You Must Keep Money Moving

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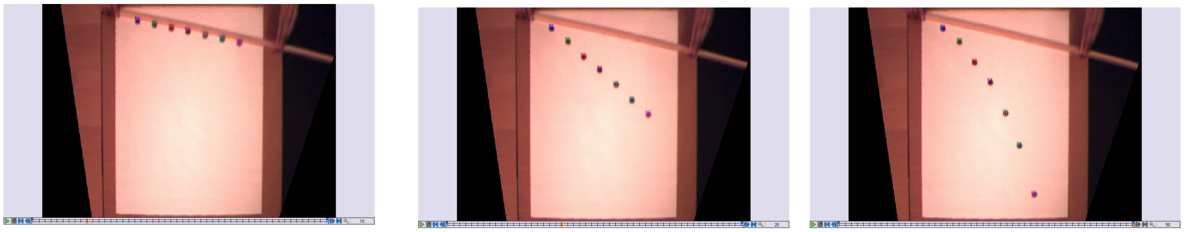

Figure 1. Moving Money.

The experiment

The experiment is simple: seven coins lie on a paper placed on a flat table, lined up at equal distances. A ruler, making a circular movement, pushes the coins. The origin of this circle is at one site of the row of coins (in this case, the left upper corner of the papers in Figure 1). The ruler stops, and the coins leave the ruler, having a velocity perpendicular to the ruler. At this point, friction comes into play. The friction is constant, because all coins are the same, and this friction does not depend on the velocity. The work done by the friction equals $W = F \cdot s$, where F is the friction and s is the distance over which the coin moves after leaving the ruler. It means that the distance a coin travels after leaving the ruler is proportional to the kinetic energy at the moment the coin looses contact with the ruler. Since the ruler makes a circular movement, the velocity v of each coin is proportional to the original distance d of the coin to the center of rotation. Because the kinetic energy of the coin E_k equals $\frac{1}{2}mv^2$ we get $F \cdot s = \frac{1}{2}mv^2$. Since F and m are constants, s is proportional to v^2. This means that the distance traveled by a coin after leaving the ruler is proportional to the square of its original distance to the center of rotation. In other words, we expect that the coins, once they stop moving, lie on a parabolic curve. The final position of the coins proves that we are right (see picture to the right in Figure 1).

Are we sure and how can we find out?

The easiest way to check whether the coins after leaving the ruler lie on a parabolic curve is to do a video measurement. Two problems arise right from the start:

1. The motion is too quick for recording with a normal camera or webcam;
2. In practice, it may not be possible to make a movie from the top.

The first problem is solved by using a high-speed camera (say at a speed of 400 fps). The second problem can be solved by perspective correction (see Figure 2).

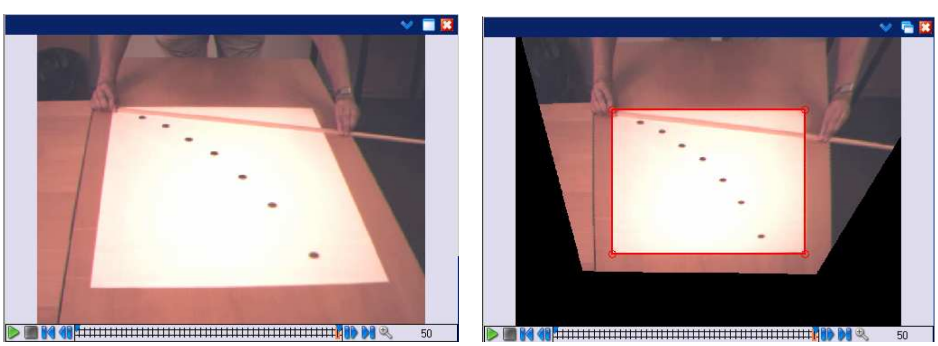

Figure 2. Before and After Perspective Correction.

