

# **AUTHENTIC STUDENT RESEARCH PROJECTS ON PHYSICS AND THE HUMAN BODY**

André Heck, Ton Ellermeijer, Ewa Kędzierska

## **ABSTRACT**

As part of the Dutch physics curriculum students are obliged to perform their own research project of approximately 80 hours. They are stimulated to choose the topic themselves (preferably with relations to two subjects, like physics and mathematics) and have a lot of freedom in the design of the research. Normally they work in small groups (2-4), and the project is carried out over a longer period parallel to the normal lessons. They are supposed to deliver a report and a presentation. Ideally students experience the stimulating aspects of doing research, and this is especially the case when the project is advanced and authentic, and even comparable with actual research done by researchers. In the last years we have developed several potential projects in the context of the human body. Examples are: the reflex of the eye on light and human gait. With the help of ICT (for data acquisition, video-analysis, modelling, data-analysis) the level and methods of the work of the students becomes quite close to the type of work of researchers in the field. This indeed has a very positive effect on students' motivation.

## **KEYWORDS**

Video recording and analysis, computer modelling, kinematics, gait analysis, electromyography, pupil light reflex

## **INTRODUCTION**

Many teachers like to provide their students with opportunities to be involved in the active process of learning mathematics and science in a context that is meaningful, motivating and challenging for the students. They want them to collect, process, and analyse data, to use digital images and videos, to develop and run mathematical models, and so on. They do this because they know from experience that motivated and active learners get a better understanding of concepts, methods and techniques than those who are taught in a traditional style of lectures, textbooks, and exercises. Technology allows students to work with high-quality, real-time data and it enables students to look at realistic applications of science and to do investigative work that resembles professional practice.

Small practical investigations (4 to 10 hours of work) and one large, cross-disciplinary research or design experiment (approximately 80 hours of work) are part of the Dutch examination programme of senior secondary education. In mathematics and science, students are expected to develop a broad range of research skills, which includes connecting real world phenomena with the scientific world, understanding the problems at hand and asking the right questions, making a project plan, designing and carrying out an experiment, and collecting, representing, analysing, and interpreting information. The basic idea is that students learn best about scientific research by actually doing research, or as Hodson (1992) has put it: "A major goal of practical work should be the engagement of students in holistic investigations in which they use the processes of science both to explore and develop their conceptual understanding and to acquire a deeper understanding of (and increased expertise in) scientific practice." For a thorough discussion of the role of scientific investigations in the science curriculum see Gott and Dugan (1995). The point made in this article is that in our opinion students need ICT tools that make authentic investigative tasks feasible and that enable them to work at an appropriate level.

However, the students' work is partly obscured because active students need many tools to do their work and they need time to learn these tools. Educational researchers and developers at the AMSTEL Institute of the University of Amsterdam put, for more than twenty years already, great effort in creating a single, versatile computer learning environment that offers many of the tools that one wants to use in mathematics and science classes in an easy and integrated way: its name is Coach. For more information and sample activities we refer to the website [www.cma.science.uva.nl](http://www.cma.science.uva.nl) and to some overview articles (Mioduszevska and Ellermeijer, 2001; Heck, 2002). Coach activities may contain:

- Texts with activity explanations and instructions;
- Pictures with illustrations of experiments, equipment, and the context situation, or to make image-based measurements;
- Video clips to illustrate phenomena or to make video-based measurements;
- Measured data presented as table, graphs, meters, or digital values
- Models (in graphical, equation, or textual mode) to describe and simulate phenomena;
- Animations of modelled phenomena;
- Programs to control devices and to do mathematical computations;
- Links to Internet sites as extra resources for students.

In addition, teachers have powerful, easy-to-learn and easy-to-use authoring tools to prepare activities for their students. They can select and prepare text, graphs, video clips, mathematical models, and measurement settings, and they can choose the right level according to age and skills of their students. Students, on the other hand, can use the same authoring tools in their research project to set up their own experiments, to create their own Coach activities and to use the results in the report and presentation of their research work.

In this paper we present illustrative examples of the use of the Coach learning environment in authentic student research projects and investigative practical work at pre-university level. In particular we discuss investigative work on human locomotion and on measurement of the pupil light reflex of the human eye. The investigative tasks and projects have been developed in our educational research and development that examines the possible contribution of ICT and real-life contexts to the realization of challenging investigations for students. This work provides input for the ongoing development of the activity-based environment Coach for learning mathematics, science and technology, and it results in study materials that stimulate and enable students to carry out investigation tasks at a high level. Software and students' activities are tested in practice. Regarding human locomotion, results come from

- a classroom experiment during regular physics hours with all students participating;
- a master class for interested pupils from various schools;
- a few case studies and research work carried out by interested students.

We interpret the authentic nature of the activities as the opportunity for pupils to work directly with high-quality, real-time data about human gait in much the same way movement scientists do. In essence, we try to make their mathematics and science learning resemble practice, in which investigations can often be characterized as being challenging, complex, open-ended, and cross-disciplinary, and as requiring a strong commitment of participants plus a broad range of skills (cf. Edelson, 1998).

Our main objectives as authors of the learning materials are to let students

- work with real data collected from video clips made by a web cam;
- carry out practical work in which they can apply much of their present knowledge of mathematics and physics in a real life context;
- practice ICT-skills, in particular making a video clip and carrying out measurements on it with a data video tool;
- experience that diagrams that are used in practice are not just pretty pictures, but contain much information about the real life phenomenon under study;
- be in contact with current research work, in our case movement science, including the nomenclature and research methods used.

