Design and Implementation of an e-Class About Continuous Dynamical Systems

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Abstract. In 2008, a small team of university and secondary school teachers in the Netherlands jointly developed an e-class for students in their final pre-university year (age: 17-18 yrs) about continuous dynamical systems. The e-class is an innovative way of teaching and learning mathematics and science by way of web-supported instruction in a blended learning approach. The e-learning ingredients are: an on-line instructional text, Java applets for students' explorative work, Java exercise applets, video clips of worked-out examples, and descriptions plus (computer) worksheets of real experiments. We present the design of the e-class plus some of the ICT ingredients of the e-class and their potential for teaching and learning dynamical systems in terms of principled design approaches to multimedia learning and pedagogical arrangements.

1 Background

The design and implementation of an e-class about continuous dynamical systems and the future plans regarding this form of blended teaching and learning cannot been seen apart from curriculum reforms and changes in the Dutch education system that took place in secondary education in the Netherlands in the last two decades, and from the reforms still under way (For details about the Dutch education system we refer to documents of the Ministry of Education, `www.minocw.nl`). In particular, the implementation in 1998 of the so-called 'Second Stage', i.e., the upper secondary education, had a strong influence because it introduced many new concepts for education and new national curricula. From 1998 on, students were only allowed to choose one of the fixed subject combinations, called 'profiles'. For our discussion of an e-class about continuous dynamical systems only the Nature profiles, that is, Nature & Technology and Nature & Health are relevant. The reason is that only students from these streams may select the newly introduced optional 'Mathematics D' subject and the 'Nature, Life, and Technology' subject (henceforth abbreviated as NLT), for which the course on continuous dynamical systems has been developed. In this paper we concentrate mostly on the intended use of the e-class in 'Mathematics D', because the focus on deep mathematics is strongest in this subject. Participants of the e-class in NLT do more practical work.

Not only the education system and the list of curriculum subjects changed in the last decades, but also the educational goals, subject matter, teaching materials, and teaching methods underwent significant changes. This holds especially in mathematics and science education. The optional subject 'Mathematics D' aims at deepening and extending students' mathematical knowledge and skills through separate modules. Statistics and probability theory, dynamical systems, or a topic like complex numbers are examples of extensions. The deepening happens the students'engagement in topics taken from a scientific or technical context such as cryptography, graph theory and discrete mathematics, or in integrated science topics, which aim to create coherence between the different subjects of the sciences and to make the natural sciences and technology more attractive. The list of possible contents of 'Mathematics D' illustrates that this optional subject aims to tackle the problems that

- exact sciences have little attraction for students because the relevant school subjects are not challenging and seem disconnected with new developments in society, mathematics, science and technology;
- too few students choose a follow-up study or profession in these fields, as a consequence of this lack of appeal.

The implementation of the new curricula and 'Mathematics D' have to contend with difficulties like a rather short preparatory period and a shortage of qualified teachers with enough scientific baggage for teaching and developing the new mathematics and science subjects. To tackle these problems and to bridge the gap between secondary school and higher education at all levels, an network was started in 2007 in the region of Amsterdam, called the 'its academy' (`www.itsacademy.nl`). Within this network, staff members from about forty secondary schools and four higher education institutes jointly design and implement the new offer of curriculum materials. Because of the real danger that not enough students would choose 'Mathematics D' or NLT, and consequently teaching can only be done cost-effectively within a cluster of schools, the potential of e-learning in conjunction with classroom activities at cluster level and practical work in the 'its-labs' at the higher education institutes has been examined. In 2007, a pilot study of e-classes for 'Mathematics D' with a study load of 40 study hours was carried out on the topic of discrete dynamical systems. We refer to [9] for a report about the experiences with one of the e-classes from the pilot project. For the purpose of this paper it suffices to mention that it was overall a great success. Most of the students appreciated that an e-class offered them the opportunity to work with and learn from screen casts, to consult their peers and the teacher in the chat room, to plan more or less their working hours, to build executable computer models, and to learn a lot in a mixed and attractive approach.