We have rescaled the rectified video in Figure 2 further to the one shown in Figure 1 in order to make dimensions fit reality better, i.e., to obtain a video in which the paper has been scaled to real proportions.

Manual collection of video data about seven coins is time-consuming, boring, and error-prone. Instead one can better track the motion of the coins and record automatically the coordinates of the objects in subsequent frames of the video clip.

Perspective correction and tracking of objects in video measurement are possible in version 6 of the Coach learning environment.

In what follows, we will check some statements made in the previous description of the experiment.

The ruler stops, and the coins leave the ruler, having a velocity perpendicular to the ruler.

Because there are no other forces besides friction, the track of each coin will be a straight line once it has left the ruler. Under these circumstances it seems natural to take the center of rotation as the origin of the coordinate system and to align the *x*-axis with the ruler when it stops moving. Using these coordinate settings and tracking, we get the next screen shot of a Coach 6 activity.

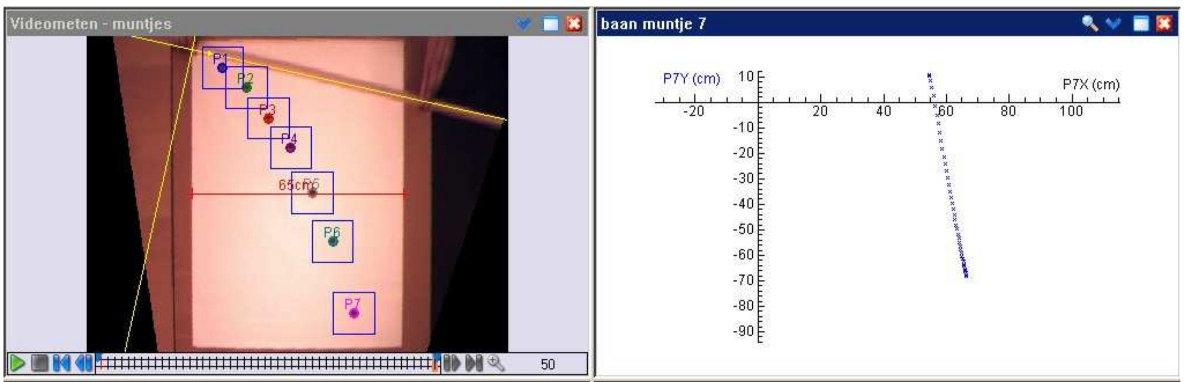

Figure 3. Tracking of Coins.

To the left, you see the yellow axis pointed out just like the ruler was at the moment that the coins lost contact. To the right, you see the track of the seventh coin. Clearly, with these coordinate settings the *x*-component of the velocity is not equal to zero. This means that the velocity at the moment when the coin looses contact is not perpendicular to the ruler shown.

We can take a closer look on this velocity component parallel to the ruler. We rewind the movie and play it frame by frame. While the ruler is rotating and is in contact with the coins, the coins turn out to move along the ruler. For example, one can measure the distance between the seventh coin and the origin in various frames. Analyzing the frames 6, 8, 10 and 11, one finds every time a slightly different value for the distance from the coin to the origin: values are 53.54, 53.71, 54.55, and 55.21 cm, respectively.

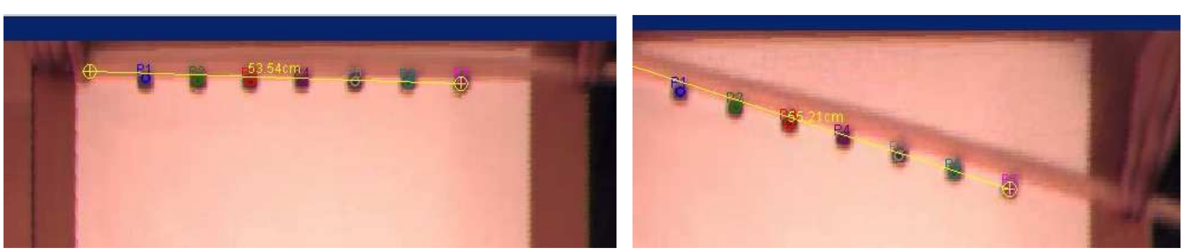

Figure 4. Measurement of the Distance of the 7th Coin in Different Frames.

