A VERSATILE ENVIRONMENT FOR ACTIVE LEARNERS AND TEACHERS

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INTRODUCTION

We all like to provide our students with opportunities to be involved in the active process of learning mathematics and science. We want them to collect and process real data, to develop and run mathematical models, to use interactive video, to work with internet-sources, and so on. We do this because we know from experience that active learners get a better understanding of concepts than those who are taught in the traditional style of lectures, textbooks, and exercises. Furthermore, technology allows for more realistic applications and we expect that this makes mathematics and science more attractive and challenging for our students, especially for those who are not motivated by abstract study.

However, this is partly obstructed because active students need many tools to do their work and they need time to learn these tools. At the AMSTEL institute of the University of Amsterdam we have developed a single versatile environment that offers many of the tools that you want in mathematics and science classes in an easy, integrated way: Coach. In addition, teachers have powerful, easy-to-learn and easy-to-use authoring tools to prepare activities for their students. They can select and prepare texts, graphs, video clips, mathematical models, and measurement settings and they can choose the right level according to age and skills of their students. In this way, Coach can be adapted for levels ranging from primary level up to undergraduate.

At present, Coach is mostly used in science and technology, where it is employed to carry out measurements, to process data, to set up computer models, to make predictions and compare these with real-world data, to control robots, and much more. Work is in progress to extend the mathematical facilities of the software so that it becomes a more complete educational environment for the exact sciences. In this paper we shall give a bird's eye view of Coach and we shall present two examples that illustrate the use of interactive video in challenging mathematics lessons.

BIRD'S-EYE VIEW OF COACH

Coach 5 is a versatile, activity-based environment for learning mathematics, science and technology at various student levels. Activities may contain:

- Texts with activity explanations and instructions.
- Pictures with illustrations of experiments, equipment, and context situation.
- Video clips to illustrate phenomena or to make video-based measurements.
- Measured data presented as graphs, tables, meters, or digital values.
- Models (in graphical or textual mode) to describe and simulate phenomena.
- Programs to control devices and to make mathematical computations.
- Links to Internet sites as extra resources for students.

Let us illustrate the variety of tools in Coach by a measurement activity, i.e., an activity in which a student collects real data with sensors. The experiment is easy to set up: the student chooses a hardware interface, e.g., CMA's CoachLab, Texas Instruments CBL™, or the CBR™ datalogger, and connects it with the computer. The chosen interface is represented graphically on the computer screen. The student connects a sensor to the hardware interface and drags the corresponding sensor icon in the software to the proper location on the virtual panel. The sensor immediately displays the measured value. Already during the measurement data can be presented in graphs and tables. In Figure 1, you see on the right side a screen dump of an activity in which Newton's Law of Cooling is explored via the CoachLab panel and a temperature sensor; the left side shows a CoachLab interface.

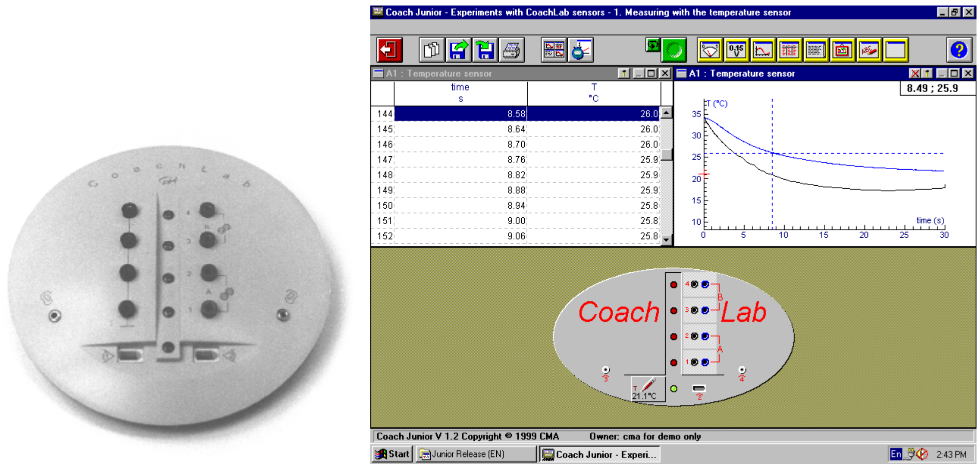

Figure 1. A sample measurement activity (right) and a CoachLab interface (left).

In this experiment, a temperature sensor is suddenly placed from a cup of lukewarm water into air. During 30 seconds, the temperature is measured. First the student has predicted in the plot window what he or she expects to happen (lower curve). Then the measurement takes place and the data are plotted (upper curve). The measured data are simultaneously tabulated in the upper left table window. A direct link between graph and table exists: the highlighted row in the table refers to the same datum as is marked in the diagram window as the intersection point of the cross hairs.