Guided by what had been learned in the pilot study and during the use of the materials in school practice in 2008, a team of two secondary school teachers in mathematics, two international master students in the Master of Mathematics & Science Education [16] at the University of Amsterdam with mathematics as their discipline specialty, and a university researcher and developer in mathematics and mathematics education set out to develop an e-class about the mathematical approach to processes of change, and in particular to continuous dynamical models. These are models of systems that can move continuously from one state into another. In the e-class the main focus is on models in the form of a differential equation

$$\frac{dx}{dt} = f(x,t), \quad x(t_0) = x_0,$$

where x stands in the model for the state of system that can be expressed as a quantity (temperature, position, concentration, etc.), which changes with time t. Lessons include not only methods for solving a differential equation in an exact or approximate manner, but also give students the opportunity to analyze systems without actually solving them and to get acquainted with modern applications of the theory.

In this paper we discuss the design and implementation of the e-class, and we elaborate on the following ICT ingredients of the web-supported blended learning approach:

- The integral on-line text explaining the basics of ordinary differential equations and aiming at an increase of students' understanding about how professionals such as mathematical biologists, physicists, and applied mathematicians use dynamical systems to investigate real phenomena;
- The Java applets for students' explorative work (for example, for plotting of line element fields, solution curves, phase portraits, etc.);
- The Java exercise applets in which students can solve randomized exercises online in a step-by-step approach while the environment checks correctness of steps made;
- The video clips that help students in their learning of mathematics by provision of worked-out examples and sample exercises;
- The video clips (screen casts) with instructions for learning to work with accompanying software such as the modeling tool of Coach and the Java applets for exploration or practicing;
- The descriptions and (computer) worksheets of real experiments that students can do at home or at school and that introduce them to applications and model building. The real experiments spice up the lessons and relate the to-be-learned mathematics with real world phenomena such as the change in time of the head of a beer, the outflow of water from a bottle, biking and sprinting, and so on.

2 What Does the e-Class Look Like?

We discuss in this section the mathematical contents of the course, the instructional set-up, and some of the ICT aspects and design choices concerning the e-class.

2.1 Mathematical Content of the e-Class

Students who participate in the e-class are expected to be mature from mathematical point of view, in the sense that it is assumed that they already have knowledge and skills about derivatives and antiderivatives. This make the mathematical content of the e-class viable for students in their final school year. The content consists of the following parts:

1. Homogeneous linear first-order differential equations in one dimension;
2. Homogeneous nonlinear first-order differential equations in one dimension;
3. Nonhomogeneous linear first-order differential equations in one dimension;
4. Nonhomogeneous linear high-order differential equations in one dimension;
5. Autonomous linear first-order differential equations in two dimensions;
6. Autonomous nonlinear first-order differential equations in two dimensions;
7. Chaotic systems.

However, it must also be noted that the content and set-up of the e-class is flexible and may consist of a selection of the above elements. Students participating in an NLT e-class will only do the basic parts 1-3. They may focus less on the mathematical theory, but instead do more practical work and applications. For example, these students can study parts 4 and 7 through computer experiments. In this set-up the minimum study load is 40 hours; Students who participate in a 'Mathematics D' e-class may extend the minimum programme (parts 1-4, 7), which has a study load of 40 hours, with a further study of autonomous 2-dimensional systems (parts 5 and 6, also with a study load of 40 hours). However, this is only possible if students are familiar with complex numbers and/or basics of linear algebra. This brings an extra study load of 40 hours, for which there is also space in the examination programme. This way, the study load of the e-class on continuous dynamical systems can range from 40 to 120 hours. A study guide for teachers explains the options, proposes a time schedule, and gives suggestions for assessment.

