Modelling Human Growth

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Abstract

Many a pupil at secondary school asks oneself questions like "Am I too fat or too thin?", "Am I short or tall in comparison with other people of my age?", and "What adult height may I expect to reach?". To answer such questions one needs real data. We have used the recent Dutch growth study to create learning material for pupils in the second stage of pre-university education (age 15-18 yr.) to carry out practical investigation tasks. A mathematical highlight is the ICP-model that models length for age within millimetres. It is used in the medical literature and yet consists of growth models that are studied at school, viz., exponential growth, quadratic growth, and logistic growth. We shall present the learning material and discuss the classroom experiences.

Introduction

Body growth of children is often used in mathematics textbooks as an example for discussing

- processes of change: here, pupils are accustomed to the discrete apparatus of increase diagram and difference quotient, as a first step to the concept of derivative.
- statistics: height growth is a standard example for introducing and practising statistical concepts such as normal distribution, mean, median, percentiles, and so on.
- discrete and dynamical models of growth.

However, the real world context is in many exercises only used for 'dressing up' a mathematical problem or as an 'ideal' illustration of a mathematical concept. What to think of an exercise in which a 15-year-old boy of height 175 cm is considered to be of short stature for his age. In Figure 1, we show the real diagrams of mean height and increase of mean height for Dutch boys together with the textbook example of a boy called Hank. Would you describe Hank's growth curve as normal? The examples come from two Dutch textbooks in the series 'Moderne Wiskunde' and 'Netwerk', but similar examples can be found in other books.

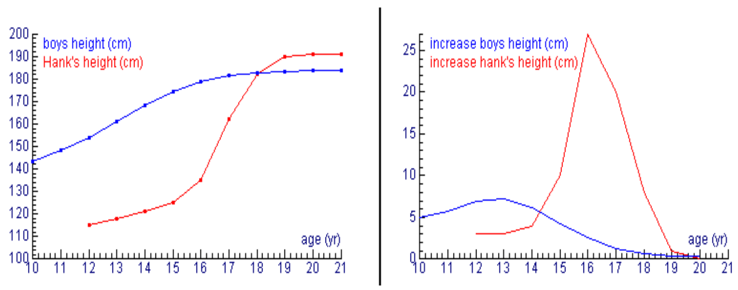

Figure 1. Real and Artificial Growth Curves.

One may argue that real data are too unmanageable or distracting for successful introduction of a mathematical subject. But this argument does not hold anymore when it comes to applications of mathematics by pupils in real contexts, assuming that the pupils have convenient tools to collect, to process, and to analyse real data. Actually, practical assignments that pupils make by themselves with the computer form a new part of the Dutch curriculum in the second stage of pre-university education [1]. In our research we examine the possible contribution of ICT and context situations to the realisation of challenging mathematical investigation tasks for pupils. It provides input for the ongoing development of the learning environment Coach [2,3] and learning materials for pupils that stimulate and enable them to carry out mathematical investigation tasks at a rather high level. Software and pupils' activities are tested in practice.

The Learning Material

The learning material is designed for pupils who are in their first year of the second stage of pre-university education (age 15-16 yr.), who have no experience with practical investigation tasks, and who have not worked with Coach before. Our main objectives are to let the pupils

- work with real data and with diagrams that are actually used in health care;
- experience how much useful information can actually be obtained from diagrams;
- see that the change of a quantity is often as important and interesting as the quantity itself;
- practice ICT-skills;
- carry out practical work in which they can apply much of their mathematical knowledge.

The learning material consists of the following three compulsory assignments:

1. The Dutch Growth Study of 1997. A quiz gives the pupils an idea of how much they already know about body growth and puberty. We use a newspaper article to introduce the 4th nation-wide growth study and to illustrate the relevance of growth data.

2. Mean Height Growth. Pupils get familiar with the main tools of Coach for studying human growth. They learn how to make data plots and increase diagrams of height in relation to age, and they learn how to interpret these diagrams in the context of child growth.

3. Secular Height Growth in 1980-1997. One understands by secular growth the changes in the development of children from one generation to another. In this task pupils use the data from the Dutch surveys of 1980 and 1997 to study the changes in mean height for Dutch boys and girls in this period. Questions stimulate pupils to formulate and underpin their conclusions.

