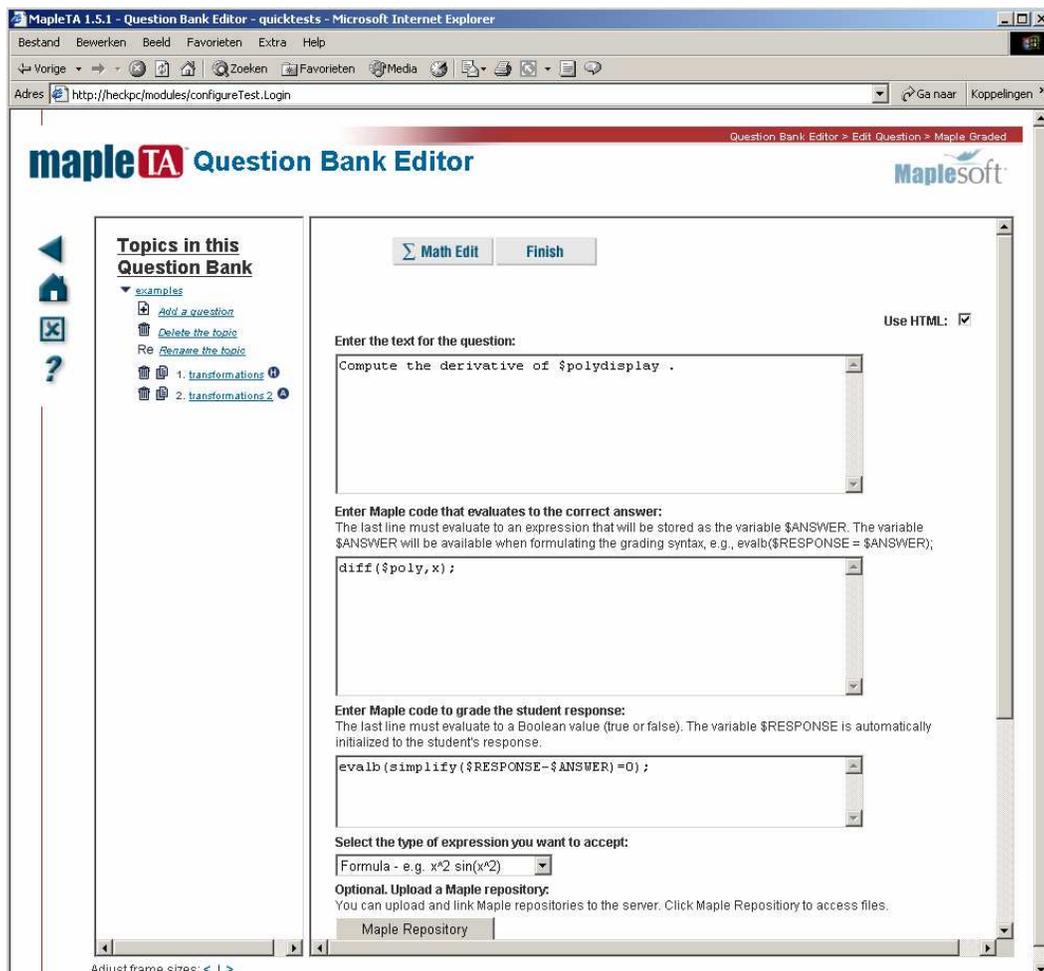
8. Type one or more lines of Maple code in the next text box that will grade the student’s response. In this example, a single line of code is sufficient:

```
evalb(simplify(($RESPONSE)-($ANSWER))=0);
```

The Maple engine executes the lines of code in order when the student submits the response. The system stores the student response in the variable \$RESPONSE, which you can use as a Maple variable anywhere inside your Maple code.

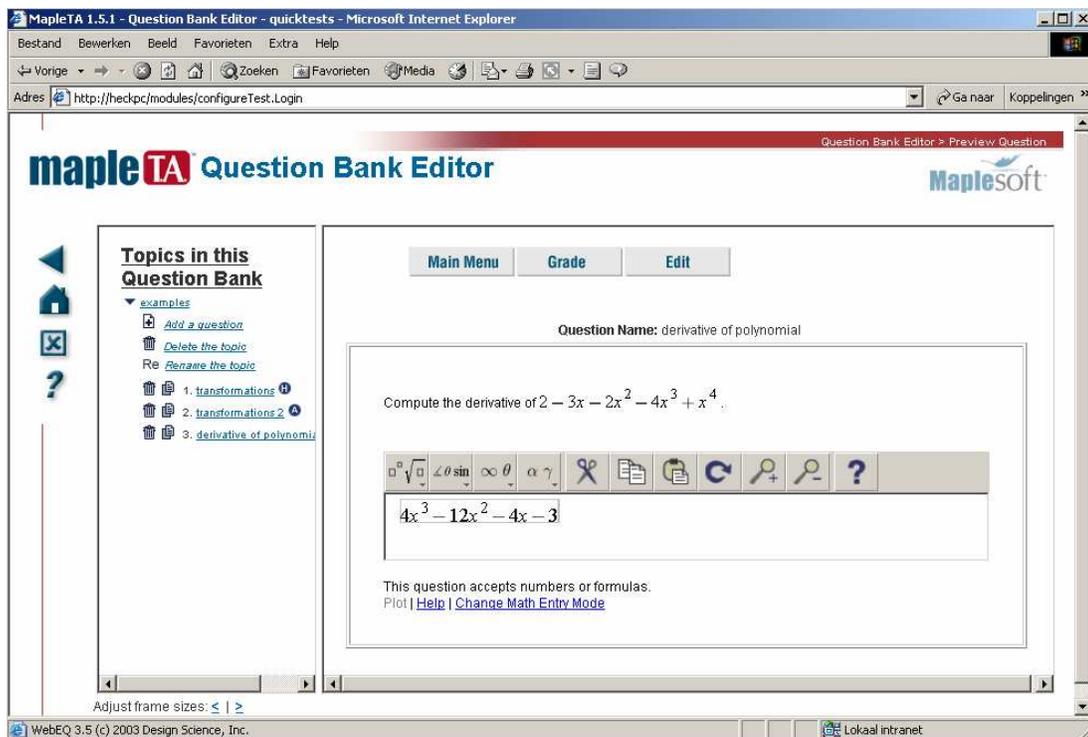
The last line of your Maple code must evaluate to **true** (for a correct response) or **false** (for an incorrect response). Maple's **evalb** command is frequently the best way to do this, but any Maple command that returns a boolean is allowed.

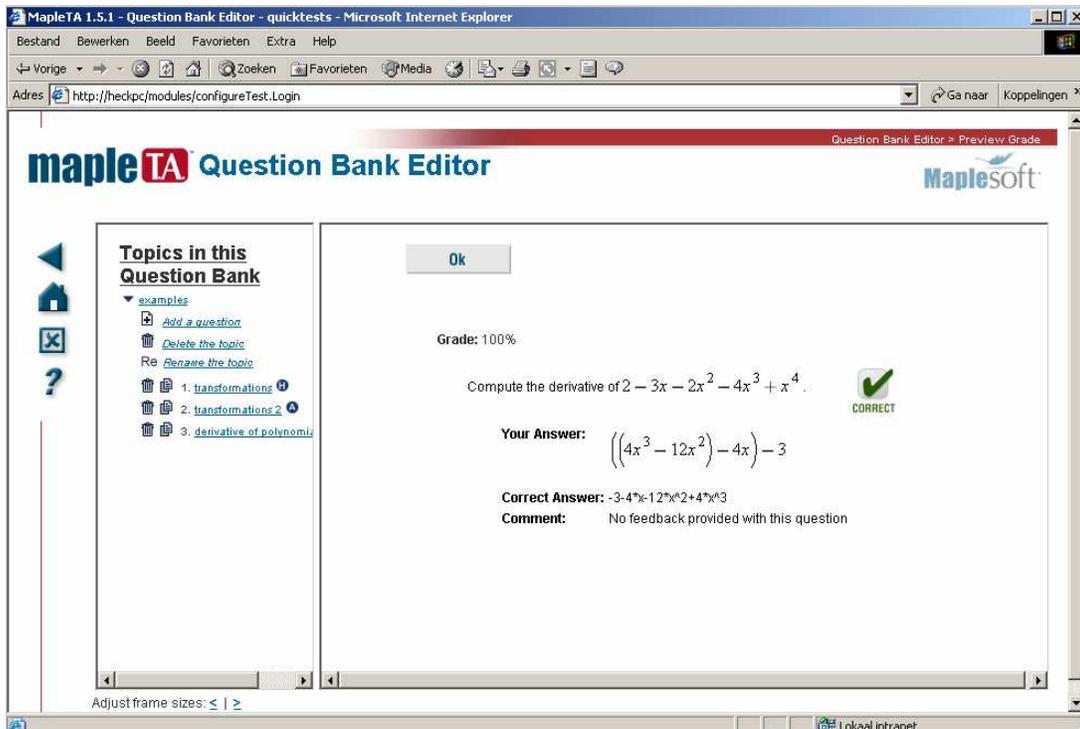
Caution: We strongly recommend that you test your Maple code in a Maple worksheet. Frequently you will find that your first version of Maple code is inadequate to give credit for all correct answers. In this example, if we had just used **evalb((\$RESPONSE) = (\$ANSWER))**, the system would only give credit for answers that are exactly the same as Maple would compute and Maple T.A. would reject all other responses, even if algebraically equivalent to the correct one. The reason is that **evalb** does not perform any simplification.