2.2 Web-Supported Instruction in a Blended Learning Approach

One of the ideas behind the set-up of an e-class was to develop a rich electronic learning environment equipped with study guides, digitized lesson materials, software for learning and doing mathematics and science, video instructions, animations, (self-)assessments, communication tools for students and teachers to chat and discuss, and so on. The blended approach, here considered as a combination of online learning and face-to-face education at school, is central. Various methods of delivery of instruction are used in such way that schools, where there are only few students who choose 'Mathematics D' or NLT, are still able to offer the course effectively by participation in a cluster of schools. Actually, the broader definition of [11] for blended learning in higher education — "learning that is facilitated by the effective combination of different modes of delivery, models of teaching and styles of learning, and founded on transparent communication amongst all parties involved with a course." — seems to us applicable to the e-class setting for secondary schools. We will adopt this definition, acknowledging that there is no universally accepted definition of blended learning [2, 23] and that there are some serious doubts about its conceptual integrity [19].

In the rest of this article we look into the different modes of delivery, models of teaching, and styles of learning.

The set-up of the e-class is both simple and complex. All course materials are available online in a Sakai-based virtual learning environment (henceforth abbreviated as VLE). There is one master copy of the e-class, from which clones are constructed for use by real classes. The teacher of a cloned class can adapt the lesson materials to the ability level of his or her students, or give it a more personal touch. The choice for Sakai [21] was based on the fact that almost full responsibility for the e-class can be transferred to the individual teacher who can, for example, make his/her own assignments, invite students and colleague teachers, and so on. Instructions for students the regarding the mathematical software for dealing with differential equation and for giving explanations of sample exercises are preferably given by means of screen casts created and made available in the VLE. A screen cast is a digital movie in which the setting is partly or wholly a computer screen and in which audio narration describes or explains the story on the on-screen action. This way, students do not need separate instructions on tool use or long manuals on how to use each tool. Students are requested to hand in their answers to some of the online assignments through the request-and-delivery system for exercises inside the VLE: this homework could be a Word document, an Excel sheet, a result file of a computer simulation made with the modeling tool of Coach [10], or whatever appropriate digital document. For personal transfer of documents between a student and the teacher (for example, to get dedicated help or advice on a task, or for the simple reasons that homework could not be delivered within the scheduled time frame), the drop box facility of the VLE is convenient. Students can use the chat tool in the e-class to keep in touch with their peers and the teacher during the course: they can discuss exercises, ask each other for further information or explanation, and so on. And last but not least, students are still expected to meet the teacher and their peers at school and ask questions, discuss homework, collaborate, and so forth. After all, face-to-face contact in education remains highly valued! The assessment of students'work is expected to be in accordance with the way they have studied the subject content, i.e., through written tests, practical work, partial credit for homework, etc.

The complexity of the e-class setting lies in the well-known fact that mere use of ICT in education does not lead to good quality of learning. The major challenge for a long time has been how to find the right blended learning arrangement with regard to content, knowledge construction, and communication within a regular curriculum setting [12, 14, 15] that leads to meaningful learning, and how to initiate, sustain, and structure interaction and enhance its quality [5]. In the design of the e-class we have based our decisions mainly on the teachers' experiences, research-based design principles of multimedia learning (for a collection of papers see [18]), and principles of learning sciences for the design of pedagogical arrangements (for a collection of papers see [22]).

Fig. 1 is a screenshot showing the content menu bar to the relevant parts of the mathematics course, which is always at the students' disposal, and a fragment of the lesson text. This fragment illustrates that the online instructional material is intended to be highly interactive. In this particular case, an applet, created with the 'Easy Java Simulations' (EJS) software tool [3], offers the opportunity to draw line element field of differential equations and plot solution curves (by clicking in the drawing canvas). A student can first predict a particular solution in the format of a mathematical formula and then can compare the plot of his or her prediction with a computed solution curve.

The last line in the screen shot in Fig. 1 illustrates the mathematical formatting in MathML. This has been realized by using ASCIIMathML [1]. Some webbrowsers need MathPlayer for the display of the mathematical formulas and users of the e-class are informed about these mild system requirements.