This means that the coins have indeed a velocity parallel to the ruler. In other words, the velocity when a coin looses contact with the ruler is in reality not perpendicular to the ruler.

We expect that the coins, once they stop moving, lie on a parabolic curve. The final position of the coins proves that we are right (see picture to the right in Figure 1).

The first thing we do is to orientate the coordinate system such that the 7th coin moves in a straight vertical line. Figure 5 shows clearly the horizontal axis is not aligned to the ruler once it has been stopped.

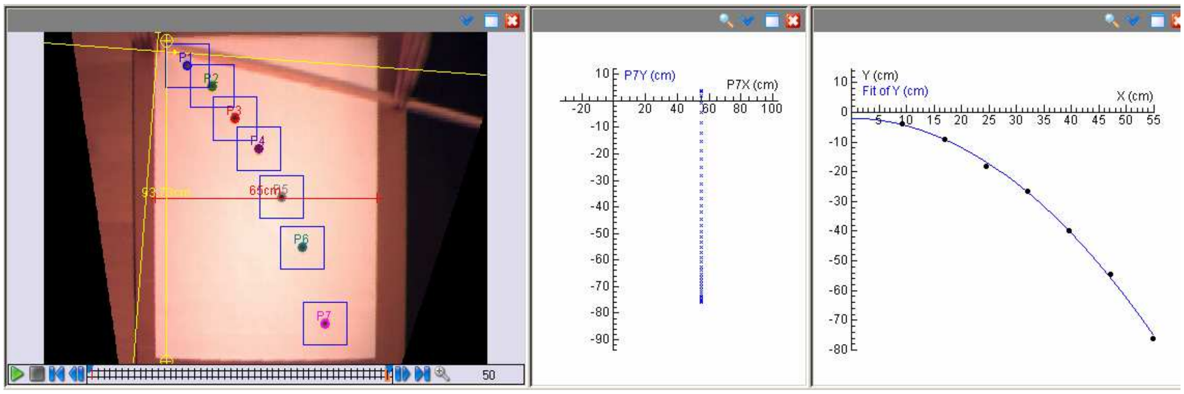

Figure 5. A More Convenient Choice of the Coordinate System.

The diagram to the right shows the final positions of the 7 coins. The coins seem to lie on a parabola, but this can only be verified by regression analysis. The curve in the diagram shows the best quadratic fit calculated by the computer program:

(-0.023976370735911*X-0.012360878869639)*X-1.999994733605468

Written in a more conventional way with less precision as

$$Y = -0.02398\,X^2 - 0.01236\,X - 1.99999$$

or

$$Y = -0.02398\,(X + 0.25777)^2 - 1.99840$$

Translation of the origin of the coordinate system over the vector (-0.25777, 1.99840) would lead to a simple relationship of the form $Y = -c\,X^2$. But it would have been mere luck if we had immediately chosen the coordinate system as such.