These objectives are rooted in our belief that the main purpose for doing practical work is to experience authentic mathematics and science, and to enjoy and become competent in it. The activity of measuring

the pupil reflex of the eye on light can also be looked upon from this point of view: on the one hand, students learn how to do simultaneously video recording, data acquisition with sensors, and control of apparatus, which is a combination of techniques that they might apply in other experiments as well, and on the other hand, students experience that cross-disciplinary work (in this case physics and biology) is quite common in actual research on an ordinary real-life phenomenon.

### GETTING AN EYEFUL OF PHYSICS BY MEASURING THE PUPIL LIGHT REFLEX

The pupil light reflex is the change of pupil area in response to a change of light. Constriction of the pupil of the eye happens quickly and automatically in response to a step-increase of light intensity. But a pupil cannot instantaneously reach its new size when the level of illumination is suddenly increased: there is a delay in reaction time ( $\approx 200\text{-}500\text{ms}$ ) and then constriction can be approximated reasonably by an exponential decay function. Hereafter, in case the step-increase was not too large, the pupil redilates slowly almost back to its original size. This response is called ‘pupil escape’. If, however, the increase in light intensity is very large, the pupil simply constricts without redilation, a response named ‘pupil capture’. The pupil light reflex can be experimentally modelled with exponential decay and a logistic regression curve of data measured in a video clip via point-tracking; see Figure 1 below. Pupil escape and capture have also been successfully modelled via nonlinear neural feedback control systems models (Sun *et al*, 1983; Krenz and Stark, 1985) and more recently via nonlinear delay differential equations (Longtin and Milton, 1989; Bressloff *et al*, 1996).

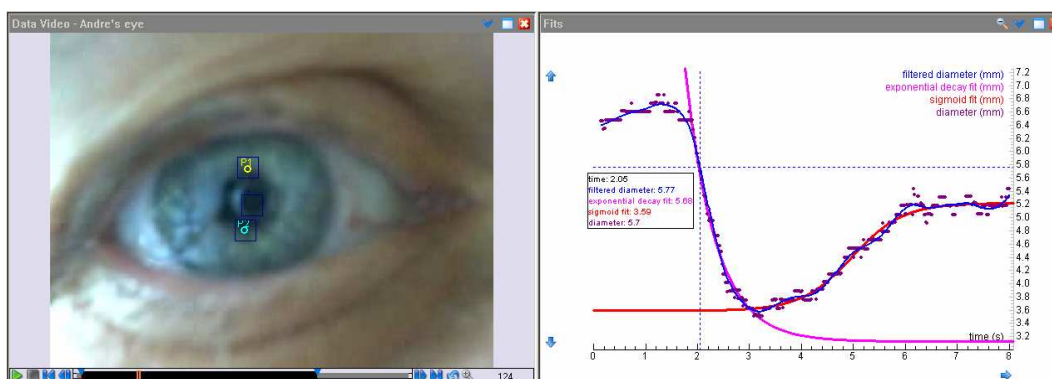


Figure 1. Experimental modelling of the pupil light reflex via exponential and logistic regression.

The pupil light reflex affects both eyes, even if only one eye is stimulated. This fact is used in the following experiment: the right eye of a person is stimulated by an oscillatory light stimulus via a bicycle lamp, which is periodically switched on and off, and the pupil motion is recorded at the same time with a web cam operating at a frame rate of 60fps. A Coachlab interface connected with the computer is used to control the lamp and to measure the light intensity in the test tube. At the beginning of the experiment, the test tube with the lamp is held in front of the web cam so that the recorded video and the light intensity measurement can be synchronized by matching the first increase of light intensity with the first time that the bicycle lamp is switched on. Hereafter the lamp is put in front of the right eye and the data collection continues. Figure 2 shows the screen shot of the Coach activity. The lower right window is a visual representation of the Coachlab interface with the connected light sensor and lamp. The upper right window shows the control program to switch the lamp on and off. The upper left window contains the recorded video and the lower left window shows the graph of the measured light intensity during the experiment. All windows are linked: so by comparing the step-increases of the light intensity with the video frames in which the pupil constricts, you can find the delay in reaction time. The measured reaction time has been 0.25s in this experiment.

The recorded video is used to measure the diameter of the left pupil during the second phase of the experiment. Point-tracking makes this measurement an easy task: two opposite points at the boundary of the pupil are selected in the starting frame and the coordinates of these points are automatically

recorded in subsequent frames. These recordings are used to compute the pupil diameter at any time during the second phase of the experiment. For more details about the algorithm implemented in Coach for point-tracking we refer to (Heck and Uylings, 2006). Figure 3 is a screen shot of the video measurement activity. The graph of the measured pupil diameter is shown in the right window. This diagram also contains the graph of the sinusoidal regression curve that fits the data. The period of the pupil oscillation is 4.3s, which is in agreement with the period of the measured light intensity.