Finally, pupils select one of the following subjects to investigate by themselves:

A. Growth Charts of Native Dutch Children (no use of ICT in this activity). Pupils get acquainted with the growth charts for native Dutch children and with the mathematical terminology. They learn how these diagrams are made, what they mean, and how they are used. They also compare their own height and weight with their peer group.

B. Mathematical Model of Height Growth for Girls. Pupils compare the mean height growth for girls with Turner syndrome with the growth for healthy girls and they make a mathematical model of the height growth for girls until puberty.

C. Mean Weight Growth. Pupils investigate the mean weight growth for healthy Dutch boys and girls in relation to age. They search for points in common and for differences, and they make a simple mathematical model of weight-for-age for children until puberty.

D. Mathematical Weight-for-Height Model. Pupils investigate the mean weight growth for Dutch children in relation to height and make a mathematical model of weight-for-stature.

For the production of the learning material, we have made extensive use of medical literature, especially the documents in which the growth data and the results of the 4th Dutch survey have been described [4-7]. Some of these texts are even readable for pupils and would give them an idea of current research. The English translation of the learning material can be downloaded from the web page `www.science.uva.nl/~heck/research/growth/`

The main role of ICT in the activities is to visualise, process, and analyse real data. Once you have found the growth data[1], you can enter them easily in the computer environment for graphical representation and mathematical processing. The fastest and easiest method is importing data in spreadsheet format. The computer screen is sufficiently large for displaying growth diagrams and increase diagrams such that you can discover differences in mean body growth between boys and girls. The diagrams on the left-hand side of Figure 2, which is a screen dump of a Coach activity, are about mean height growth of Dutch children in relation to their age. Two things catch the eye in the increase diagrams: the growth spurt during puberty and the fact that this occurs for boys later than for girls, and with higher intensity.

However, it is also instructive to investigate diagrams for children with growth disorders. The diagrams on the right-hand side of Figure 2 are about mean height growth of healthy Dutch girls and girls with Turner syndrome. Two symptoms of Turner syndrome can be found in the diagrams: slow growth and no pubertal growth spurt. For girls with Turner syndrome the growth rate looks after 3 years like a linear function of age. In other words, the mean height of girls with Turner syndrome in relation to age can be described mathematically by a parabola. As a matter of fact, the quadratic model describes it up to a millimetre. Who still dares to say that working with real data in mathematics lessons is too troublesome? In the assignments B, C, and D, pupils use the regression tool of Coach to do similar experimental modelling of growth curves.

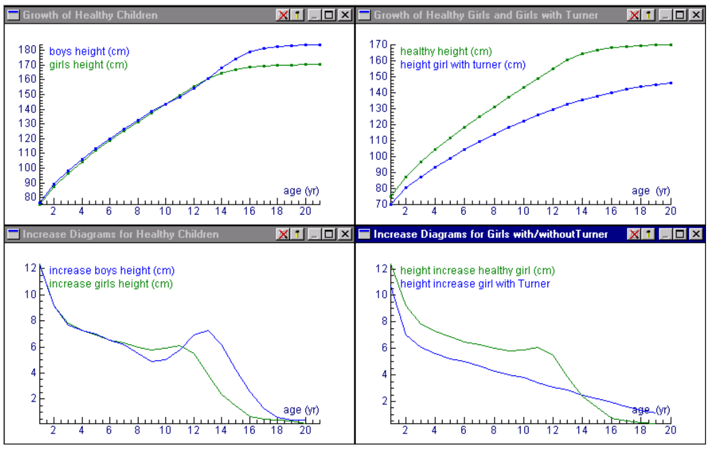

Figure 2. Mean Height and Increase in Mean Height for Dutch Boys and Girls.