9. Select **Formula - e.g. $x^2 \sin(x^2)$** from the menu under the Maple code box (see the above screen shot). This option lets students enter basic algebraic expressions, either by using a symbol palette or by typing them in standard calculator

- syntax, e.g. $(2x/y + \sin(x y))(4x^2-1)$. When the student submits the response in this form, the system translates it into Maple syntax so that the Maple engine can apply your Maple code to it. If you want the student to enter equations, differential operators, integral signs, summations, vectors, matrices, DE's, or other advanced mathematical expressions, then select the option **Maple syntax – e.g. diff(2*f(x),x)**. This option allows a much wider range of student response types. However, it requires the student to enter the response using proper Maple syntax, because the system runs your Maple code directly on what the student types.
- Click the Finish button. This will take you to a screen in which you can test your question or start a new one.
 - Now pretend you are a student doing the assignment and enter a typical response in the box. The subsequent screens may look as shown below.





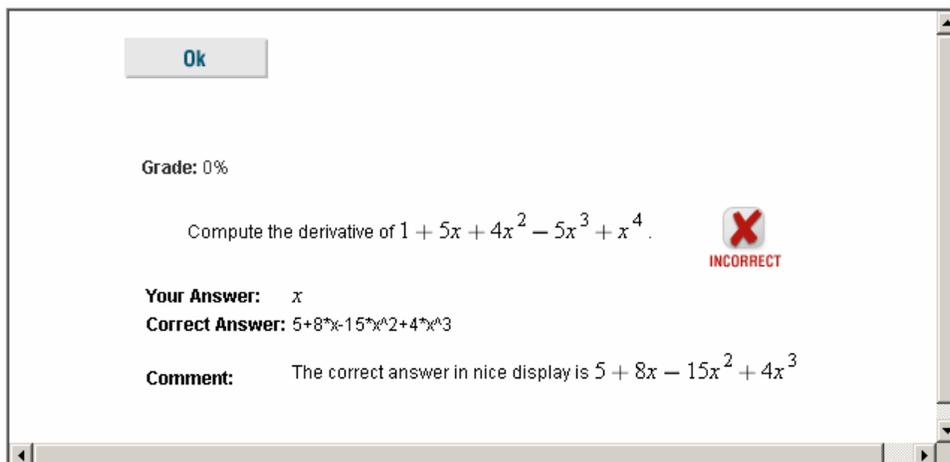
This example illustrates that we might want to have a nice MathML-formatted display of the answer. This can be arranged by adding an algorithmic variable for this purpose via the statement

```
$answerdisplay = maple("printf(MathML:-ExportPresentation(
                        diff(($poly),x)))");
```

In the textbox where you enter feedback you can put the line

```
The correct answer in nice display is $answerdisplay
```

The figure below shows a possible feedback in the grading, carried out in this way:



3.2. *LateX* Authoring

As was noted in the previous section about question types in Maple T.A., authoring a test bank via LaTeX can be done through the following five steps:

1. Write your question bank in LaTeX using a text editor;
2. Run LaTeX on your text files. Preview the output to ensure it is correct. You may perform multiple iterations of the above steps;
3. Using the Web-based conversion service at <http://qa.brownstone.net/latex2edu/>, convert your LaTeX-format question bank to Maple T.A. format;
4. Upload the converted question bank;
5. Install the question bank into your class Web site.

It is our experience that LaTeX authoring for an experienced LaTeX user is a quicker and simpler way to generate questions than by the Question Bank Editor. For access to the Maple T.A. plotter or for cross-platform formatting of scientific symbol systems (for example, chemical expressions) the LaTeX authoring method is preferred. It is also the last resort to create advanced questions, before starting to edit the plain-text script files. The ease of use of LaTeX authoring is illustrated by re-doing the examples that were given before in this section. By the way, the structure of a LaTeX document is almost always as follows:

```
\documentclass[12pt]{article}

% load style file
\usepackage{ed}

% optionally set the location for graphic files
\setImageBase{../classes/classname/figures/}

% questions are organized in topics; enter topic name
\begin{topic}{name}

% enter the code of the questions of given type
\begin{question}{type}
...
\end{question}

\begin{question}{type}
...
\end{question}

\end{topic}

\end{document}
```

In the examples below we only explain the coding within the question environment. All questions can be given a name, a comment, and a set of properties.

Notes:

- The `\name` field is used as an identifier for a question. It is recommended that you assign a name to every question. This will help you to identify questions when creating assignments from your question bank.
- If present, the `\comment` field is shown to the student instead of the `\answer` field if the question is answered incorrectly.
- The `\property` field takes two arguments. The first is the name of the property. The second is its value. You can choose any useful name. There can be more than one `\property` statement. When converted to the Maple T.A. format files, properties are converted to do-called information fields.

Example 1 & 2: a basic question on transformations

Given $f(x) = (x+3)^2$, find $f(x+5)$

- $(x+8)^2$
- $x^2 + 6x + 14$
- $x^2 + 7x + 14$
- none of these

The LaTeX code is pretty simple:

```
\begin{question}{NonPermutingMultipleChoice}
\name{example 1}
\qutext{\$Given \$f(x)=(x+3)^2, find \$f(x+5)\$}
\choice*{\$(x+8)^2\$}
\choice{\$x^2+6x+14\$}
\choice{\$x^2+10x+24\$}
\choice{none of these}
\end{question}
```

The **MultipleChoice** and **NonPermutingMultipleChoice** question types create a question and a list of choices, which includes one correct answer.

- You can have as many (or as few) choices as desired. Mark the correct choice with an asterisk.
- The order of the choices is permuted for the student. To force the choices to appear in the order you specify, use the **NonPermutingMultipleChoice** question type.

Example 3: a “Mathematical formula” question type

The LaTeX code that presents our question as a mathematical/scientific free-response question type for which the student’s response is automatically checked against the correct answer is as follows:

```
\begin{question}{Formula}
\name{example 3}
\qutext{Given \$f(x)=(x+3)^2\$, find \$f(x+5)\$}
\answer{(x+8)^2}
\end{question}
```

The **Formula** question type is the most general math question type. Answers can be any kind of formula, entered in calculator syntax, or using the system Symbolic Editor entry mode. Independent of entry method, answers are encoded as MathML, enabling the system to grade equivalent answers correctly. Answers can include variables and standard functions.

Notes:

- The `\answer` field is a calculated field. It must be in calculator syntax, not in LaTeX. Enter the answer as you expect the student to enter it (although equivalent answers are graded correct).
- If the answer is a number (that is, does not have any variables), it is recommended that you specify in the question text whether an exact response or an approximation is required. In the case of an approximation, the margin of error must be specified in your correct answer. This can be done by:
- Specifying a margin of error using the `?` operator. For example, if the answer field were `{2.34 ? 0.01}` any response between 2.33 and 2.35 (inclusive) would be graded correct. In other words, the number following the `?` operator is the margin of error.