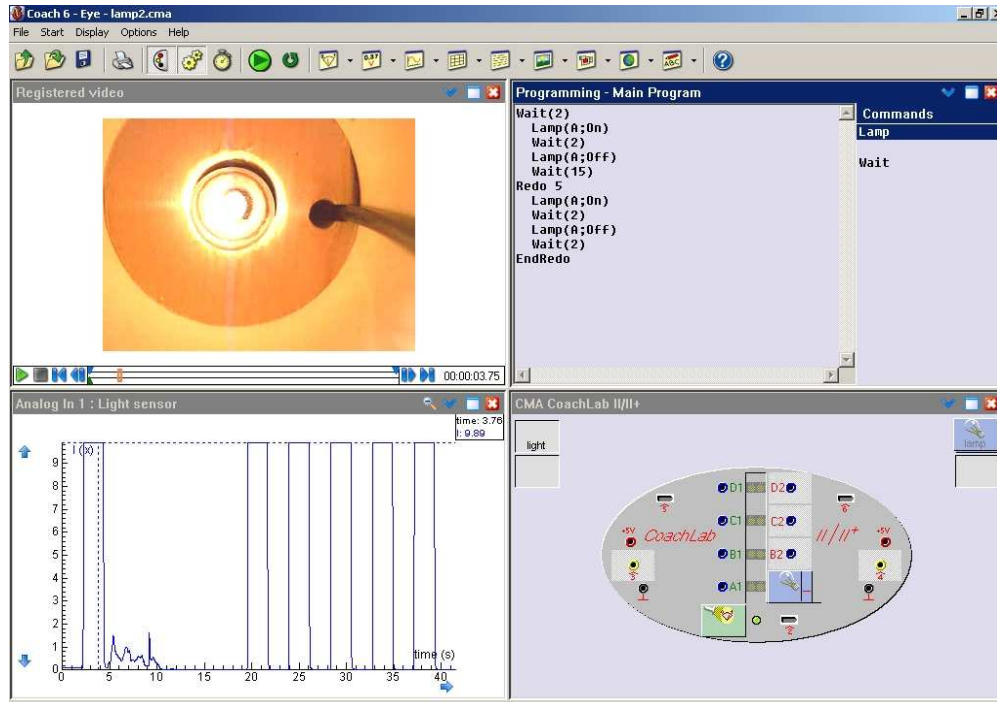


Figure 2. Screen shot of simultaneous data collection, video recording, and control of light in Coach.

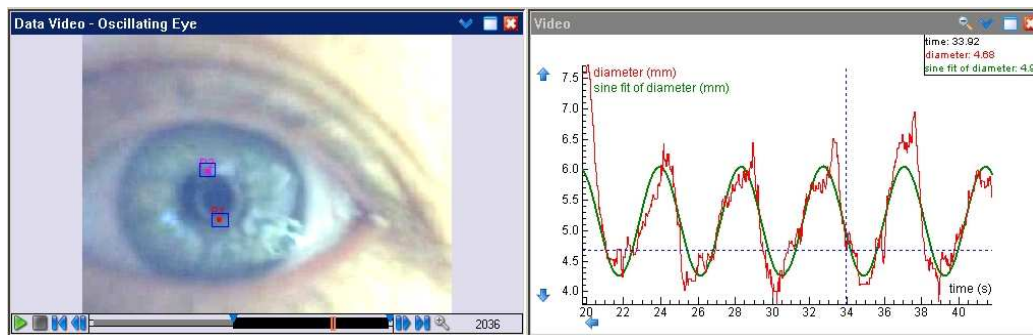


Figure 3. Screen shot of a video analysis activity in Coach 6.

This example shows that students doing experiments that resemble professional research practice often need a computer environment in which various tools can be used simultaneously or in combination with each other. It is not good enough to focus in ICT enhanced learning on separated experimental control, data collection, and data analysis.

### WALK LIKE A MOVEMENT SCIENTIST

One learns walking at the age of one. Hereafter walking goes naturally and one does not give it much thought anymore until you are getting on a bit or in case you get an injury of your legs. Then you

realize how many times you actually use your limbs and how many way of human locomotion you practice: normal walking, sauntering, jogging, running, hopping, skipping, tiptoeing, race walking, crutch walking, and so on. You may start wondering what the differences and similarities between the gait patterns. It turns out that walking and running are complicated but interesting periodic motions from mathematical and biomechanical point of view. Movement scientists have developed various research methods to study gait. In the last years we have developed several practical investigative tasks and potential projects in the context of the human locomotion that enable upper pre-university students to act like human movement scientists. The students collect and analyse high-quality gait data in much the same way movement scientists do via recording and measurement of motions with a video analysis tool (for kinematical analysis of the motion) and via electromyography (for registration of muscle activity). Physics, mathematics and biology come together in these scientific investigations by students. We have published about the students' work (Ellermeijer and Heck, 2002; Heck and Holleman, 2003; Heck and van Dongen, 2008; Heck, n.d.) In this article we discuss part of this work to illustrate that science learning at school can resemble science practice in research labs, provided that students have adequate tools. The chosen subjects: (1) motion analysis of human gait and in particular of the hip and knee angle during locomotion, (2) electromyography of normal walking, and (3) models of sprinting. In all examples, Coach 6 provides the required tools in a single computer learning environment.

### Motion Analysis of Gait

Normal walking is a periodic movement of each foot from one position of support to the next position of support in the direction of progress. The gait cycle is defined as the period of one foot strike to the next foot strike of the same foot. Figure 4 illustrates the two phases of the gait cycle and the main gait events.