[1] Actually, we could not find the data from the Dutch growth studies on the Internet. They seem to be only available in the professional medical literature. Results of surveys for other populations are more publicly accessible. For example, US data can be found at `http://www.cdc.gov/growthcharts/`

The Classroom Experiment

The experiment took place in a class of 26 pupils matching the learner profile that was described at the beginning of the previous section. The practical assignment was not part of the pupils' examination portfolio, but it was graded as a regular test in the semester. The pupils worked in pairs for two weeks; estimated study load is 6 to 8 hours. In the first week, work took mainly place in the computer lab during the regular mathematics lessons (of 45 minutes). The mathematics teacher and the authors were present as assistants. In the second week, the pupils could make use of the computer facilities at school to work all by themselves on the assignments. They had to hand in the report of their work (written with a text editor), a questionnaire, and a diskette with their Coach activities and results.

In our research experiment we wanted to get answers to the following questions:

- Do the learning material and the chosen instructional setting enable pupils, who have no prior practical experience with Coach, to acquire the skills and experience to use the tabular and graphical tools effectively in their study of human body growth?
- What do pupils think of the use of the software?
- What do pupils think of the subject, of the learning material, and of the use of real data?
- Since it is the first time that pupils do practical work in the context of mathematics all by themselves with a computer, how do they like it and what difficulties do they encounter?

The tools used to get answers to these questions are classroom observations, video recordings (including video capturing of computer work), a questionnaire, and the reports of the pupils. From the questionnaire we learn about the opinions of nine teams.

Pupils' opinions about the use of Coach. All pupils report that they like the quick and easy way to create graphs from tables. They also appreciate that they can copy and paste graphs and tables into their report. They find it convenient that the instructions are not only presented on paper, but also inside the Coach activity. On the negative side, they mention that saving results causes sometimes runtime errors[2] and that explanations are sometimes too vague for them. Some pupils report difficulties with getting the graphs exactly in the way they want them: choosing unique names for quantities and labelling axes seem to be the biggest problems.

Pupils' opinions about the subject and the learning material. Almost all pupils find that the quiz and the newspaper article form a nice introduction to the subject. Half of the pupils find the 15-minutes demonstration of the software at the beginning too short. From the questionnaire we cannot decide whether the pupils rate the Coach instructions as clear and sufficient. They have no complaints about the number of hints. All pupils report that the structure of the second activity, which consists of explicit questions before the task of answering the research question, helped them to formulate their summary of height growth of Dutch children. Half of the pupils are of opinion that the third activity about secular height growth links up well with what they have learnt about Coach before that. But also half of the pupils find that this assignment does not link up well with their mathematical knowledge.

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[2] The installation of the software on the computers in the computer lab differed from the way the Coach developers had intended. This was the main cause of the problems that pupils had with saving results.

Pupils' experiences. All pupils indicate that they like it that they may choose the last assignment. Some admit that they choose the task that they guess will be easiest, others (mostly girls) give more personal reasons. For example, Gitta and Caroline motivate their choice of assignment C by: "We have come ourselves to an age where we get heavier. Therefore we find it interesting to investigate the mean weight." Inge and Annemieke choose assignment A because "the report is without Coach; we are not very good with computers." Linda and Joni, who investigate height growth for girls, write: "Turner syndrome looks to us an interesting subject. You have to compute something about girls and then it is nice to see if it matches with yourself." None of the pupils find the assignments easy, but their biggest complaint is that time was too short[3]. No surprise that pupils' appreciation of the practical work range from "We do not like it much. We need more time for it." to "We like it very much. You get a lot of freedom. Only time was too short." Some pupils prefer the normal mathematics lessons in which the teacher is always around to help them; others enjoy the freedom in this kind of activities or simply like the subject. We quote Inge and Annemieke: "It was fun to do; something different from the textbook and an interesting subject."

We continue with our main findings from classroom observations and reports.

