

# **Walk like a Mathematician; An Example of Authentic Education.**

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## ***Abstract***

A child learns walking at the age of one. Later on walking goes naturally and most people do not give it much thought anymore, even though it is an interesting periodic motion from mathematical and biomechanical point of view. The fact is, every person walks differently and can do this in various ways: normal walking, jogging, running, skipping, tiptoeing, etc. Gait analysis deals with a scientific description of human locomotion. Video measurement is a much-used tool in the empirical research work of movement scientists.

We have created learning materials for pupils in upper pre-university education to investigate human movement in the same style as biomechanists do. The pupils record video clips of their own ways of walking, e.g., on a treadmill in a fitness centre, and they use the computer learning environment Coach for measuring these movies and analysing the collected data. They apply their knowledge of mathematics and science to answer research questions like “When does normal walking move into running?” and “How does your body motion change at various gait speeds?”.

We shall discuss the video technology, our learning material, and our experiences. In this way we hope and expect to present an inspiring example of how ICT and context situations can contribute to the realization of challenging, cross-disciplinary investigation tasks that resemble professional research practice.

## ***Introduction***

In the Dutch examination programme of senior secondary education, pupils are required to build up an examination portfolio by carrying out some small practical investigation tasks and one rather large, cross-disciplinary research or design assignment. Many departments of universities seize the opportunity to build up relationships with secondary schools and to attract prospect students by giving guest lectures at schools and by allocating time to support pupils with their research or design project. At the Faculty of Science of the University of Amsterdam this is organised in the format of pupils visiting a laboratory for carrying out their science experiments under professional guidance and for using technical facilities that are normally not present at schools [1].

In this paper we shall discuss practical investigation tasks on human locomotion carried out by pupils in their last two years of upper secondary education (age 16-18 yr.), either as small-size practical work (study load of 10 hrs.) or as a research project (up to 80 hrs. work) in the ‘UvA Lab Work’ framework. The investigation tasks have been developed in our educational research work that examines the possible contribution of ICT and real-life contexts to the realization of challenging mathematical investigation tasks for pupils. This work provides input for the ongoing development of the activity-based environment Coach for learning mathematics, science and technology [2], and it results in study materials for pupils that stimulate and enable them to carry

out investigation tasks at a high level. Software and pupils' activities are tested in practice. In this study, results come from

- a classroom experiment during regular physics hours with all pupils participating;
- a master class for interested pupils from various schools;
- a few case studies and research work carried out by interested pupils.

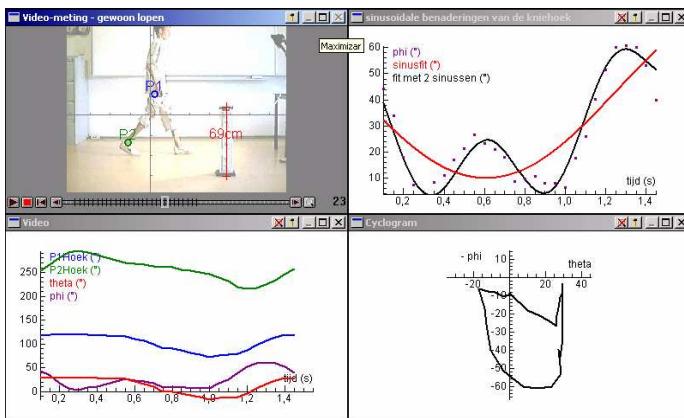
We interpret the authentic nature of the activities as the opportunity for pupils to work directly with high-quality, real-time data about human gait in much the same way movement scientists do. In essence, we try to make their math & science learning resemble practice, in which investigations can often be characterized as being challenging, complex, open-ended, and cross-disciplinary, and as requiring a strong commitment of participants plus a broad range of skills [3].