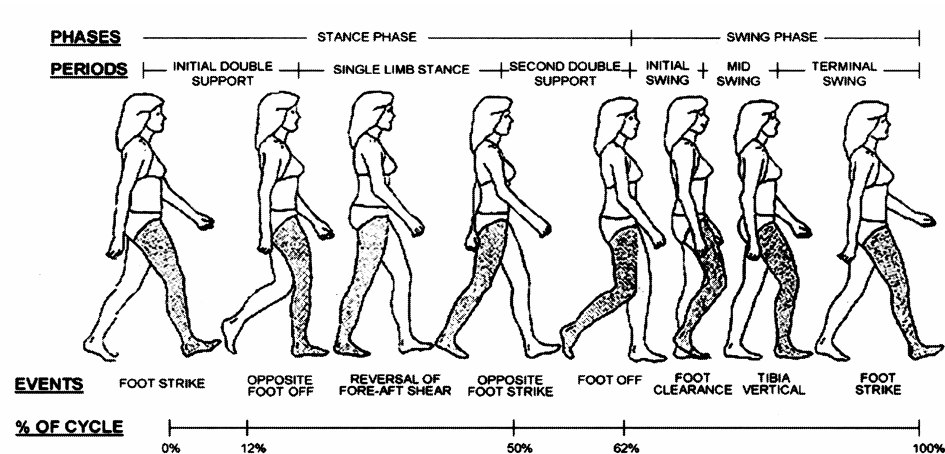


Figure 4. Normal walk cycle illustrating the event of gait (Rose and Gamble, 2006).

Motion analysis is the science of analysing video clips of body and body part motions in order to study the kinematics and the forces involved. In the practical investigative tasks students record the motion on video with a web cam and use Coach to analyse the planar motion of the upper and lower part of the leg around the hip and knee joint. The leg movement of normal walking can be pretty well described by a lateral and dynamically coupled oscillator model (Yam *et al.*, 2002) and this holds for many human gait patterns. A result of this is for example that the angular motion curve of the knee joint can be pretty well described by a sum of two sine functions in which one frequency is twice as large as the other one. The natural period for the leg (or the preferred stride frequency) based on a force-driven oscillator model (Holt *et al.*, 1990) is given by

$$\text{natural period} = 2\pi \sqrt{\frac{I}{2mgd}},$$

where  $I$  denotes the moment of inertia of the leg (including the foot),  $g$  represents the acceleration of gravity,  $m$  stands for the mass of the leg, and  $d$  is the distance from the centre of gravity of the leg to the hip joint. Another useful tool in gait analysis, which more or less characterizes the gait pattern in a

quantitative and objective way, is the so-called hip-knee cyclogram in which the knee joint angle is plotted against the hip joint angle. The left diagram in Figure 5 shows the hip-knee cyclogram of a normal equal-level walk taken from the research literature (Goswami, 1998). Points marked with an 'o' belong to important events of the stride. Students obtain similar diagrams in their practical work and are able to label the main gait events in their diagrams as well. See the right part of Figure 5 for an example taken from the research project of Caroline van Dongen (Heck and van Dongen, 2008).

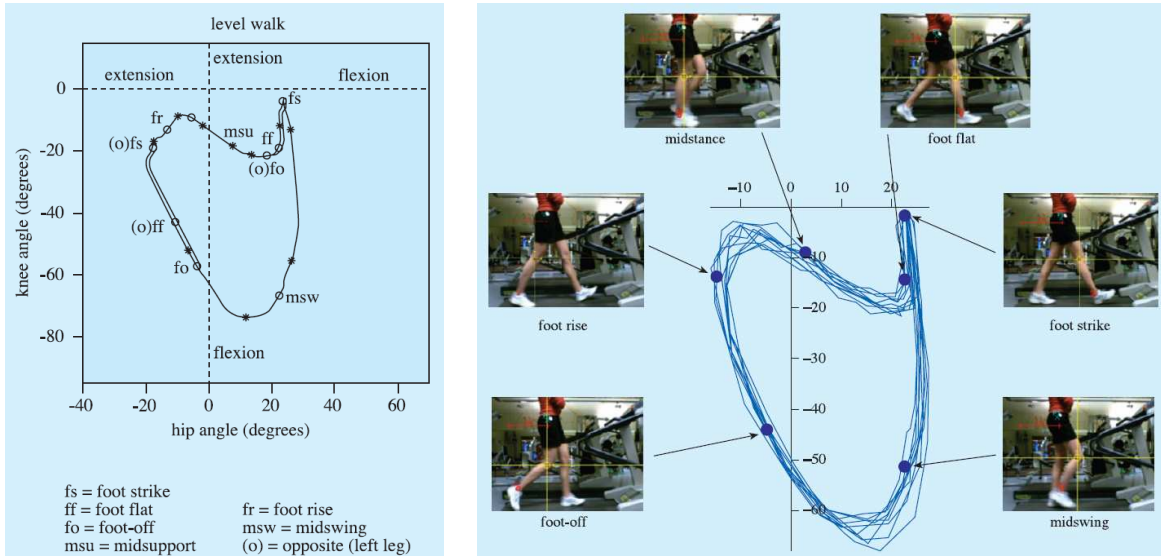


Figure 5. Hip-knee cyclogram from research literature (left) and from student research (right).