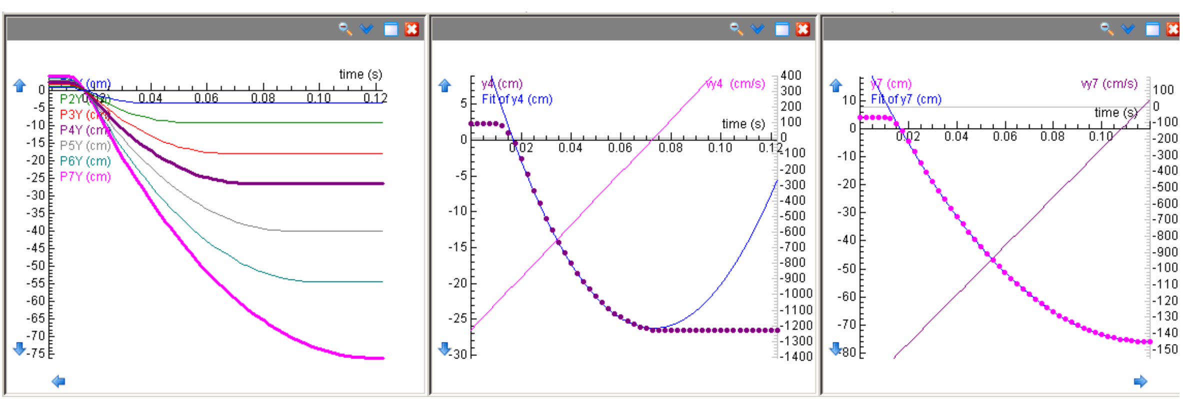

Figure 6. Y-t graphs of Moving Coins and Quadratic Regression Curves

The Y-t graphs of the coins as long as they move freely on the table also look like parabolas. See Figure 6 for the quadratic regression curves of the 4th and 7th coin. In each diagram is also displayed the velocity curve of the function fit. Its intersection with the horizontal axis shows at what time the particular coin stopped moving.

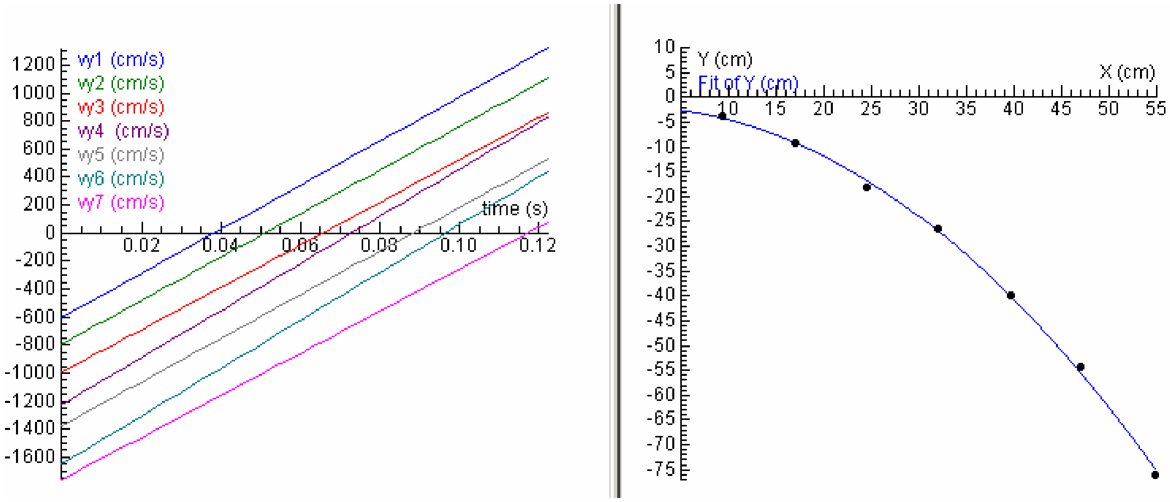

Figure 7. v-t graphs of Freely Moving Coins and a Quadratic Fit of the Final Positions.

To the left in Figure 7 we show the velocity curves of the coins in one diagram: it seems that the 1st, 3rd, 5th and 7th coin move such that their velocity curves during free motion are parallel equidistant lines, as theory could explain. The 4th and the 6th coin also move such that their velocity curves during free motion are parallel equidistant lines, but their deceleration occurs to be greater than those of the other coins. This is reflected in the diagram to the right on Figure 7, which shows a quadratic fit of the final positions of the coins: the 4th and 6th coin are indeed a little above the regression curve. Apparently friction was not of equal size for all coins or could it make a difference whether head or tail is down? With hindsight, did we pay attention to this in the experiment and would it have made a difference? Reality turns out to be more complex than can be foreseen and that theory predicts. It is a good experience for students to deal with such subtle issues in linking theory and practice.