Example 4: randomizing a question via algorithmic variables

We randomize the multiple-choice question to the following template

Question: given $f(x) = (x+a)^2$, find $f(x+b)$

Correct answer: $(x+a+b)^2$

where a and b are chosen to be integers ranging from 1 to 6.

For this purpose Maple T.A. provides *algorithmic variables*. Algorithmic variables may be incorporated into any other question. Main rules and recommendations for effective use of algorithmic variables are given below, illustrated by our example.

- The code which creates and manipulates these variables is contained within a `\code{ }` section of the question. Each variable in the code section is designated with a `'$'` symbol, such as `$m`, `$ans`, `$b2`, etc. The first character after the `'$'` symbol must be a letter. Each code statement is of the form

`variable = expression;`

The final semicolon is not optional. Note also that LaTeX comments (`%`) may not appear within the argument to `\code;` comments may be included using the `'comment'` control.

The usual arithmetic operations and functions can be used in the `'expression'` portion of the code statement. However, unlike most other programming languages, variables cannot be redefined in terms of themselves (for example, the statement `$a=$a+1;` is not allowed).

- The variables are inserted into other sections of the question using the `\var` control sequence. For example, `\var{a}` would refer to variable a .

We now show the code of our example:

```
\begin{question}{Formula}
\name{example 4}
\qutext{Given  $f(x)=(x+\text{\var{a}})^2$ , find  $f(x+\text{\var{b}})$ }
\answer{(x+\$a+\$b)^2}
\code{\$a=range(1,6); \$b=range(1,6);}
\end{question}
```

The above code chooses random integer coefficients a and b between 1 and 6. The answer is calculated in the `\answer` section.

Although the above example will work fine, there are several effects that may be undesired. First of all, the answer will appear to the student in the form " $(x+3+5)^2$ " (for example). This will provide some indication of the method for obtaining the answer, which may or may not be what the instructor desires. It is probably more common that only the final answer, $(x+8)^2$ in this case, is desired. To remove this problem, we'll rewrite this example as follows, using a new variable s for the sum $a+b$:

```
\begin{question}{Formula}
\name{example 4}
\qutext{Given  $f(x)=(x+\text{\var{a}})^2$ , find  $f(x+\text{\var{b}})$ }
\answer{(x+\text{\var{c}})^2}
\code{
  \$a=range(1,6);
  \$b=range(1,6);
  \$c=\$a+\$b;
  \$ans=mathml((x+\$c)^2);
}
\comment{The correct answer is \var{ans}.}
\end{question}
```

In the above code we have also introduced the algorithmic variable ans in which we store the MathML code of the correct answer as a comment, which is feedback provided to students upon grading.

In conclusion, the following recommendations should be followed:

- Make sure first of all that all choices of algorithmic variables will make sense in the question text and the answer, and that the responses will be graded correctly.
- Care must be taken if you want integers to appear in the question text or answer. Also, choose coefficients carefully to avoid expressions like "1x", "0x", "x+-3", or "-3/-4".
- Try to arrange the code so that if the student answers incorrectly, then the correct answer appears in a readable and aesthetic form. This will often involve the careful use of strings, as shown in the final example above.

Example 5: A randomised Maple-graded question with plots

We will present the LaTeX authoring of our previous example of computing the derivative of a 4th degree polynomials of type $x^4 + \text{random } 3^{\text{rd}} \text{ degree polynomial in } x$. We will extend the question (taken from (Sangwin, 2004a)) by showing the student in the question part the graph of the randomly generated polynomial on the interval $(-5,5)$. The student can also compare the graph of his/her response with the graph of the polynomial in the question part.

```
\begin{question}{Maple}
\name{example 5}
\type{formula}
\qutext{
  Compute the derivative of \var{polydisplay}.\newline
  The graph of this polynomial on the interval  $(-5,5)$  is given
  below.
  \drawMaplePlot*[500][500]{
    plot($poly,x=-5..5,y=-100..100,color=blue);
  }\newline
  You can also plot the graph of your answer on this interval and
  compare it with the original polynomial.
}
\maple*{expr := $RESPONSE; evalb(simplify(expr-($answer))=0);}
\code{
  $p = maple("randomize():RandomTools:-Generate(
    polynom(integer(range=-5..5),x,degree=3))");
  $poly = maple("($p)+x^4");
  $polydisplay = maple("printf(MathML:-ExportPresentation(
    $poly))");
  $answer=maple("diff(($poly),x)");
  $answerdisplay = maple("printf(MathML:-ExportPresentation(
    diff(($poly),x))");
}
\plot*{expr := $RESPONSE; plot([$poly,expr], x=-5..5, y=-100..100,
  color=[blue,red]);}
\comment{The correct answer is \var{answerdisplay}}
\end{question}
```

The Maple-graded question type adds the capabilities of the Maple 9 mathematical computation package to the testing and assessment. In Maple type questions, the student's response can be passed directly to Maple for grading. Moreover, questions can include Maple-generated plots, and students can preview plots of their answers (provided that the answer is an expression suitable for plotting).

Notes:

- Maple-graded questions use the **\maple** macro instead of **\answer** to specify an answer. The argument to **\maple** is the expression that stands for the correct answer, expressed in Maple syntax.
- Use the starred macro **\maple*** in case you want to provide raw Maple code. This macro may contain more than one Maple statement, but the last instruction

- must evaluate to a Boolean (hence the `evalb` command in the example). In this example we used the auxiliary variable `expr` in order that it does not matter whether the entered student's response ends with a semicolon or not.
- Maple syntax is much more stringent than syntax normally used in Formula type questions. For example, in such question `2x` is recognized as a product, but Maple requires `2*x`. To allow students to use EDU syntax in their responses, include `\type{formula}` in the question body. The student's response, will than be preprocessed, accepting the same syntax as 'formula' mode, before sending it to Maple. This means that the syntax is more forgiving, so that the response `2x` would be accepted as well as `2*x`, but advanced Maple commands will probably be rejected.
 - Authors may insert Maple-driven plots in the `\qutext` field with the `\drawMaplePlot` macro. Actually, we prefer to use the starred macro `\drawMaplePlot*` because it allows you to enter raw Maple code for the production of graphics. In this plot command you must specify the algorithmic variables again by `$(name)` instead of `\var{<name>}`. Optional arguments of the unstarred and starred `drawMaplePlot` macro are the width and the height of the plot (in pixels; default is 250)
 - Algorithmic variables are defined in the `\code` macro and you can use a Maple statement for this purpose by encapsulating it in the following way:


```
\maple( "<Maple statement>" );
```
 - The `\plot` and `\plot*` can be used to specify the plotting command to be used in plotting a student's result.

In the appendix we show a short test, called a quicktest, at Dutch high-school level (3vwo) that can be done by pencil and paper, and LaTeX code of the corresponding on-line test in Maple T.A. In this way we provide some more examples of real test questions. The next section also contains many examples of test items authored via LaTeX.

4. Advantages and Limitations of Computer-Assisted Assessment

The following lists of commonly accepted advantages and limitations of computer-assisted assessment (CAA) is compiled from the CAA Centre website in the United Kingdom www.caacentre.ac.uk and from (Rawles et al, 2002)

4.1. Advantages of Computer-Assisted Assessment

The use of CAA in education has a number of benefits. Here are some of the commonly expressed pedagogic advantages. CAA

- allows more frequent coursework assessment and many repetitions of formative assessments to aid student learning;
- gives rapid and specific feedback to students and lecturers;
- can be used to identify discrepancies between actual and expected performance;
- integrates well with web-based methods in delivering material;

- facilitates students to monitor their own progress and it promotes student self-assessment;
- enables new types of assessment that are impossible by pencil and paper (e.g. testing of ICT skills, use of multimedia);
- allows rapid testing of a wide range of topics within a body of knowledge;
- is a key step towards the use of adaptive testing to match the test to the student's ability;
- makes it possible to provides students with clues while making a test and to mark accordingly.