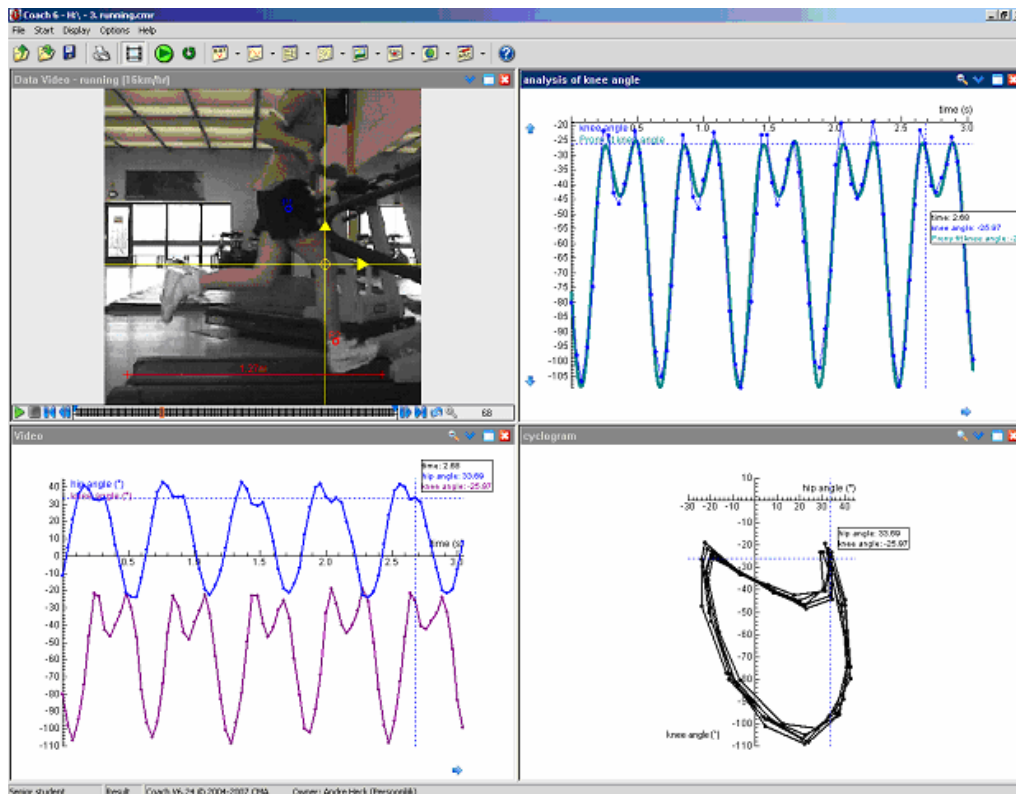


Figure 6. Motion analysis of a student running on a treadmill.

The hip-knee cyclogram is a parametric curve with respect to time that alters when gait conditions change. Figure 6 shows a screen shot of a Coach activity in which the motion of a student running at a speed of 16km/hr on a motor-driven treadmill is analysed. The lower right diagram is the hip-knee cyclogram of this motion and the crosshairs indicate the position of foot strike. The shape of this diagram resembles the previously shown hip-knee cyclograms but also differs in important aspects: for example, during running the hip and the knees bend more. The video clip shown in the upper left window of the Coach activity shown in Figure 6 had been recorded with a web cam at a frame rate of 30fps. The polar coordinates of the hip and ankle joint of the right leg have been collected and used to determine the hip and knee joint angles. Students can figure out themselves what formulas are needed to this end and they can almost immediately see if they have it right because otherwise they obtain physiologically impossible hip or knee motions. The graphs are shown in the lower left diagram of Figure 6. The best function fit of the knee joint angle as a function of time with a sum of two sine functions is shown in the upper right diagram: it matches well with the collected data. This fit can be found automatically with the signal analysis tool of Coach 6, which has high-resolution methods for sinusoidal regression on board. The Prony method leads in this particular case to the following formula:

$$\text{knee joint angle} = 4.1 + 32.4\sin(601.6\text{time} - 136.8) + 22.3\sin(1201.8\text{time} - 179.3)$$

where the angle is in degrees. Clearly, one of the frequencies is about twice as large as the other. This experimental approach confirms the theoretical model of a force-driven harmonic oscillatory motion.

### Electromyography of normal walking

Muscle activity is typically studied using dynamic electromyography, abbreviated as EMG (Perry, 1992; Rose and Gamble, 2006). In case of measurement of activity of superficial muscles, surface electrodes are placed on the skin surface to detect the muscle action potential, i.e., the electric activity responsible for contraction of muscles or muscle groups. Muscle force cannot be estimated directly from the intensity of the signal. Figure 7 illustrates EMG activity during normal walk as simple on-off diagrams for certain muscles.

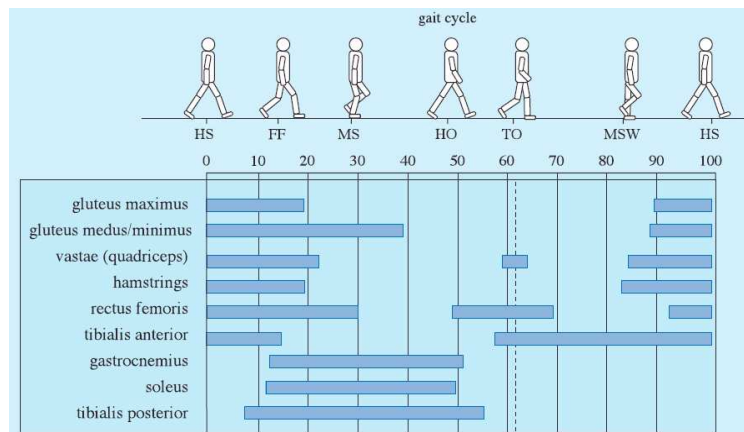


Figure 7. EMG activity during gait (extracted from Decker and Beckers, 1996).