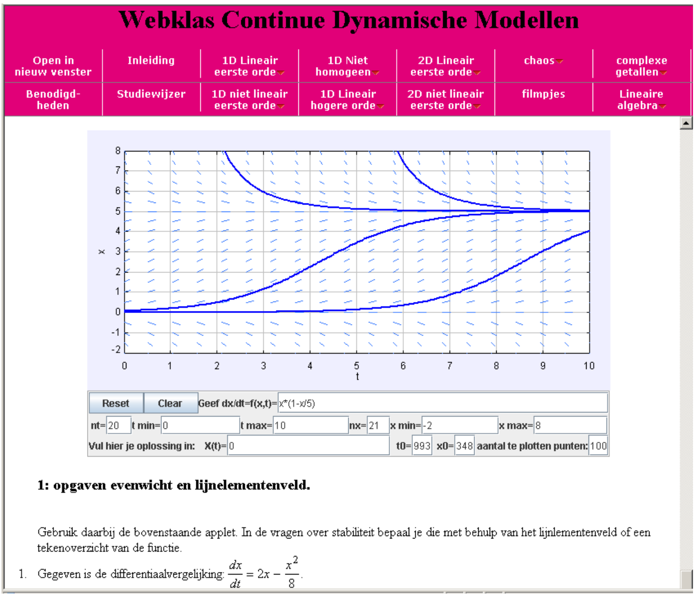

Fig. 1. Screenshot of a lesson fragment in the e-class illustrating the use of an EJS applet.

Another illustrative example of user interactivity is shown in Fig. 2, in which GeoGebra [4] is used to create a graphical representation of a adjustable complex number. The hope and expectation is that students get a better understanding of the geometrical meaning of complex numbers by exploration with such applets. The screenshot also illustrates the authors' intention to make the layout of the instructional material similar to a classical textbook: for example, frames with colored backgrounds are used to summarize or highlight the essentials.

For online learning and practicing algebra and calculus, the e-class makes use of exercise applets, developed in collaboration with researchers and developers at

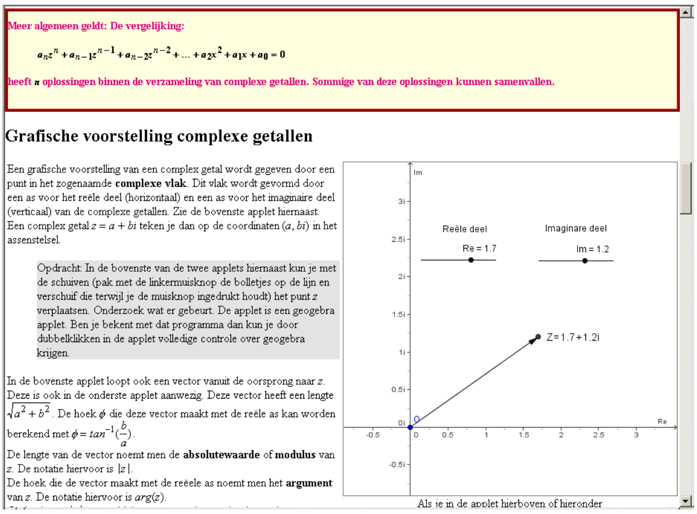

Fig. 2. Screenshot of a lesson fragment in the e-class illustrating the use of a GeoGebra applet.

the Freudenthal Institute in the Netherlands. The reader is referred to [7] for a detailed discussion of the design and use of such applets. Here it suffices to show some screenshots. Fig. 3 illustrates how students can do some calculation step by step on the computer; in this particular case it is a rehearsal of the substitution technique in computing an integral. In each step the student's answer is validated so that the student knows whether (s)he is still on the right track.

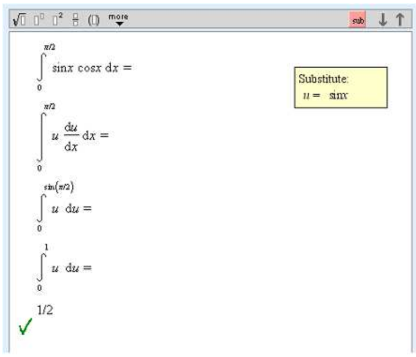

Fig. 3. Screenshot of an exercise applet about integration.