The strongest impression makes the good quality of the pupils' work in general. For example, Gitta and Caroline describe the change of mean weight for Dutch children as follows: "The weight of boys and girls increases smoothly until the age of 8 years, although we must add that girls weigh on average less than boys in this period. From the age of 8 years, girls are on average heavier than boys, possibly because they reach puberty earlier than boys and many changes occur in their body. This weight increase lasts until the age of 14; from this age on boys are heavier than girls. This you can see in the diagram, because the green curve is below the blue one. The weight of boys increases for some years and they stay in the end heavier than girls." The fact that they relate a difference in the graph to earlier puberty for girls than for boys is certainly good for a pat on the back. Another example of good work is the description of Linda and Joni of the difference in height and height increase between healthy girls and girls with Turner syndrome: "The difference in height for girls with and without Turner syndrome is not so big until the age of two. You can also see that they skip puberty more or less. Where a healthy girl starts growing faster during puberty and you can see a peak in the diagram, this is not the case for girls with Turner syndrome. They grow as it were constant." The pupils make good sense of the height diagram and of the increase diagram, regardless of the weaknesses in the formulation from mathematical point of view.

The abilities of pupils to create and interpret graphs has been focus of numerous research studies (e.g., see [8] and references therein). What we observe in our group of students is that they usually have no problems with interpretation tasks up to intermediate level. Local processes, i.e. point reading, are easy for this group of pupils. However, most of the questions in the learning material are interpretation tasks on a global level and can be characterised as

[3] The pupils are right: the authors underestimated the time that pupils need to learn Coach in the first activities, especially when errors in the learning material or pupils' reading mistakes disturb the learning process.

interval reading. In general, pupils have no difficulties with these tasks. We do not notice interval/point confusion. Maybe that our attention to unambiguous phrasing of questions paid off or that our focus on interval tasks simply avoids the confusion.

Only one question in the activity about height-for-age, viz., “Who grows faster, boy or girl, and at what age?”, puts a few pupils on the wrong track. They do not use the increase diagrams to answer the question, but they instead mix up the statement “boys growing faster than girls” with “boys being taller than girls” and use the graph of the difference in height between boys and girls for their answer. This is an example of the slope/height confusion, meaning that pupils confuse the quantity and the change of a quantity. However, it may well illustrate that pupils are not used to think of change of a quantity as an interesting quantity itself. We notice this in assignment C, which is essentially the same as the activity about height growth (weight instead of height). Yet no pupil investigates increase diagrams.

The pupils have no problem with a second vertical axis in the diagram. They use it for the first time when they plot the difference in height of boys and girls. Bianca and Floor explain the use of a second axis as follows: “The second vertical axis is for scaling the difference. This way, you can see the line better and it is not so small.”

Name clashing troubles pupils who follow instructions literally and who forget that names of quantities must be unique. For example, these pupils read that the increase in height for boys is given the name ‘increase’ in the instruction. When asked to create the increase diagram for height of girls, they redo the instructions literally and re-use the name ‘increase’ instead of a more specific new name. The software refuses for good reasons, but to the surprise of the pupils. Fortunately, pupils’ difficulties with the software do not hinder them to get good results in the end. Teams often help each other out.

The learning environment has a function-fit tool that enables pupils to work with various regression models. We notice that they feel free to try any function type in the tool. For example, the weight-for-length curve in assignment D is fitted by pupils as a modified rational function and as a modified exponential curve. When asked a simple function-fit, most pupils interpret this as a fit with a straight line; a quadratic fit is not ‘simple’ for them. They have difficulty in analysing a curve in pieces or at least they do not do this on their own.

We end with some general remarks. Girls perform better than boys in this experiment do and weak pupils have a chance to do better. Girls seem to be more interested in the subject of body growth and they pay more attention to the report (even decorate it with pictures).

Extensions: ICP-model, Studies at School, …

Henceforth we restrict ourselves to height growth in relation to age. One can hardly expect that pupils discover a mathematical model for height growth by themselves. But it is already nice if they can validate a proposed mathematical model. A fine model to test is the infancy-childhood-puberty (ICP)-model [9]. This model breaks down growth mathematically into three partly superimposed components:

1. *Infancy* (0-3 years). Restricted growth, in which the growth rate is a linear function with respect to height. It is represented by the modified exponential curve:

   $H_1 = a_1 - b_1 e^{-c_1 t}$, where the symbol t stands for age.