EMG recording is rather difficult because correct placement of electrodes is critical. Nevertheless, high school students are able to get with sensors experimental results that are qualitatively in agreement with results known from the research literature. Figure 8 gives an impression of the equipment used and the results that are obtained in a student research project. In this experiment it is extremely useful that Coach allows simultaneous measurement with sensors and video capture. The video clip and the measured data are synchronized: this means that pointing at a graph or a table entry automatically shows the corresponding video frame and that selecting a particular frame highlights the corresponding points in diagrams, when the scanning mode is on. This makes scrubbing, i.e., advancing or reversing a clip manually, an effective means to precisely identify and mark interesting events in the video clip and to relate them to graphical features. This feature is used to link maxima and minima in EMG diagrams with gait events visible in the video clip. The lower right diagram in Figure 8 shows the graph of the

processed EMG signal during one gait cycle: processing means here that we have taken the absolute value of the raw signal with respect to the zero level of the signal (full-wave rectification) and smoothed the rectified signal. Peaks correspond with muscle activity. We have marked some gait events, which have been found by scanning the graph while watching the corresponding video frames. This diagram can be interpreted as follows: activity of the gastrocnemius is mostly during stance phase as in agreement with the on-off diagram shown in Figure 7. Peak values are at terminal stance and when the foot is taken off the ground. This can be understood because during this part of the stride, the muscle first undergoes eccentric contraction, which controls the forward movement of the tibia over the fixed foot; hereafter the gastrocnemius undergoes a concentric contraction which initiates plantar flexion, which begins the toe-off phase. At terminal swing, the muscle might be used during foot descent in order to provide stability to the ankle joint in preparation for foot contact. Beginning at foot strike and ending at midstance, the gastrocnemius is probably used for shock absorption, stability and progression of the limb.

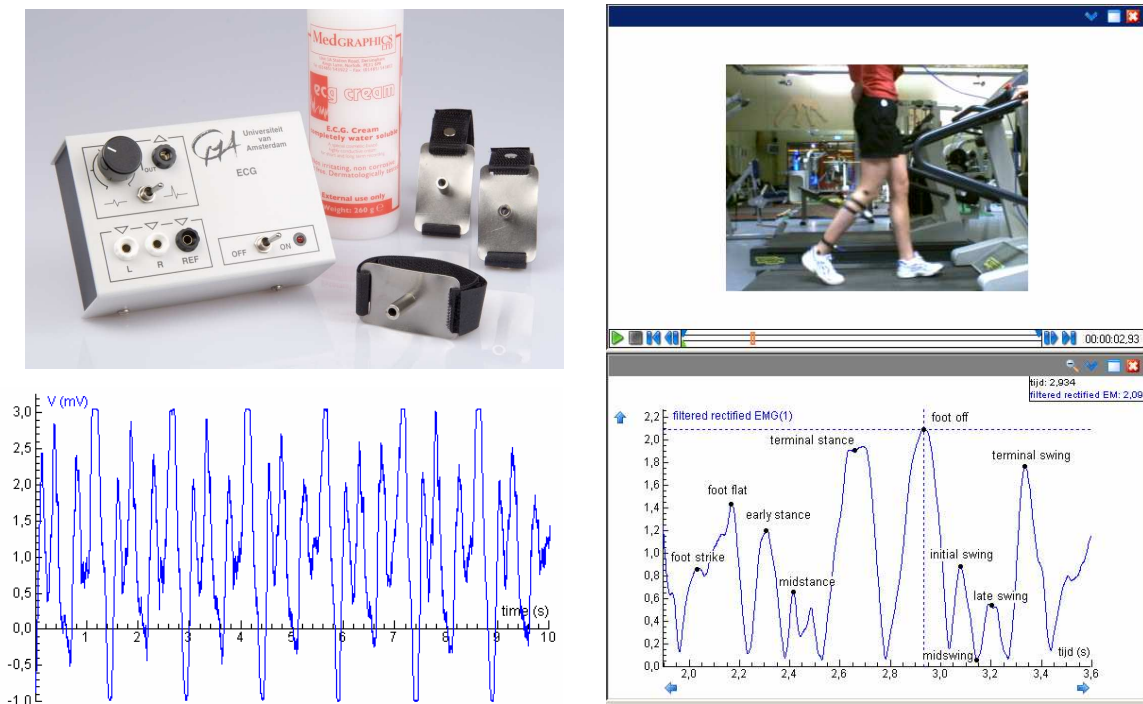


Figure 8. The EMG equipment (upper left), a recorded raw EMG signal of the gastrocnemius for a normal walk at low speed (lower left), and the corresponding processed EMG signal (lower right) that can be linked directly to the motion of the leg recorded simultaneously with a web cam (upper right).

The EMG activity and the motion analysis activities illustrate that affordable ICT can bring high school students into contact with the field of gait analysis. In the investigations described, focus was on the leg motion during normal walk and running. But other aspects such as arm motion and other gait patterns could be investigated as well. Animal locomotion, motion of the lower limbs during (power)cycling, and sports motions are research topics that could be studied in a similar way. The authenticity of the student activities is realized by the opportunity for high school students to investigate a real-world motion in much the same way as movement scientists would do. Student can learn how physics, mathematics, and biology come together in such work.