Common administrative advantages of CAA are:

- Marking is not prone to human error.
- Computerized marking of tests saves staff time.
- Large groups can be assessed quickly.
- Diagnostic reports and analyses can be generated.
- Aids with the transmission and administration of marks that can be automatically entered into information management systems and student records databases.
- Cheating can be reduced through the randomization of questions.
- CAA eliminates the need for double marking.
- Better re-use of assessment materials is encouraged.

4.2. Limitations of Computer-Assisted Assessment

Some of the limitations of CAA are:

- Construction of good objective questions and tests plus adequate feedback to students requires skill and practice, and so it is (initially) time-consuming (cf. (Greenhow and Gill, 2003)
- Because of the above, testing of higher order skills (commonly identified with the upper three levels of Bloom's taxonomy of learning objectives, see Figure 2 below) is not easy. But it is doable (see (McCabe et al, 2002), (Sangwin, 2003a,b and 2004b)).

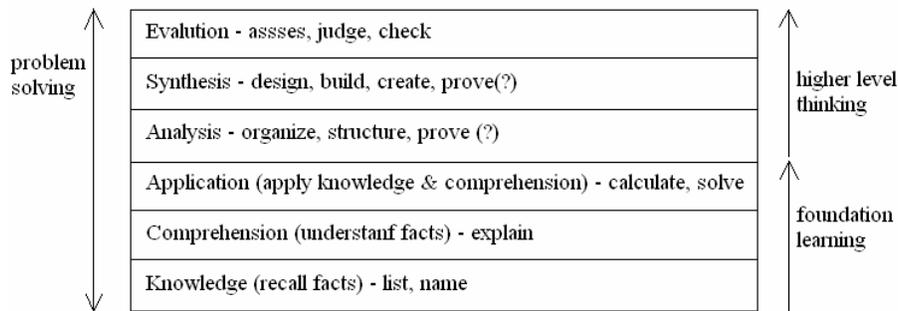


Figure 2. Bloom's Cognitive Learning Levels

- Implementation of a CAA system can be costly and time-consuming.
- Hardware and software must be carefully monitored to avoid failure during examinations.
- Students require adequate ICT skills and experience of the assessment type.
- Assessors and invigilators need training in assessment design, ICT skills and examinations management.

- o A high level of organization is required across all parties involved in assessment (academics, support staff, computer services, quality assurance unit, administrators.)

4.3. Strong Points and Weaknesses of Computer Algebra Based CAA, and in particular of Maple T.A. about the Creation of Test Items

As was noted before computer algebra based CAA broadens the scope of mathematical questions and the kind of feedback that can be given to students and teachers. It allows the creation of algorithmically generated symbolic expressions in questions, answers, and solution, a smart evaluation of students' responses that involve formulas and other complex mathematical structures typed in by students, and a display of mathematical expressions in a variety of output formats. On the other side, the design of high quality item banks is demanding work that has its own peculiarities. We give some more examples to illustrate these aspects.

Example 6: dealing with an infinite number of correct answers

Give an example of a function in one variable that is increasing on the interval (0,1)? Only specify the function body.

You can also plot the graph of your answer on this interval to verify your answer.

```

\begin{question}{Maple}
\name{example 6}
\type{formula}
\qtext{
  Give a function in one variable that is increasing on the interval
  (0,1)? Only specify the function body.\newline
  You can also plot the graph of your answer on this interval to verify
  your answer.
}
\maple*{
  expr := $RESPONSE;
  var:= remove(type, indets(expr,name), realcons);
  if nops(vars)<>1 then
    check := false;
  else
    var := op(var);
    check := verify(diff(expr, var), 0, 'greater_equal')
    assuming var::RealRange(Open(0),Open(1));
  end if;
  evalb(check);
}
\plot*{expr := $RESPONSE; plot(expr,x=-1..2);} \comment{
  If your answer is marked as wrong, you could have figured this out
  yourself by verifying that the derivative is positive on the given
  interval.
}
\end{question}

```

As you see, answer evaluation requires quite some Maple programming in this example. Response processing flow charts as suggested in (Naismith & Sangwin, 2004b) can make authoring of response handling easier in test items that require diverse feedback on incorrect answers.

You must realize that in principle the CAS-based method of equivalence testing of the student's response with the correct answer can also be used to check the student's answer for equivalence with an incorrect answer derived from a common mistake. In such cases a tailored feedback can be given to explain the student's particular misconception (cf. (Naismith and Sangwin, 2004a)). In Maple T.A., version 1.5, such tailored feedback is not built into the system (yet). But again, one must realize that providing tailored feedback is demanding work for the author of a test item.

On the positive side, all this work of evaluating free text answers with a computer algebra system in cases where many answers are correct leads in the end to a rather objective testing of students' free text answers for specific properties. Other examples of questions of this type, which can be implemented in MapleT.A. (see LaTeX coding below), are “Give an example of an even function on the interval $(-1,1)$ ”, “Give an example of a differentiable function $f(x)$ with a stationary point at $x = 1$ ”, “Give an antiderivative of the function $f(x) = \cos(x)\sin^2(x)$ ”, “Give an example of a real 2×2 matrix with trace 0 and determinant 1”. Note that in case of the last question about creating a matrix with certain properties you may want to check them separately and provide partial credit and/or tailored feedback as appropriate; see e.g. (Sangwin, 2003a and 2004a). This is not possible (yet) in Maple T.A.

Example 7: more sample questions with an infinite number of correct answers

We give the LaTeX code for the above test items

Give an example of an even function on the interval $(-1,1)$.

```
\begin{question}{Maple}
\name{example 7.1}
\type{formula}
\qutext{
  Give an example of an even function on the interval  $(-1,1)$ . Only
  specify the function body.\newline
  You can also plot the graph of your answer on this interval to verify
  your answer.
}
\maple*{
  expr := $RESPONSE;
  var:= remove(type, indets(expr,name), realcons);
  if nops(vars)>1 then
    check := false;
  else
    var := op(var);
    check := evalb(simplify(expr - eval(expr, var=-var))=0);
  end if;
  evalb(check);
}
\plot*{expr := $RESPONSE; plot(expr,x=-1..1);}
\comment{
  If your answer is marked as wrong, this is because the vertical axis
  is not a symmetry axis for the graph of your function.
}
\end{question}
```

Give an example of a differentiable function $f(x)$ with a stationary point at $x = 1$.

```
\begin{question}{Maple}
\name{example 7.2}
\type{formula}
\qutext{
  Give an example of a differentiable function  $f(x)$  with a stationary
  point at  $x=1$ . Only specify the function body.\newline
  You can also plot the graph of your answer to verify your answer.
}
\maple*{
  expr := $RESPONSE;
  evalb(eval(simplify(diff(expr,x)), x=1)=0);
}
\plot*{expr := $RESPONSE; plot(expr,x=0..2);}
\comment{If your answer is marked as wrong, this is because
  the derivative of your function in  $x=1$  is not equal to 0.}
\end{question}
```

Give an antiderivative of the function $f(x) = \cos(x)\sin^n(x)$, for $n = 1, \dots, 5$.

```
\begin{question}{Maple}
\name{example 7.3}
\type{formula}
\qutext{Give an antiderivative of the function \var{fdisplay}.}
\maple*{
  expr := $RESPONSE;
  evalb(eval(simplify(diff(expr,x)-cos(x)*sin(x)^($n))=0));
}
\code{
  $n = range(1,5);
  $f = maple("cos(x)*sin(x)^($n)");
  $fdisplay = maple("printf(MathML:-ExportPresentation($f))");
  $answer = maple("simplify(integrate(($f), x))");
  $answerdisplay =
    maple("printf(MathML:-ExportPresentation($answer))");
  $Int = maple("Student:-Calculus1:-Rule[change, sin(x)=u, u](
    Integrate(($f), x))");
  $integral = maple("printf(MathML:-ExportPresentation($Int))");
  $intermediateresult = maple("printf(MathML:-ExportPresentation(
    value(rhs($Int))))");
}
\comment{
  If your answer is marked as wrong, this is because the derivative of
  your function is not equal to \var{fdisplay}. \newline
  A correct answer is \var{answerdisplay}. It is computed by
  the method of substitution.\newline
  We try  $\sin(x)=u$  as a substitution. So,  $\frac{du}{dx}=\cos x$ 
  and  $\cos(x)dx=du$ . Hence, \var{integral}. The integral on the
  right-hand side can be computed; the result is
  \var{intermediateresult}. The final answer is obtained by
  backsubstitution  $u=\sin(x)$ : one antiderivative is
  \var{answerdisplay}, by adding a constant you get another one.
}
\end{question}
```

Note that the system only tests differentiability in the point $x=1$. A function like $|x|(2-x)$, which has a local maximum at $x=1$ but is not a differentiable, is marked as correct. The grading scheme actually only considers the question “Give an example of a function $f(x)$ that satisfies $f'(1)=0$ ”.