2. *Childhood* (from 3 years of age to the onset of puberty). A simple quadratic function fits growth during this period very well:
$$H_2 = a_2 t^2 + b_2 t + c_2$$
3. *Puberty*. The contribution of the pubertal growth spurt to the final height can be modelled using a logistic function:
$$H_3 = \frac{a_3}{1 + e^{c_3 - b_3 t}}$$

Here, a_1, b_1, c_1, a_2, b_2, c_2, a_3, b_3, c_3 are positive parameters, which must be estimated from the growth data. The mean height for each age, *H*, is given by $H = H_1 + H_2 + H_3$.

One can use the following curve fitting procedure. First, note that the childhood component is the only part of the model that is described by an unrestricted growth process. Therefore, it makes sense to begin searching for a parabola that on the one hand fits well the height between 3 and 10 years of age, and that on the other hand reaches its maximum at the age of 20 years, when height growth usually stops. This curve fitting process is supported in Coach by a manual function fit, as shown in Figure 3.

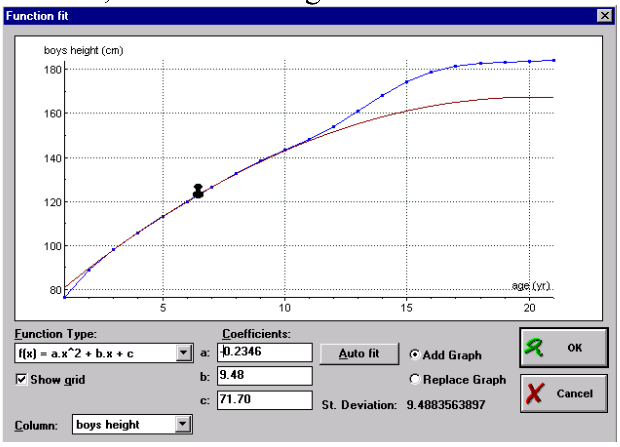

Figure 3. Manual Function Fit of Childhood Component of ICP-model.

The above screen dump shows a manual function fit in action. The formula of a parabola, $y = ax^2 + bx + c$, has been selected as function type; the selected column corresponds with the mean height of Dutch boys. The icon of the pin on the screen dump is such that the approximation has been fixed at that location. By dragging another point of the parabola with the mouse you can re-scale the graph. When you release the fixed pin by double clicking, then you can translate the parabola, i.e., independently change the parameter *c*.

After subtraction of the extrapolated values of the childhood component from the observed values during the periods before and after this phase, two additional components are extracted and modelled. The modified exponential regression model and the logistic model can be applied for these two periods, respectively. The results for Dutch boys and girls are:

Boys: $H_1 = 76.4 - 19.4e^{-1.56t}$ $\quad H_2 = -0.235t^2 + 9.5t - 4.7$ $\quad H_3 = \dfrac{16.1}{1 + e^{16.4 - 1.2t}}$

Girls: $H_1 = 74.3 - 18.7e^{-1.65t}$ $\quad H_2 = -0.256t^2 + 9.8t - 4.8$ $\quad H_3 = \dfrac{8.6}{1 + e^{12.4 - 1.1t}}$

If you plot the sum of the three contributions to the mean height of boys plotted in the same diagram with the data plot of the mean height of Dutch boys you would see that the difference between computed and measured values is everywhere less than 0.5 centimetres!

This is a beautiful result for a formula that is completely built up from mathematical models that are studied at school. However, do not think that the ICP-model is the only successful mathematical model for height growth of boys and girls. In the literature [10-14] you can find various models that could be investigated by pupils. You can also apply the mathematical models to study other growth parameters. For example, the ICP-model works fine for weight in relation to age. But there are many more topics that pupils can investigate. They suggest themselves the following new research questions for further practical work. How can it be that humans get taller every generation? Will this process ever stop and does this hold for animals, too? When does the mean height usually begin to decline? How does your own height and weight or the data of your classmates compare with the reference data?

The last suggestion is probably the most interesting and most instructive. Pupils will encounter as a matter of course many problems in doing research, and not only mathematical problems. Some of the questions are. How do you set up the research study? Are you going to measure growth parameters or just ask participants? How do you deal with personal data? What if people refuse to collaborate? How do you present your results? And so on. In this way pupils can indeed experience many aspects of doing research.

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