### Giving high school students the run of models of sprinting

Sports science is another attractive area for student research projects. High school students can construct computer models of running and use their programs to compare results of simulations with



experimental results obtained from a video analysis of the start of a sprint. All models that are based on Newton's second law of motion use as basic equation

$$a(t) = F_{\text{propulsive}}(t) - F_{\text{resistive}}(t),$$

where  $a(t)$  represents the acceleration of the runner at time  $t$ ,  $F_{\text{propulsive}}$  the horizontal component of the propulsive force per unit mass at time  $t$ , i.e., the force that the runner applies at the given moment to be in motion, and  $F_{\text{resistive}}$  the force per unit mass that the runner has to overcome at time  $t$  in order to be in motion. Keller (1973) assumed in his model of competitive running that the propulsive term is constant for sprinting and that the dominant resistive effects from running mainly result from frictional losses within the body of the runner and can be modelled by a term linear in speed. This model, in which air friction is neglected, can be written for a runner who is initially at rest as

$$v'(t) = F - v(t)/\tau, \quad v(0) = 0,$$

where  $F$  is the constant force per unit mass that the runner can exert in the horizontal direction and  $\tau$  is a time constant. This initial value problem can be solved analytically:

$$v(t) = F\tau(1 - e^{-t/\tau}).$$

The product  $F\tau$  equals the maximum speed of the sprinter. The distance  $D(t)$  covered in time  $t$  is found by integration of the above equation:

$$D(t) = F\tau^2(t/\tau + e^{-t/\tau} - 1).$$

The mathematical formulas are not just a hobby of the teacher, but they play in these models an important role in the determination of good estimates for the model parameters. In this article we describe briefly the use of a graphical modelling tool by high school students to create representations of quantities and relationships about running where analytic solutions are intractable. Another motive to do computer modelling in practical work is to bring students into contact with modern approaches in human movement science. The need for computer models is quickly felt when one wants to take several factors into account for models of running. For example, the effect of head- or tailwind can be translated mathematically into an aerodynamic term derived completely from air resistance that is proportional to the square of the relative speed.

But (computer) modelling does not replace empirical work. Validation of models with experimental results is necessary. In the context of students investigating their own running performances Coach 6 provides the tools to record the start of a sprint with a web cam and to analyze the collected data with a video analysis tool. Figure 9 is a screen shot of a video analysis of a 25-metre sprint of a high school student. The lower left corner of Figure 9 immediately reveals a problem when one wants to measure the position of the sprinter in the recorded video clip: perspective distortion. A front-view of the plane of motion is actually needed. Luckily, Coach 6 provides a perspective correction tool that properly transforms the frames of a video clip, in this case into the video shown in the upper left corner of Figure 9. Here, the video clip was also flipped horizontally to get a motion from left to right, which matches a traditional coordinate system with a positive axis pointing to the right. Since the 25-metre sprint takes about 5s and the frame rate of the recorded video is 30fps, manual data collection by clicking on points of interest in the video clip is too time-consuming, error prone and boring work to do. Fortunately, the Coach 6 video tool provides its user with the facility of automated data collection in the video clip via a technique that is also known as point-tracking. This is how the distance-time graph in the upper right corner of Figure 9 was obtained. For other inspiring applications of perspective correction and point-tracking in video analysis activities for students we refer to (Heck and Uylings, 2006).

The velocity-time graph in the lower right corner of Figure 9 was obtained by computing the derivative of the covered distance with a penalized quintic spline smoothing algorithm. In addition, the speed data were approximated by an exponential regression curve according to the Keller model of sprinting with a nonzero initial velocity:

$$v(t) = 6.38(1 - e^{-0.802t}) + 1.25.$$

A simulation of the Keller model, implemented via the graphical modelling tool of Coach 6 (similar to STELLA and Powersim), with measured data points in the background is shown in Figure 10.

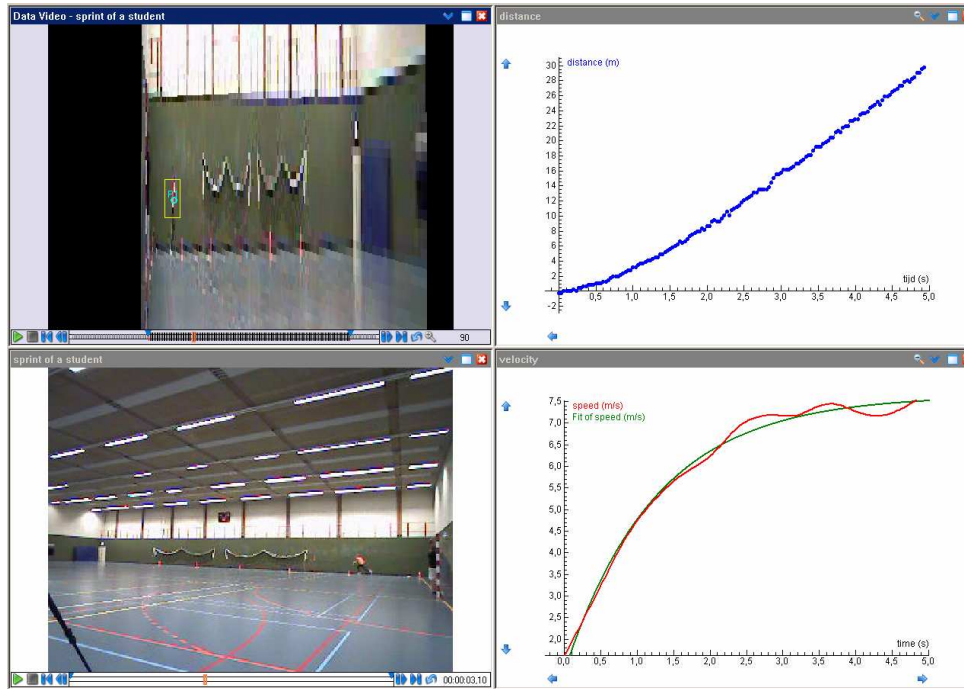


Figure 9. Video analysis of a 25-metre race of a student.