Such exercise applets are used in the introduction to verify if the students have the required mathematical knowledge and skills, and further on in the course to practice newly learned techniques of solving differential equations. Fig. 4 shows a screenshot of an applet in which the student is taken by the hand to go through solution steps; Fig. 5 shows a screenshot of an exercise where the student is free to choose how to solve a differential equation analytically.

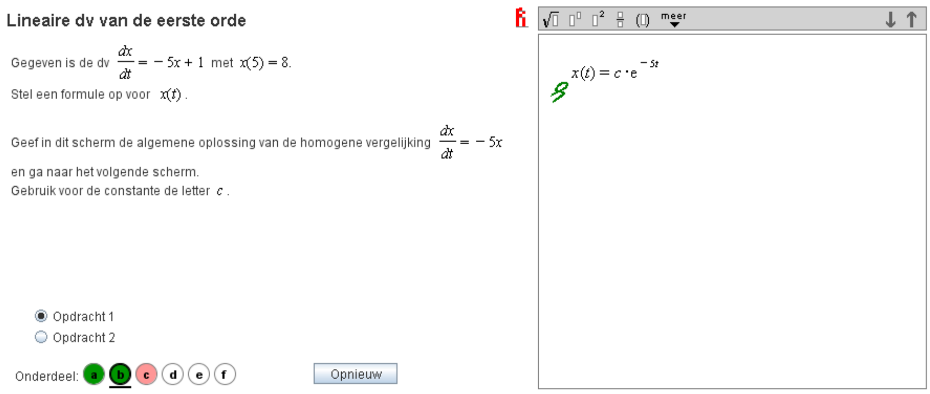

Fig. 4. Screenshot of a guided exercise applet about differential equations.

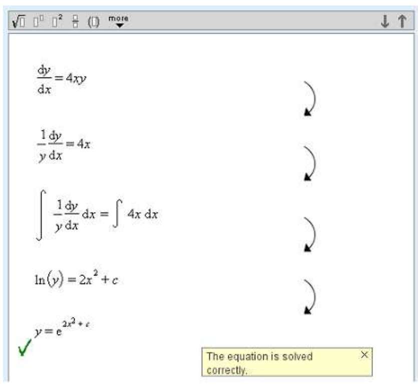

Fig. 5. Screenshot of a free-hand exercise applet about differential equations.

2.3 Use of Executable Computer Models

The focus of the e-class about continuous dynamical systems is not solely on analytical solution methods. Students should realize that the most interesting mathematical models of real phenomena that can be expressed in the form of differential equations can only be solved numerically. Easy Java Simulations [3] software has been used in illustrative examples in the lesson text, but when students have to create computer models from scratch they are expected to use the modeling tool of Coach [10]. Fig. 6, taken from [8], shows a graphical model representing the Keller model of sprinting [13], which is the following initial value problem for the speed of the sprinter:

$$\frac{dv}{dt} = F - \frac{v}{\tau}, \quad v(0) = 0.$$

Here v is the sprinter's speed represented by a rectangular icon, and F and τ are constants. The acceleration a in the graphical model is represented by an inflow that depends on v, F, and τ. Our experience is that high school students can work well with such system dynamics based modeling tool, certainly when they can make the link between the graphical representation and the mathematical notation of differential equations. They also enjoy doing this.

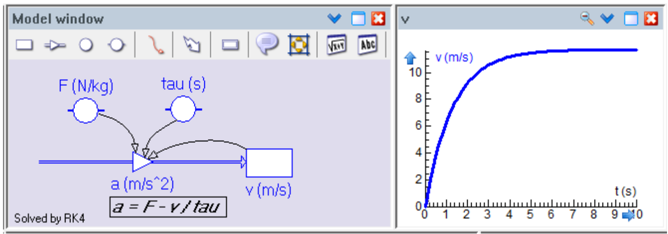

Fig. 6. Screenshot of a graphical model and simulation of a sprint.