Further processing and exploring of measured data can be done by the student on his own: curve fitting shows immediately that the curve is not a parabola; exponential curve fitting gives on the other hand an excellent result that motivates a student to study exponential functions. Newton's Law of Cooling can be verified or rediscovered by computing numerical derivatives of temperature and plotting physical quantities against each other. A student can also do other experiments to answer questions like "Is there a similar law of heating?'', "What difference does it make whether cooling takes place in air or in ice-cold water?", or "How does the temperature graph depend on initial and final temperature?". In all these questions mathematics and physics go hand in hand.

INTERACTIVE VIDEO: BASKETBALL SHOOTING

A video activity allows collecting position and time data from digital video clips. Data are gathered by clicking on the location of items of interest in each frame of a movie. Data can be plotted and used for further analysis. We believe that video measurement offers great opportunities to study mathematics and science on the basis of real-world situations in challenging activities. Advantages of interactive video, compared to measurements with sensors, are:

- No experimental setup is required. This saves time, takes away many practical issues that must be dealt with in real experiments, and lowers costs of equipment.
- Processes that are difficult or impossible to measure with sensors can still be studied.
- It is not necessary to determine in advance what and how you are going to measure.
- Measurements can be done in an easy and quick way. Data can be verified later on and, if necessary, be corrected.

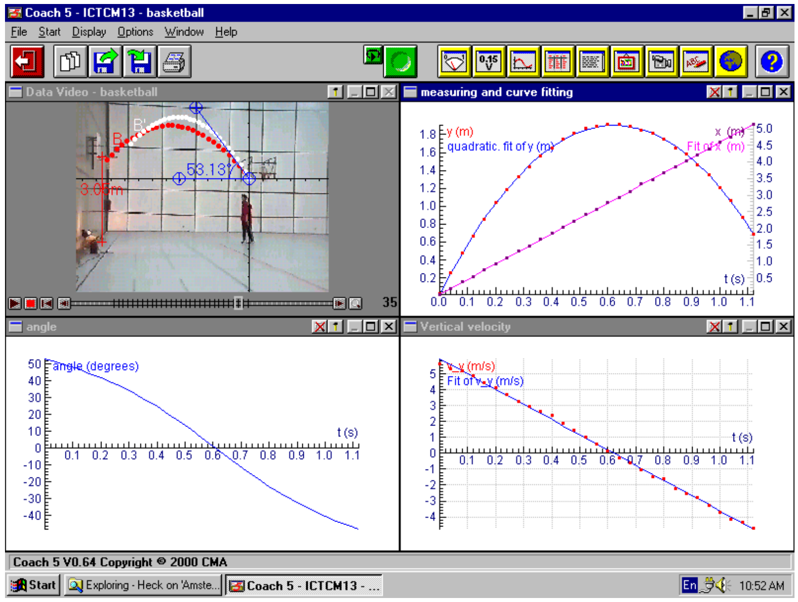

Figure 2. A sample video activity: exploring basketball shooting

In Figure 2, you see a sample activity in which basketball shooting is investigated. In the video clip, various locations of the basketball after the shot have been measured (lower trace of ball locations). But more has been done: the trajectory of the ball has been computed for the case it would have been thrown with the same velocity, but at a different starting angle. Several points of the computed trajectory have been drawn in the video clip (upper trace of ball locations). The other windows show what kind of mathematical tools come into play in this study: numerical differentiation to obtain velocity and acceleration of the ball, curve fitting to verify the parabolic shape of the trajectory, formula manipulation to get formulas of interest, and graphing functions. Students are really motivated in this activity to use mathematical formulas, graphs and functions. These are not abstract notions or a hobby of the teacher. On the contrary, they have become concrete and effective notions. They allow to answer questions like “How accurately does a player need to throw the ball in order to score from the free shot?”, “What parameters are important?”, and “Has a tall player an advantage over a short player in the free shot?”. Summarized in two words: meaningful mathematics.

MEASURING STILL IMAGES: THE CATENARY

Interactive video can also be used to investigate still images. Figure 3 shows a screen dump of a sample activity in which the shape of a perfectly flexible chain hanging under gravity is investigated. By collecting positions on the chain and trying a quadratic curve fit on the measured data, a student quickly finds out that the form of the chain is not a parabola (as Galileo erroneously claimed). At once, the simple question “How does a chain hang?” becomes a challenging problem. Students can follow in the footsteps of Huygens, Bernoulli, and Leibniz. These mathematicians solved the problem of the catenary at the end of the 17^{th} century, when differential calculus was discovered. It was one of the key problems in which they worked out and tested new mathematical techniques.