### ***Outline of the Learning Material***

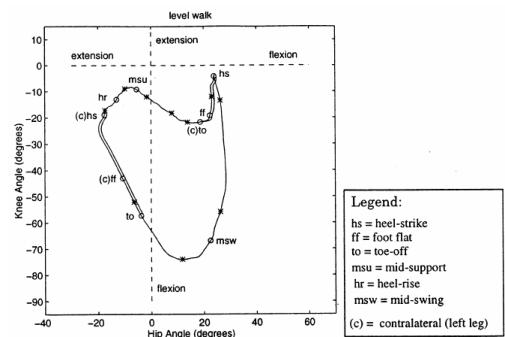
The learning material has been designed for a classroom experiment with pupils in upper pre-university education (16-18 yr.) who have already some experience with practical investigation tasks and with Coach, including the data video tool. A detailed description can be found in [4]. The learning material can be downloaded from [www.science.uva.nl/~heck/research/walking](http://www.science.uva.nl/~heck/research/walking) and it consists of the following four assignments:

*Activity 1. Mathematical Analysis of Human Gait.* Pupils are introduced into the typical normal walk cycle and the events of gait. They also practice their skills in using the graphical and video facilities of Coach and its curve-fitting tool. In the first activity we concentrate in particular on the movements of the arms during normal walking. The arm motion with respect to the shoulder joint is similar to that of a pendulum. The horizontal position of the hand,  $x(t)$ , is modelled mathematically as a combination of a straight line and a sinusoid, i.e., by the formula  $x(t) = at + b \sin(ct + d) + e$ , with parameters  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $e$  that must be estimated. This task prepares pupils to the use of a moving reference frame in video measurement and to multi-step curve-fitting (first a straight-line fit of the measured data and then sinusoidal regression applied to the residual values).

*Activity 2. Swing Phase in Sauntering Gait.* Pupils analyze the motion of the swing leg during a slow walk. They record the coordinates of the hip and the ankle of one leg with respect to the knee joint and they derive from these data the hip angle and the knee angle of the leg as a function of time. On the left-hand side of the screen shot below you see the recorded video clip of a pupil sauntering in the classroom and the time-graphs of the measured angles. In the upper-right window of the screen shot you can see that a sum of two sinusoids with different frequencies turns out to be a rather good description of the knee angle as a function of time. This means that the leg movement is well described by a lateral and dynamically coupled oscillator model [5]. This holds for many human gait patterns.



Screen shot: Jordi sauntering in the physics classroom.



Cyclogram of a normal equal-level walk.

**Activity 3. The Gait Cycle in Sauntering Gait.** This activity is a continuation of the previous one, but now the complete gait cycle is considered, i.e., the sequence of motion occurring from heel strike to heel strike of the same foot. The pupils investigate how the hip and knee angle can still be described by a sum of two sinusoids. Furthermore, they investigate the leg motion via the hip-knee cyclogram, in which the joint angles of knee and hip are plotted against each other. This diagram is a parametric curve with respect to time. Characteristic points on the curve correspond with distinct events during the gait cycle. Cyclograms change when gait conditions alter. For example, cyclograms of uphill or downhill, and level walk have different shape. In the lower-right window of the above screen shot you see the cyclogram obtained by a pupil who studied sauntering. The shape of the cyclogram and the points marking important events in the stride such as heel-contact, heel-rise, and toe-off are roughly the same as the diagram to the right, which is taken from the literature [6].

**Activity 4. Investigating one's own motion.** The first three assignments are video-based laboratories that can be done during regular lessons. The main purpose of these activities is to familiarize the pupils with the theoretical framework, terminology, research methods and techniques of movement scientists and to prepare the pupils for the fourth assignment, which is a small investigation task that they can carry out independently. In this activity the pupils perform a gait of their own choice and record the motion with a web cam. Hereafter they collect and analyze data on their own video clip. To limit the practical work to a rather short assignment, one can let the pupils only construct the hip-knee cyclogram of their motion, let them compare their result with one obtained by a fellow pupil, and ask for a short note about their findings. The main mathematical issue in this activity is graph interpretation: pupils must relate the diagrams that they create with real-world events.