Also note that we have randomized the sample question and provided a comment that gives the student an appropriate worked out answer. This is the tailored feedback that can be provided with Maple T.A. You can also put the worked out solution in a solution field (`\solution{...}`), but then it is only available in a so-called *Study Session Assignment*. Study Sessions (which do not require students to register or log in) draw from a bank of questions and serve repeated practice to students one question at a time. These assignments are principally a teaching tool, whereas other types of assignments might be considered testing tools with results recorded in the student’s gradebook.

Give an example of a real 2×2 matrix with trace 0 and determinant 1

```

\begin{question}{Maple}
\name{example 7.4}
\qutext{
  Give an example of a real  $2 \times 2$  matrix with trace 0 and
  determinant 1. \newline
  Use a list-of lists to specify the matrix. For example, the matrix
   $\left(\begin{array}{cc} 1 & 2 \\ 3 & 4 \end{array}\right)$ 
  must be entered as \verb|[[1,2],[3,4]]|.
}
\maple*{
  M := $RESPONSE;
  if type(M, listlist) then
    M := convert(M, Matrix);
    check := evalb(
      [0,0]=StringTools[Search](["Matrix","matrix"], "$RESPONSE") and
      [LinearAlgebra[Dimensions](M)]=[2,2] and
      LinearAlgebra[Trace](M)=0 and
      LinearAlgebra[Determinant](M)=1);
  else
    check := false
  end if;
  evalb(check);
}
\comment{
  If your answer is marked as incorrect, this is maybe because you did
  not enter a  $2 \times 2$  matrix in the right style or because the
  matrix did not have the requested properties.
}
\hint{
  Consider the matrix  $M = \left(\begin{array}{cc} a & b \\ c & d \end{array}\right)$ .
  Then the trace of  $M$  is equal to  $a+d$  and the
  determinant of  $M$  is equal to  $ad-bc$ .
}
\end{question}

```

In this sample question we included a check whether a student uses in his/her response a Maple command containing the substring `Matrix` or `matrix`. We do this to prevent students from entering a Maple command to compute the matrix.

Example 8: avoiding computer algebra syntax in student's response

One point of consideration with free text answers in computer algebra based CAA is that you must arrange yourself that a user may not use an answer in the syntax of the computer algebra system. For example, when you ask a student to enter the derivative of say $\sin(x^2)$, then you take not `diff(sin(x^2), x)` for an answer, unless it is a test of Maple skills. In Maple T.A. the following example shows how you can write specific grading code to avoid acceptance of answers that use Maple syntax as correct.

Compute the derivative of $\sin(x^2)$

```
\begin{question}{Maple}
\name{example 8}
\qutext{Compute the derivative of  $\sin(x^2)$ .}
\maple*{
  evalb([0,0]=StringTools[Search](["diff","D"], "$RESPONSE")) and
  evalb(simplify($RESPONSE-$answer)=0);
}
\code{
  $answer = maple("diff(sin(x^2),x)");
  $answerdisplay = maple("printf(MathML:-ExportPresentation(
    $answer))");
}
\comment{Use the chain rule to differentiate this formula.
  The correct answer is \var{answerdisplay}.}
\hint{Use the chain rule:  $(f(g(x)))' = f'(g(x))g'(x)$ ,
  for differentiable functions  $f$  and  $g$ .}
\solution{
  The chain rule is:  $(f(g(x)))' = f'(g(x))g'(x)$  for differentiable
  functions  $f$  and  $g$ . In this exercise  $f(x) = \sin x$  and  $g(x) = x^2$ .
  So,  $f'(x) = \cos x$  and  $g'(x) = 2x$ . Therefore
   $(\sin(x^2))' = \cos(x^2) \cdot 2x = 2x \cos(x^2)$ .
}
\end{question}
```

If the response type is just an algebraic expression and you want to prevent Maple-savvy students from entering a Maple command to compute the expression, then you do not really need to program appropriate checks in the Maple code: it suffices to include the statement `\type{formula}`. When Maple syntax is needed, as it was in the last sample question of the previous example (giving a matrix with certain properties), then you must preprocess the student's response to check whether unwanted Maple routines are used.

The trick with `\type{formula}` to restrict the student's response to a Maple T.A. formula works because such a formula is far more restricted than a formula in Maple. We list the differences between these two kinds of formulas in Table 4 below:

<i>Maple T.A. Formula Syntax</i>	<i>Maple Formula Syntax</i>
<i>General:</i>	<i>General:</i>
Only numbers, symbols, and algebraic expressions built by arithmetic operators, trigonometric functions, inverse trigonometric functions, the square root function, natural and common logarithm, and the absolute value function.	Any Maple expression is accepted: also an equation, an inequality, a list, a boolean expression, a set, a vector, a matrix, etc.
<i>Numbers and Constants:</i>	<i>Numbers and Constants</i>
e (≈ 2.72), pi ($\pi \approx 3.14$), i (complex unit) 2.4E-3 (scientific notation, = 0.0024)	exp(1) (≈ 2.72), Pi ($\pi \approx 3.14$), I ($= \sqrt{-1}$) 2.4e-3 or 2.4E-3 (scientific notation) More constants: γ and Catalan (Euler-Mascheroni and Catalan's constant)
<i>Multiplication:</i>	<i>Multiplication:</i>
Multiplication operator * may be left out: 2x, (x+1)e^x, x y	Multiplication operator is always required: 2*x, (x+1)*exp(x), x*y
<i>Mathematical Functions:</i>	<i>Mathematical Functions:</i>
ln is the natural logarithm (base e) log is the common logarithm (base 10)	ln and log stand both for the natural logarithm; log[10] is common logarithm (base 10)
Exponential function exp is absent; use instead e^x	Exponential function is denoted by exp
Hyperbolic functions are: hypsin, hypcos, hypatan, hypsec, hypcsc, hypcot	Hyperbolic functions are: sinh, cosh, tanh, sech, csch, coth
Inverse hyperbolic functions are: archypsin, archypcos, archyptan, archypsec, archypcsc, archypcot	Inverse hyperbolic functions are: arcsinh, arccosh, arctanh, arcsech, arccsch, arccoth

Table 4. Differences between a Maple T.A formula and a Maple formula

If you incorrectly use a trigonometric function in the answer to a restricted mode question, you will usually be warned about this when you go to grade the test. For example, if you enter an equation in the answer of a formula question, you will get the message The character = cannot be used in a formula when you request grading of the answer.

Maple T.A. also has a *Restricted Formula* at its disposal: This simply means that the formula is not allowed to involve any trigonometric functions or their inverses. The main reason for introducing this formula type is because if a student is asked a question like “What is the sine of π ?” he or she could type either 0 or $\sin(\pi)$. The latter answer is probably not what is wanted in mathematical assessments; in a question of restricted formula type, such an answer is not graded as correct. If you incorrectly use a trigonometric function in the answer to a restricted formula question, you will usually be warned about this when you go to grade the test. In a Maple-graded question such unwanted

students' answers can only be prevented from grading as correct by writing more or less error-prone Maple code to track down unwanted answers. Such coding is time-consuming and has a steep initial learning curve: in (Delius, 2004), an average of 2 hours work per test item is reported for the AiM assessment system with undergraduate students as authors.