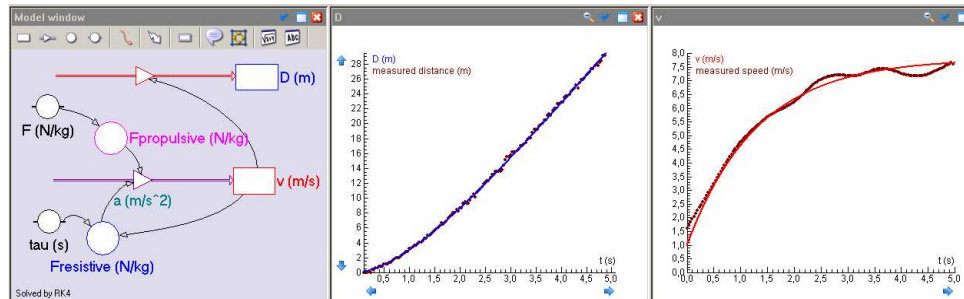


Figure 10. Simulation of the Keller model for the 25-metre sprint of the student.

The descriptive power of the Keller model seems to be good. But when it is used to compute the expected result of a 100-metre sprint of this particular student, the race time of 14s seems a bit unrealistic. In other words, the predictive power of the Keller model is not so strong. It is important that high school student get acquainted with this kind of working and reasoning with (computer) models.

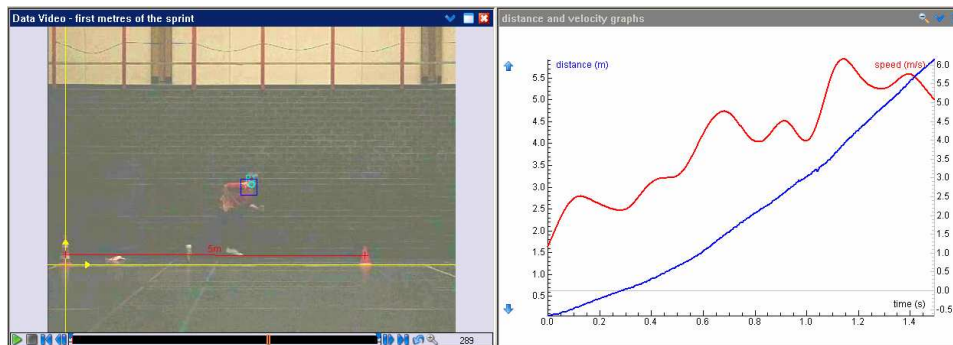


Figure 11. Video analysis of the first 6 metres of the sprint of a student.

More details of the motion of the sprinting student during take-off can be found in the video recording of the first 6 metres with the high-speed camera with a frame rate of 300fps. With the appearance of the Casio EXILIM Pro Ex-F1 digital camera in 2008, moderately priced and allowing video recordings at a frame rate up to 1200fps, high-speed video technology has become available at consumer level and it enables students to record fast motions. In Figure 11 are shown the obtained distance and velocity graphs of the start of the sprint. The velocity graph is the most interesting curve: increasing and decreasing parts of the graph correspond with propulsion during foot contact and with the flight phase, respectively. Differences in consecutive increases in the velocity-time graph indicate that the propulsive force of the right leg is stronger than that of the left leg. It may be connected with left- or right-handedness of the student, but it may also have to do with search of a good balance during take-off.

## CONCLUSIONS

In our point of view, the real educational issue in the investigative student work that we described in this article is the ICT-supported interaction between experimental work and modelling, in which the interpretation of results is based on methods from mathematics, physics, and biology. The role of ICT is to allow students to collect high-quality, real-time data, to construct and use computer models of dynamic systems in much the same way as professionals do, and to compare results from experiments, models, and theory. Furthermore, students can develop and practise through their activities research abilities, and the fact that they must apply their knowledge of mathematics and physics in a meaningful way in a concrete context leads at the same time to deepening and consolidation of this knowledge. Important research abilities that students practise through this kind of practical investigations are:

- formulate good research questions that guide the work;
- design and implement an experiment for collection of relevant data;
- apply mathematical knowledge and physical concepts in new situations;
- construct, test, evaluate, and improve computer models, and have insight in their role in science;
- interpret and theoretically underpin results;
- collaborate with others in an investigation task and reflect on the work.

ICT plays an important role in enabling students to carry out investigations at a high level of quality. It also brings the real world into mathematics and physics education in an attractive way. Computer models can be constructed by students themselves on the basis of general physical concepts in situations where algebraic formulas are out of reach or even impossible. In short, ICT provides the students with tools to be actively involved in the process of finding solutions to challenging problems that they come up with themselves. The fact that these tools are bundled and integrated in a single computer learning environment comes in handy: this makes it for students easier to apply these tools in their attempts to solve problems or to get a better understanding of given situations. Actually, we consider the modelling process, the underlying thinking processes, and the discussions with fellow students during the research as more important in the students' work than the obtained results. All the same it is joyful when experiment, model, and theory are in full agreement, as is the case for example in the study of the human locomotion.

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André Heck, Ton Ellermeijer, and Ewa Kędzierska  
AMSTEL Institute, Universiteit van Amsterdam  
Kruislaan 404, 1098 SM Amsterdam, The Netherlands  
Email: A.J.P.Heck@uva.nl, A.L.Ellermeijer@uva.nl, E.Kedzierska@uva.nl