The learning material has also been used in the master class and case studies for interested pupils to familiarize the participants with video measurement, sinusoidal regression, and the topic of human locomotion before they do their own experiments and answer their own research questions. In these studies, pupils write more elaborate reports and they often give a presentation of their work.

## ***Objectives***

Our main objectives are to let the pupils

- work with real data they collect from video clips self-made with a web cam;
- carry out practical work in which they can apply much of their present knowledge of mathematics and physics in a real life context;
- practice ICT-skills, in particular setting up an experiment, making a video clip and carrying out measurements on it with a data video tool;
- experience that diagrams that are used in practice are not just pretty pictures, but contain much information about the real-life phenomenon under study;
- be in contact with current research work, in our case movement science, including the nomenclature and research methods used.

These objectives are rooted in our belief that the main purpose for doing practical work is to experience authentic mathematics and science, to enjoy and become competent in it, and to experience that mathematics and physics have some bearing on real life. We choose on purpose a complex, real-life phenomenon for which a complete mathematical and biomechanical description fails and certainly would be out of reach of pupils, because pupils can experience in this way that simplified mathematical and physical models are nonetheless useful in the sense that they can still yield interesting results and provide qualitative answers to research questions.

We deliberately let the pupils do the experiments themselves, following more or less their own route and choosing their own gait pattern. One reason is that we hope that this motivates them and makes them more strongly committed to their task. Another reason is that the pupils get in this way first-hand experience of mathematical and physics concepts and of techniques used in research work. We hope and expect that it will give meaning to these concepts as well as to the research methods, tools and techniques. Therefore we try to let the pupils' tools resemble the professional tools as much as possible. The practical work is meant to contribute to pupils' understanding of what video analysis means, what it is good for, and how it can be applied. What they have learned in this small investigation task, they can utilize in the larger research project that they carry out in the final year at school.

In the activities we try to pay much attention to reading and interpreting graphs. Our main motive for this is that pupils tend to interpret any graph shown to them in a narrow-minded way. In physics lessons it is a distance or velocity vs. time graph and in mathematics lessons it is almost always a graph of a formula. In this way pupils do not experience that a single picture can convey much information about a phenomenon and that plotting a graph may help to interpret measured data. This provides a serious handicap to their understanding of many subjects of study [7].

Mathematics and science is not just investigation. It is a human activity in which scientists share results, concerns, ideas, plans, and questions among collaborators. To mimic this as much as possible we choose in the classroom experiment an instructional setup in which pupils work in pairs and in which teams are obliged to work with results of others. Furthermore, we organize that pupils can work not only during regular lesson hours, but also outside the classroom. For the master class and the case studies we take pupils to a fitness centre to record several video clips of body motions.

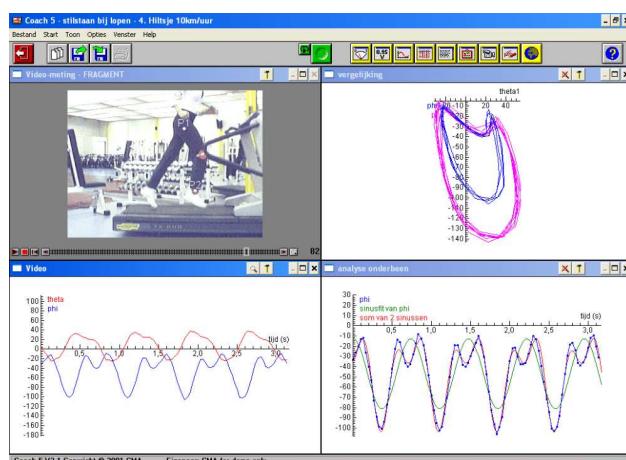
The benefits of making mathematics and science learning better resemble mathematics and science practice are clear. But how does one implement it at school? In this paper we report about such attempts on a small scale and we describe how our instructional setup in the authentic learning model works in practice. Our main research questions are:

- Is the computer learning environment Coach [2] a useful tool in the sense that it supports pupils in obtaining, organizing, displaying, manipulating and analyzing data?
- Does the instructional strategy of first familiarizing pupils in a classroom setting with the topic and the way professionals work before engaging them in doing their own gait analysis work well, i.e., does it make authentic science feasible at pupil level?
- What difficulties do pupils encounter in their investigation task?