2.4 Screen Casting in the e-Class

The chatroom in the VLE is a communication channel in the e-class in which students help and motivate each other. The main idea is that students working at home could overcome difficulties (for example, with the ICT tools or with the underlying mathematics) through discussions with their peers and the teacher in the chatroom.

Besides the chat facility, screen casting is the killer application, appreciated by all students. The aim of screen casting is to allow students to:

- see and hear the thinking and explanation of a teacher when doing a task;
- watch whenever and as many times they want;

- go stepwise through an instruction;
- look at the whole process and not only at the final result of an activity.

The main benefits of the screen casts in the e-class are that they support student learning outside the classroom, function in case of software instruction better than user manuals or support pages, and help the teacher in the sense that (s)he can make optimal use of the face-to-face contact time with students. The success of this form of screen casting can be underpinned by research-based principles of multimedia learning. We refer the interested reader to [9].

2.5 Real Experiments in the e-Class

The e-class about dynamical systems also emphasizes doing real experiments regarding phenomena that can be mathematically modeled by differential equations. The experiments must be simple enough so that students can do them outside school. An example is the collapse of beer foam, which can be modeled by an exponential decay model. When also the liquid content of the glass is taken into account a linear system of differential equations comes into play. The interested reader is for a detailed description of the modeling of beer collapse referred to [6]. Here it suffices to mention that a video recording of the phenomenon with a webcam will do the job in combination with video analysis and modeling in the Coach environment. Fig. 7 shows a screenshot of a video analysis of the head of an alcohol-free beer in a cylindrical glass. Data collection took place by automated point tracking and an exponential regression curve is plotted with the measured data in the background.

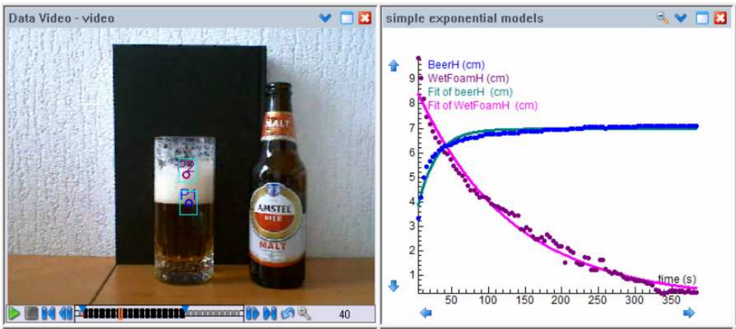

Fig. 7. Screenshot of a video analysis of the head of an alcohol-free beer.

The exponential decay model of beer foam corresponds with the following ordinary differential equation:

$$\frac{d}{dt}\text{WetFoamH}(t) = -\frac{1}{\tau} \cdot \text{WetFoamH}(t),$$

where τ is the lifetime of the beer foam. The graphical representation in the modeling tool of Coach 6 is shown in Fig. 8. The decay model with the most suitable values of initial height and lifetime matches the measured data rather well, except for the first stage.

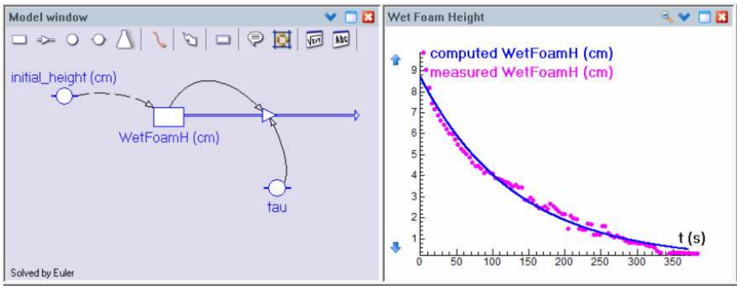

Fig. 8. Screenshot of the exponential decay model of beer foam height.

It goes without saying that many a student is enthusiastic about such applications of mathematics. We are of opinion that motivating and engaging students in mathematics activities, and having them think about the problem and try to test their ideas matter more than whether a model is perfectly correct from scientific point of view.

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