Example 9: avoiding the generation of impossible or trivial questions

When algorithmic variables come into play, attention must be paid to the choice of random parameters, in order to prevent the generation of impossible or trivial questions. Suppose that you want to present students a randomized question about the roots of a quadratic polynomial. There are various question subspaces: polynomials with real roots, with rational roots, or with integer roots, polynomials with distinct roots, zero roots, or repeated roots. If you wish to avoid some of these question subspaces, you must pay special attention to this in the authoring of a randomized question. The example below shows a technique of creating a question with certain properties, viz. a quadratic polynomial with one or two integer roots.

Determine the roots for the given polynomial

```

\begin{question}{Maple}
\name{example 9}
\qutext{
  Determine the roots for the polynomial \var{polydisplay}.\newline
  Write the roots in a set, using curly brackets with elements
  separated by commas.}
\maple*{
  expr := $RESPONSE;
  if not type(expr,set) then
    check := false
  else
    check := evalb(expr=$answer)
  end if;
  evalb(check);
}
\code{
  $a1 = range(-2,2);
  $a = if(ne(($a1),0),($a1),1);
  $b = range(-3,3);
  $c = range(-3,3);
  $poly = maple("expand(($a)*(x-($b))*(x-($c)))");
  $polydisplay = maple("printf(MathML:-ExportPresentation(
    $poly))");
  $factoreddisplay = maple("printf(MathML:-ExportPresentation(
    factor($poly)))");
  $answer = maple("{ $b, $c }");
  $answerdisplay = maple("printf(MathML:-ExportPresentation(
    $answer))");
}
\comment{
  Correct answer is \var{answer}.\newline
  Check that \var{factoreddisplay} is equal to \var{polydisplay}.}
\hint{Suppose that the polynomial can be written in the form
  $a(x-b)(x-c)$. Expand this expression and see what equality with

```

```

    the given polynomial means for the parameters $a$, $b$, and $c$.)}
\end{question}

```

But even when one is aware of the authoring problem of randomized questions, (Sangwin 2003a) confesses that setting mathematically impossible problems and choosing parameters, which subsequently result in a zero denominator, pop up regularly.

Partial Credit

If you want to give partial credit to a student's response, you must split in Maple T.A. the test items into multiple parts that can be marked individually. This may even tempt you to change the style of testing. Let us consider the following question template, i.e., the parameters a and b in the question are randomly selected, say $a \in \{-2, -1, 0, 1, 2\}$ and $b \in \{a+1, a+2, a+3\}$. The example is taken from (Sangwin, 2003b).

Give an example of a univariate quadratic polynomial with roots in a and b

You may want to give partial credit for the fact that a student

- enters a univariate polynomial p of degree 2
- $p(a) = 0$
- $p(b) = 0$

In Maple T.A. you can only give full credit or no credit to a test item; there is nothing in between. But you may change the test item into a multipart test item in the following style and give non-uniform weighting when the question is graded:

Example 10: constructing examples of objects satisfying certain properties

In the following questions we ask you to consider a univariate polynomial p and the following properties:

- (a) p is quadratic
- (b) $p(a) = 0$ (randomly chosen $a \in \{-2, -1, 0, 1, 2\}$)
- (c) $p(b) = 0$ (randomly chosen $b \in \{a+1, a+2, a+3\}$)
- (d) $p(0) = ab$ (generated by $a \times b$)

Give an example of a polynomial satisfying

1. (a)
2. (a) and (b)
3. (a), (b), and (c)
4. (a) – (d) inclusive
5. only (b) – (d)

Now you can grade each sub item on its own in Maple T.A. The LaTeX authoring of the question may be as follows.

```

\begin{question}{Multipart}
\name{Example 10}
\numbering{arabic} % (1), (2), (3), ...
\weighting{1,2,2,2,3}
% part (2)-(4) will be worth twice what part (1) is worth

```

```

% part (5) will be worth three times what part (1) is worth
\qutext{
  In the following questions we ask you to consider a univariate
  polynomial  $p$  and the following properties:
  \begin{description}
  \item{(a)}  $p$  is quadratic
  \item{(b)}  $p'(a)=0$ 
  \item{(c)}  $p'(b)=0$ 
  \item{(d)}  $p(0)=c$ 
  \end{description}
  Give an example of a polynomial satisfying
}
% (1)
\begin{question}{Maple}
\type{formula}
\qutext{(a)}
\maple*{
  expr := $RESPONSE;
  if not type(expr,polynom) then
    check := false
  else
    expr := expand(expr);
    var:= remove(type, indets(expr,name), realcons);
    if nops(var)<>1 then
      check := false
    else
      var := op(var);
      check := evalb(degree(expr,var)=2)
    end if
  end if;
  evalb(check);
}
\comment{A correct answer would be \var{ans1}.}
\end{question}
% (2)
\begin{question}{Maple}
\type{formula}
\qutext{(a) and (b)}
\maple*{
  expr := $RESPONSE;
  if not type(expr,polynom) then
    check := false
  else
    expr := expand(expr);
    var:= remove(type, indets(expr,name), realcons);
    if nops(var)<>1 then
      check := false
    else
      var := op(var);
      check := evalb(degree(expr,var)=2 and
                    simplify(eval(expr, var=$a))=0)
    end if
  end if;
  evalb(check);
}
\comment{A correct answer would be \var{ans2}.}
\end{question}

```

```

% (3)
\begin{question}{Maple}
\type{formula}
\qutext{(a), (b), and (c)}
\maple*{
  expr := $RESPONSE;
  if not type(expr,polynom) then
    check := false
  else
    expr := expand(expr);
    var:= remove(type, indets(expr,name), realcons);
    if nops(var)<>1 then
      check := false
    else
      var := op(var);
      check := evalb(
        degree(expr,var)=2 and
        simplify(eval(expr, var=$a))=0 and
        simplify(eval(expr, var=$b))=0)
      end if
    end if;
    evalb(check);
  }
\comment{A correct answer would be \var{ans3}.}
\end{question}
% (4)
\begin{question}{Maple}
\type{formula}
\qutext{(a)-(d) inclusive}
\maple*{
  expr := $RESPONSE;
  if not type(expr,polynom) then
    check := false
  else
    expr := expand(expr);
    var:= remove(type, indets(expr,name), realcons);
    if nops(var)<>1 then
      check := false
    else
      var := op(var);
      check := evalb(
        degree(expr,var)=2 and
        simplify(eval(expr, var=$a))=0 and
        simplify(eval(expr, var=$b))=0 and
        simplify(eval(expr, var=0)-($c))=0)
      end if
    end if;
    evalb(check);
  }
\comment{A correct answer would be \var{ans4}.}
\end{question}
% (5)
\begin{question}{Maple}
\type{formula}
\qutext{only (b)-(d)}
\maple*{
  expr := $RESPONSE;

```

```

if not type(expr,polynomial) then
  check := false
else
  expr := expand(expr);
  var:= remove(type, indets(expr,name), realcons);
  if nops(var)<>1 then
    check := false
  else
    var := op(var);
    check := evalb(
      degree(expr,var)<>2 and
      simplify(eval(expr, var=$a))=0 and
      simplify(eval(expr, var=$b))=0 and
      simplify(eval(expr, var=0)-($c))=0)
  end if
end if;
evalb(check);
}
\comment{A correct answer would be \var{ans5}.}
\end{question}
\code{
$a = range(-2,2);
$b = range(($a)+1,($a)+3);
$c = int(($a)*($b));
$ans1 = maple("printf(MathML:-ExportPresentation(x^2))");
$ans2 = maple("printf(MathML:-ExportPresentation((x-($a))^2))");
$ans3 = maple("printf(MathML:-ExportPresentation(
  2*(x-($a))*(x-($b)))");
$ans4 = maple("printf(MathML:-ExportPresentation(
  (x-($a))*(x-($b)))");
$ans5 = maple("printf(MathML:-ExportPresentation(
  (x+1)*(x-($a))*(x-($b)))");
}
\end{question}