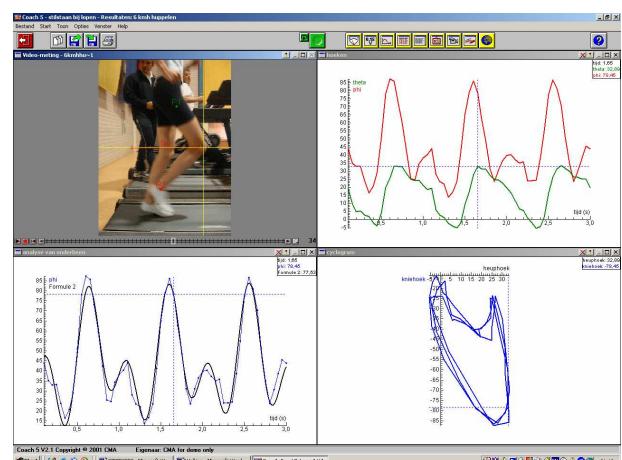
Note that the chosen instructional strategy closely resembles authentic science practice: scientists usually assemble prior knowledge about their subject of study from expert sources and they acquaint themselves with the common methods and techniques. If necessary, they invest time to acquire the skills needed for using new and promising technology in the field of study.

### **Video Measurement**

Our first research question can be specialized into the question whether pupils using a web cam, a computer, and Coach can obtain good hip-knee cyclograms. If the screen shot of Jordi sauntering in the classroom does not convince you, below we show two more screen shots coming from case studies, viz., Hiltsje running (master class) and Laurine skipping on a treadmill (research project).



Screen shot: Hiltsje running on a treadmill.



Screen shot: Laurine skipping on a treadmill.

Let us first explain how the pupils collect the motion data with Coach. In the right screen shot you see in the upper-left window a video clip of Laurine skipping on a treadmill. The pupils gather their data simply by clicking on points on the video clip: first they inform the system that they want to reposition the origin of the coordinate system for each frame of the video clip on the knee joint of the right leg and also that they want to measure in each frame the positions of two points, viz., the hip joint and the ankle joint. By measuring positions in polar coordinates, the hip and knee joint angles can easily be computed from the measured quantities. In the upper-right diagram the hip angle and the knee angle are plotted against time. Using a treadmill the pupils can easily record more than one gait cycle. This has the advantage that they can verify periodicity of the movements and can filter irregularities if they wish. In the lower-left diagram you see how well the model of two sinusoids works for the knee joint angle as function of time in skipping gait. The periodicity of the motion is nicely visualized in the cyclogram in the lower-right window. The cross-hairs in the diagrams indicate that Coach is in scanning mode: This means that pointing at a graph or a table entry automatically shows the corresponding video frame and that selecting a particular frame highlights the corresponding points in diagrams. This makes scrubbing, i.e., advancing or reversing a clip manually, an effective means to precisely identify and mark interesting events in the video clip and to relate them with graphical features.

### ***Findings***

Only in practice one can find answers to the questions whether the free choice of gait pattern by the pupils will result in useful cyclograms and whether the mathematical analyses that the pupils practice in the first part of the project are applicable to their own body motions. One of the most important messages coming from the classroom experiment, master class, and case studies by pupils is that it goes surprisingly well, taking into account that body motions during locomotion are complex and that only simple mathematical and physical models are applied. Pupils agree: e.g., Dries and Edwin write in their report “It is cool to see that your legs make such a nice mathematical curve, the stroke when your foot makes contact with the ground, and so on.” They add to this: “Furthermore, it is something different from the usual standard lesson. This was nice.” This remark agrees with our field observations of pupils seemingly enjoying their work and being committed to their tasks. The fact that very few pupils have difficulties with using the video tool, with creating the graphs, and with applying a given regression model contributes to the pupils’ satisfaction. All of them get results!