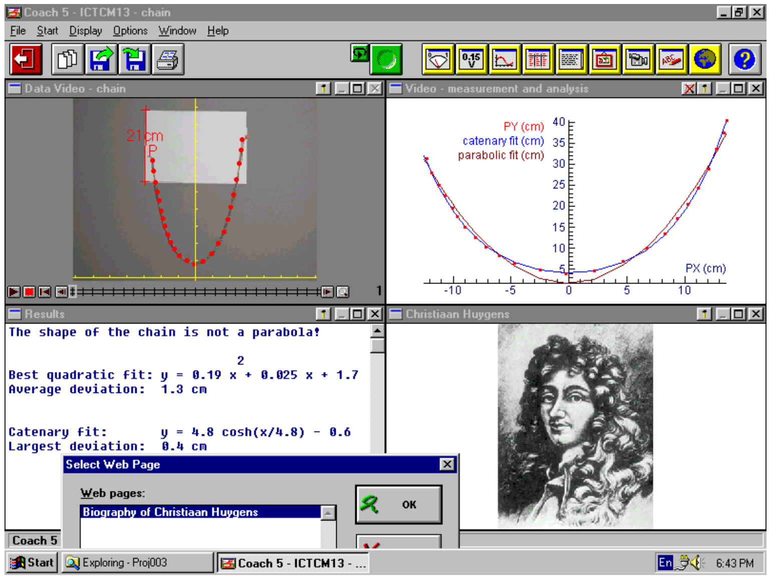

Figure 3. Measuring and computing the form of a chain.

The function $y(x)$ that describes the vertical position of a point on the chain as a function of the horizontal displacement x satisfies the following differential equation:

$$\frac{d^2y}{dx^2} = \frac{1}{c}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\ ,$$

for some positive constant c. If the coordinate system is chosen such that the origin equals the lowest point of the chain, the solution is as follows:

$$y(x) = c\left(\cosh(\frac{x}{c}) - 1\right) = c\left(\frac{e^{x/c}+e^{-x/c}}{2} - 1\right).$$

Of course, a student could try to match the data with this formula or try to find a value of k such that the numerical solution of the differential equation with initial values $y(0)=0$, $y'(0)=0$ matches the measured data. In the latter case, it is convenient to rewrite the 2^{nd} order ODE as a linear system of 1^{st} order ODEs. A modeling activity in Coach offers an environment to create this model and to do numerical computations. For creating models two types of editors are available: a graphical and a text editor. Based on the given model iterative computations are carried out. The results can be compared with experimental data from video- or sensor-based measurements.

One can take a different approach to investigate the catenary and first study a similar, but simpler problem: “How does a linkage system consisting of some identical bars hang under gravity?”. Figure 4 shows a screen dump of an activity in which a symmetric four-bar linkage system is explored. Measurements in the video clip reveal that the slopes of the two rightmost bars have a fixed ratio, viz., 1:3. Measuring in other video clips will convince a student that this does not depend on the length of the bars or how far they hang apart from each other. It turns out that many-bar linkage systems have the following fixed ratio of positive slopes: 1:3:5:7:9:… Elementary physics can explain this: the sum of forces and moments acting on a bar add up to zero. This simple observation allows to apply elementary geometry and to compute how a symmetric linkage system would hang.

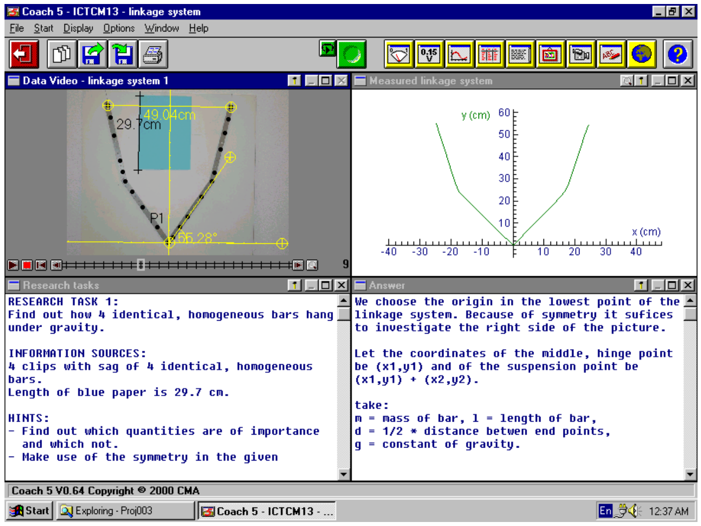

Figure 4. Exploring the shape of a four-bar linkage system.

The next step would be to approximate the chain with a linkage system of $2n$ bars, for large n, where the bars are of small length so that the total length of the linkage system remains the same. This could lead a student again to the differential equation that describes the catenary and to an algorithm to solve the equation numerically. We leave it to the reader to fill out the details.

The last approach offers an opening to the investigation of other systems of masses acting under gravity on a rope. The scope of investigation can be broadened to anchor catenaries or to shapes of suspension bridges and of arches of bridges and buildings. This would illustrate the use of common mathematical shapes and functions such as straight lines, parabola, exponentials and logarithms, and it would reinforce some of the ideas of calculus. But more important, it would bring the real world into mathematics lessons.

REFERENCES

For further information about Coach, sample activities, and educational research related to Coach we refer to the website www.cma.science.uva.nl/English/ .

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