```

Question chaining in a test is possible in Maple T.A., but it can only be realized by plain script text authoring, which is an awkward thing to do. We give one example in which a student is asked to compute an antiderivative of a monomial and then to differentiate his/her answer. If the antiderivative that the student enters is incorrect, the student's response to the request to differentiate the previous result can still be marked OK.

```

qu.1.topic=Integration and Differentiation@

qu.1.1.mode=Maple@
qu.1.1.type=maple@
qu.1.1.name=<b>Exercise 1</b>@
qu.1.1.id=ex1@
qu.1.1.editing=useHTML@
qu.1.1.algorithm=
  $n=range(2,5);
  $fcn=mathml(x^$n);
@
qu.1.1.question=
  <p class="noindent">Compute the antiderivative of $fcn .</p>
@

```

```

qu.1.1.maple=
  expr:=$RESPONSE;
  evalb(type(expr,polynom(anything,x)) and
        simplify(expr-$ANSWER)=0);
@
qu.1.1.maple_answer=integrate(x^($n),x);@
qu.1.1.comment=
  <p class="noindent">Your answer is marked incorrect
  when it is not the antiderivative of $fcn .</p>
@

qu.1.2.mode=Maple@
qu.1.2.type=maple@
qu.1.2.name=<b>Exercise 2</b>@
qu.1.1.editing=useHTML@
qu.1.2.question=compute the derivative of $response.ex1 .@
qu.1.2.maple=
  expr:=$RESPONSE;
  evalb(simplify(expr-$ANSWER)=0);
@
qu.1.2.maple_answer=diff($response.ex1,x);@
qu.1.2.comment=
  <p class="noindent">Your answer is marked incorrect
  when it is not the derivative of $response.ex1 .</p>
@

```

The trick is that we capture a student's response to the first question in the special variable `$response.ex1`, where `ex1` is the *id* tag that is set for question 1, and then include that response as part of the second question. Drawback of this method is that the special variable can only be used inside the question statement and answer/comment definition of the test item, but not inside the algorithm section (because the algorithm is initialized only once when the assignment is created). This makes it hard to use response variables in complex computations: in our example, this is the very reason why only Maple syntax can be used in the second question. For the very same reason, adaptive testing, i.e. changing the difficulty of follow-up questions based on answers of a student to earlier problems in a test, cannot be realized by response variables in Maple T.A.

Security issues

Server attacks by CAS instructions that take enormous amount of computer resources or even use system calls to hack the system are an issue for the design of a computer algebra based CAA systems (see also (Naismith & Sangwin 2004b)). In Maple T.A., for example, calling the Maple routine `system` or `ssystem` results in a message *Maple computation security error* at the Maple T.A. server. The system administrator can control the number of Maple processes that the server maintains for evaluations and he or she can limit the amount of computer resources available for grading a test item.

4.4. Summary of strong points of CAS-Based CAA

In summary the strong points of computer algebra based CAA are:

- Incorporating a computer algebra system (CAS) into a computer aided assessment (CAA) system allows students' free text responses to be tested objectively. Equivalence testing of expressions is well-developed in a modern CAS like Maple to

go well beyond the level of traditional test items like multiple-choice and multiple selection.

- A rich set of mathematical objects is available in questions and feedback. E.g. (interactive) mathematical graphs may be used in test items.
- A modern CAS like Maple has sufficient facilities for import, export, and display of mathematical expressions in an electronic learning environment.
- A CAS also allows for the instantiation of questions with random parameters and mathematical preprocessing;
- Tailored feedback is in principle possible; e.g., a CAS may be used to test properties of a student's answer and to provide appropriate feedback and (partial) credit.

4.5. Summary of Weak Points of CAS-Based CAA

In summary the weak points of computer algebra based CAA are:

- Authors of test item must be familiar with the underlying CAS.
- Intelligent feedback requires rather sophisticated programming.
- It is difficult to foresee the construction of an unsolvable or trivial problem when algorithmic question instantiation is applied.
- Questions must sometimes be rephrased in order that CAS-verification of answers is possible.
- It is difficult to guarantee correct marking of free-text responses of students and this requires rather sophisticated programming.

4.6. Some Weak Points and Bugs of Question Creation in Maple T.A. 1.5

Weak points or bugs are listed below:

- If you are an experienced LaTeX user you can use LaTeX authoring to quickly generate test items. But in this case, you depend on the conversion facility of *Brownstone EDU Campus*¹⁵. Alas, this facility does not recognize Maple-graded question type and give an error statement. In most case, this does not do any harm because the generated question bank will work fine if your Maple coding does not contain syntax errors. However, the author noticed a problem with the inequality operator $<>$: this operator is translated into HTML code `<>` which must be reset manually (or by preprocessing the test bank before uploading into Maple T.A.) to the Maple operator. Also, you cannot use the % symbol in the LaTeX sense. This would not be so important if most mathematics and physics teachers at universities would not use LaTeX as their primary method of authoring mathematics. Besides, for a commercial product one would expect that this service is provided by the producer MapleSoft, Inc. and not by a business partner. Who else can guarantee that this conversion service actually works and is maintained for its customers?
- Tailored feedback is limited by the fact that in the Maple code to verify an answer, you cannot define variables that can be used in other parts in a test item, e.g., in the comment field. If this was possible, you could program the feedback from within Maple.

¹⁵ www.brownstone.net/support/edu/latex2edu/

- o Question chaining is limited for the same reason as given before: you can hardly define variables that can be used across test items, e.g. in question and answer processing field.
- o Partial credit for answers that are incorrect only in a small detail (such as a sign) is not supported; you cannot set the grading otherwise than correct or incorrect.
- o Writing Maple code to rule out forbidden Maple syntax in a student's free-text response is a delicate thing and error-prone. A simple mechanism in which you specify forbidden word is not available.
- o One of the system requirements on the client side is the use of a Java-enabled browser. Nonetheless, the author did not get the advanced question type of *Sketching a Graph* working in Internet 6.0. This is probably a problem with the Java Runtime Environment.
- o Using Maple T.A. at a Dutch secondary school raises a language issue: there seems to be no facilities for translating Maple T.A. to different languages.

These above weak points and bugs with regards to the creation of test items just reflect that Maple T.A. is a system that is still under continuous development. These weaknesses, of which some are technical failures that can easily be repaired, will probably disappear as time goes. But even the current version of Maple T.A. has already wonderful things to offer to teachers who want to use intelligent on-line assessment of students in the context of mathematics education, be it at secondary school level or at university level. And we have not even mentioned all of the facilities! For example, the fact that Maple T.A. allows you to work easily with scientific constants and units of quantities makes the system useful for assessments in natural sciences, too.

5. Internet Resources.

We list some useful resources at the Internet that we have used for the preparation of this report..

www.caaconference.com The website at Loughborough University, UK, about the International Computer Assisted Assessment (CAA) Conferences.

<http://mathstore.ac.uk/> The website of the Maths, Stats & OR Network part of the [Higher Education Academy](#). Search for CAA.

<http://www.pass-it.org.uk/> The website of the Project on Assessment in Scotland – using Information Technology

<http://www.scrolla.ac.uk/> The website of the Scottish Centre for Research into On-Line Learning and Assessment

http://elearning.surf.nl/six/toetsen_en_assessment/ The Dutch SURF E-learning website of the SURF SiX expertise group on testing and assessment.

www.mapleapps.com The Maple Application Center; go to the *Maple TA* link under the *Education* header.

Acknowledgements

The author thanks CANDiensten and MapleSoft for letting him try out Maple T.A. An 'anonymous practice' assessment with all examples in this report can be done at URL <http://mapleta.can.nl/classes/heck>. Other assessments are also available at this website.

Appendix: Quicktest A (Translation of a Dutch Test)

Pencil & Paper: Quadratic Equations and the Quadratic Formula

Exercise 1

Solve: $(x - 11)^2 = 81$

$$x = \dots \text{ or } x = \dots$$

Exercise 2

“Any quadratic equation can be solved with the quadratic formula”. This statement is:

- ◇ true
- ◇ not true

Exercise 3

Which of the following equations has $x = 1$ as a solution?