But interpreting graphs in the context of body movements turns out to be different story. For many a pupil, comparing cyclograms just means writing down the differences between the diagrams without coupling them to the motions in the video clips. We suspect that many a pupil does not

quite grasp the meaning and purpose of a cyclogram, even though the learning material (after a first adaptation) discusses the diagram extensively. This may also be the reason that pupils do not bother about the orientation hip and knee joint angles so that their cyclograms occasionally are rotated or mirrored with respect to the ones presented in the learning material. Especially the convention of negative knee joint angles seems to be not understood or looked upon as irrelevant. Discussing in class a prototypical example of the use of a cyclogram is probably the recommended remedy here.

Luckily there are still many pupils' reports in which successful attempts are made to find differences between cyclograms and to explain them in terms of body motion. For example, Manon and Marleen compare in their case study cyclograms of level walking on a treadmill with walking on a 10% uphill slope. They write in their report: "Full extension of the leg, i.e., a knee angle of zero degrees, occurs less in walking against a 10% slope. This is so because firstly you make more steps and consequently your leg has less contact with the ground floor. Secondly, if you have put your foot flat on the floor, you still do not have a knee angle of zero degrees because you walk uphill. The positive hip angle is greater when you walk uphill, because then you have to raise your legs more, with the consequence that the hip angle gets larger. The negative hip angle becomes larger, because you leave your leg behind for a longer time since your front leg needs more time to put enough force for getting uphill." Compare this with the conclusion in [6] that the total range of hip movements (knee movements) is a linearly increasing (decreasing) function of the slope inclination. The pupils' results are quite close to the conclusions in the scientific report. Their terminology lacks precision, but in fact they do very well in their case study. They do more than just writing down an empirical result: they also try to explain the results obtained from their data in terms of body motion. It is clear that for Manon and Marleen the graphs are about something and that many things read in the diagrams can be connected with the gait process.

The fact that pupils can work well with the video analysis tool does not mean that there are no problems. Some pupils have conversion problems with recorded video clips because Coach allows only a limited number of compression/decompression techniques. It is a matter of luck whether there is enough video-technology knowledge present at school to find help. Doing the video measurements manually quickly becomes a time burden in a research project. To get an idea: a measurement of the leg motion during one gait cycle of about 1 second, recorded at 30 frames per second, means that one has to click altogether at least 90 times on the video clip. This screams for a facility to track moving objects in a video clip automatically.

Comparison of graphs is limited in Coach to two graphs in one diagram; this hinders pupils when they study the influence of a gait parameter on the shape of a cyclogram. E.g., Laurine and Rosa solve this problem in their study on the effect of gait speed in the following way: "...The only solution is to let all graphs have the same 'graph size'..." The vague term 'graph size' means that

the ranges of the plotted quantities are chosen the same for all diagrams. Because of lack of experience, pupils do not always make optimal use of the learning environment. E.g., Laurine and Rosa apply sinusoidal regression twice to get a sum of two sinusoids model for the knee motion. But they do not seize the opportunity to apply the signal analysis tool for verification of their results. Pupils often concentrate on the diagrams alone and they do not make much use of scrubbing the video clip and inspecting the synchronized tables and diagrams at the same time.

Pupils in general underestimate the time and effort needed for setting up an experiment at adequate quantitative precision. Kristian, Yvonne, and Jordy add to this in their report: "...writing the report and ordering the collected data took more work than expected. This was a lesson for us!"

### ***Conclusion***

In summary, we are quite satisfied by the motivation and performance of the pupils. They are able to produce similar results as reported in the scientific literature about gait and they learn a lot about the possibilities of graphics to convey information about a real-life phenomenon. Affordable technology like a web cam and the learning environment Coach makes this possible at student level.

### ***Acknowledgements***

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