- ◇ $x(x - 1) = (x - 1)(x - 6)$
- ◇ $x^2 + 5x - 1 = 6x - 1$
- ◇ $x(x - 2) = 0$
- ◇ $x^2 + 5(x - 1) = 1$

Exercise 4

The general solution of the (linear) equation $ax + b = 0$ is equal to:

$$\diamond x = \frac{a}{b} \quad \diamond x = -\frac{a}{b} \quad \diamond x = \frac{b}{a} \quad \diamond x = -\frac{b}{a}$$

Exercise 5

In the table below $y = f(x)$ is a quadratic function of x . The graph is a parabola with symmetry axis $x = \frac{1}{2}$. Fill in the blanks.

x	-4	...	2	5
y	9	3	3	...

Exercise 6

Solve the puzzle to the right.

length = ...
width = ...



Perimeter = 54
Area = 180

Exercise 7

How many solutions has the following equation: $(x + 3)^2 + 11 = 9$?

- ◇ 0 ◇ 1 ◇ 2 ◇ more than 2

Exercise 8

What are the solutions of the quadratic equation $ax^2 + bx + c = 0$?

$$\diamond x = \frac{b + \sqrt{b^2 - 4ac}}{a} \qquad \diamond x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$\diamond x = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \qquad \diamond x = \frac{b - \sqrt{b^2 - 4ac}}{a}$$

Exercise 9

How many solutions has the following equation: $x^2 + 2x + 2 = 0$?

- ◇ 0 ◇ 1 ◇ 2 ◇ more than 2

Exercise 10

The solutions of the equation $ax^2 + bx + c = 0$

- ◇ ... are the *zeros* of the function $f(x) = ax^2 + bx + c$
- ◇ ... are the solutions of the equation $ax^2 + bx = c$
- ◇ ... are the solutions of the equation $ax^2 + bx = -c$

Exercise 11

Give an example of a quadratic polynomial with exactly one root.

...

Exercise 12

Is the discriminant of the equation $(x - 11)(x + 36) = 0$ positive, zero or negative?

- ◇ $D > 0$ ◇ $D = 0$ ◇ $D < 0$

LaTeX code of online Quicktest A

```
\documentclass[12pt]{article}
\usepackage{ed}

\begin{document}

\begin{topic}{Quicktest A}

\begin{question}{Multi Formula}
\name{Exercise 1}
```

```

\qutext{Solve:  $(x-11)^2=81$ .\newline
      Omit in your answer the text  $x=$ .\newline
      In case there is more than one solution,
      Separate the solutions by semicolons.}
\answer{2;20}
\hint{Rewrite the right-hand side of the equation as a square.}
\end{question}

\begin{question}{TrueFalse}
\name{Exercise 2}
\qutext{``Any quadratic equation can be solved with the
      quadratic formula''. This statement is }
\false
\comment{If you allow complex numbers as solutions, then the quadratic
      formula provides all roots of a quadratic polynomial.}
\end{question}

\begin{question}{MultipleSelection}
\name{Exercise 3}
\qutext{Which of the following equations has  $x=1$  as a solution?}
\choice*{ $x(x-1)=(x-1)(x-6)$ }
\choice*{ $x^2+5x-1=6x-1$ }
\choice{ $x(x-2)=0$ }
\choice*{ $x^2+5(x-1)=1$ }
\end{question}

\begin{question}{MultipleChoice}
\name{Exercise 4}
\qutext{The general solution of the (linear) equation  $ax+b=0$  is
      equal to:}
\choice{ $x=\displaystyle\frac{a}{b}$ }
\choice{ $x=-\displaystyle\frac{a}{b}$ }
\choice{ $x=\displaystyle\frac{b}{a}$ }
\choice*{ $x=-\displaystyle\frac{b}{a}$ }
\comment{The adjective ``general'' in ``general solution'' means that
      special cases like  $a=0$  and  $b=0$  are not taken into account.}
\end{question}

\begin{question}{Inline}
\name{Exercise 5}
\qutext{In the table below  $y=f(x)$  is a quadratic function of  $x$ .
      The graph is a parabola with symmetry axis  $x=1/2$ .
      Fill in the blanks.

      \begin{center}
      \begin{tabular}{|c|c|c|c|c|}\hline
       $x$  & -4 & \blank & 2 & 5 \\ \hline
       $y$  & 9 & 3 & 3 & \blank \\ \hline
      \end{tabular}
      \end{center}
}
\begin{blank}{Numeric}
\answer{-1}
\end{blank}
\begin{blank}{Numeric}
\answer{9}
\end{blank}

```

```

\hint{Write for  $f(x)=ax^2+bx+c$  down what conditions the parameters
     $a$ ,  $b$  en  $c$  must satisfy under the given conditions of
     $f(-4)=9$ ,  $f(2)=3$  and  $x=\frac{1}{2}$  being a symmetry axis.
    Subsequently, solve the corresponding system of equations.}
\end{question}

\begin{question}{Inline}
\name{Exercise 6}
\setImageBase{../classes/quicktests/figures/}
\qutext{
    \begin{center}
        \begin{tabular}{cc}
            Solve the puzzle to the right.\\
            length = \blank, width = \blank & \image{rectangle_puzzle}
        \end{tabular}
    \end{center}
    Note: we assume that the length is not greater than the width.
}
\begin{blank}{Numeric}
\answer{12}
\end{blank}
\begin{blank}{Numeric}
\answer{15}
\end{blank}
\end{question}

\begin{question}{MultipleChoice}
\name{Exercise 7}
\qutext{How many solutions has the following equation:  $(x+3)^2+11=9$ ?}
\choice*{0}
\choice{1}
\choice{2}
\choice{more than 2}
\comment{A quadratic polynomial has never more than two roots.
    The sign of the discriminant is a good indicator
    for the number of solutions.}
\end{question}

\begin{question}{MultipleSelection}
\name{Exercise 8}
\qutext{What are the solutions of the quadratic equation
     $ax^2+bx+c=0$ ?}
\choice{\math{x}=\frac{b+\sqrt{b^2-4ac}}{a}}
\choice{\math{x}=\frac{b-\sqrt{b^2-4ac}}{a}}
\choice*{\math{x}=\frac{-b+\sqrt{b^2-4ac}}{2a}}
\choice*{\math{x}=\frac{-b-\sqrt{b^2-4ac}}{2a}}
\comment{The quadratic formula gives the roots of a quadratic
    polynomial.}
\end{question}

\begin{question}{MultipleChoice}
\name{Exercise 9}
\qutext{How many solutions has the following equation:  $x^2+2x+2=0$ ?}
\choice*{0}
\choice{1}
\choice{2}
\choice{more than 2}

```

```

\comment{A quadratic polynomial has never more than two roots.
        The sign of the discriminant is a good indicator
        for the number of solutions.}
\end{question}

\begin{question}{MultipleSelection}
\name{Exercise 10}
\qutext{The solutions of the equation  $a\,x^2+b\,x+c=0$ }
\choice*{\ldots are the \emph{zeros} of the function  $f(x)=ax^2+bx+c$ }
\choice{\ldots are the solutions of the equation  $ax^2+bx=c$ }
\choice*{\ldots are the solutions of the equation  $ax^2+bx=-c$ }
\end{question}

\begin{question}{Maple}
\name{Exercise 11}
\type{formula}
\qutext{Give an example of a quadratic polynomial with exactly
        one root.}
\maple*{
expr := $RESPONSE
if not type(expr,polynom) then
    check := false
else
    vars := remove(type, indets(expr,name), realcons);
    check := evalb(nops(vars)=1 and
                    degree(expressie,op(vars))=2 and
                    nops({solve(expressie)})=1 and
                    type(op(1,{solve(expressie)}), realcons) );
end if;
evalb(check);
}
\comment{If your answer is marked as incorrect, it may be that you
        have not entered a quadratic polynomial or because the
        polynomial has 0 or 2 distinct roots.}
\end{question}

\begin{question}{MultipleChoice}
\name{Exercise 12}
\qutext{Is the discriminant of the equation  $(x-11)(x+36)$ 
        positive, zero or negative?}
\choice*{$D>0$}
\choice{$D=0$}
\choice{$D<0$}
\hint{The discriminant of the polynomial  $ax^2+bx+c$  is equal to
         $b^2-4ac$ .}
\end{question}

\end{topic}

\end{document}

```

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