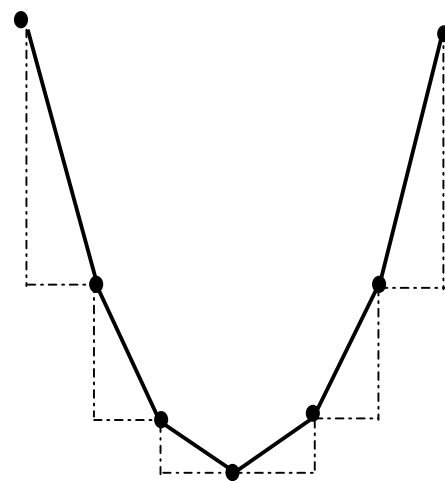
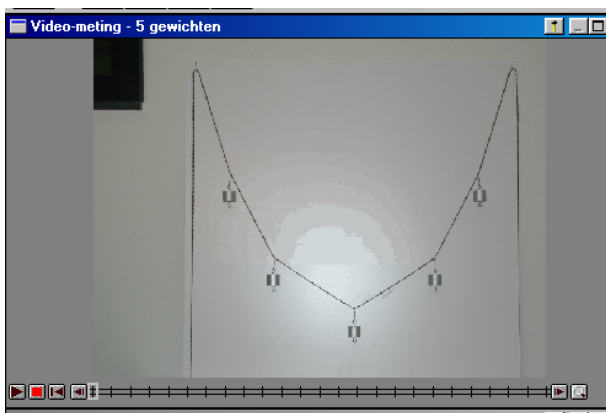


# Practical Investigation Task: Modelling with Coach

Author: André Holleman (Bonhoeffer college, “teacher in research” at the AMSTEL Institute)  
© 2001 AMSTEL Institute. Further usage is only permitted with quotation of sources.

A practical investigation task for pupils at upper secondary school level who have no experience with Coach. Estimated working hours: 6.

Below you see a photograph of a hanging chain to which five equal weights have been attached. Next to it is a schematic drawing of the chain and the weights. In this investigation task you will use this drawing to practise modelling with Coach.



## Table of Contents

Introduction .....	2
Measuring and Modelling .....	2
The Report.....	2
Contents of the Report .....	2
Getting Started with Modelling.....	3
Some Practice First.....	4
Executing the Model .....	4
The Diagram Window.....	4
The First Model .....	5
Where Does the Model come from? .....	5
Task 1 .....	5
Practising without a Closed Formula .....	6
Saving Results.....	6
Opening Results.....	6
Task 2 .....	6
Task 3 .....	7
Task 4.....	7
Task 5.....	7
Appendix .....	8
Installing Coach .....	8
Mathematical Operations in Coach.....	8

## **Introduction**

This practical investigation task is part of what becomes later a more thorough investigation task. At the same time, this task is part of research work about usefulness of Coach 5 in mathematics education. The experiences will contribute to improvements of this learning environment.

## **Measuring and Modelling**

You may have seen already that you can use the computer and Coach to collect data in measurement experiments. You can also process and analyse in Coach measurements that have been carried out by others. The latest development in Coach is that you can collect data from digital videoclips and photographs, which you or others have recorded.

One of the possible ways to analyse collected data and to understand the results (and that is what it is all about): *mathematical modelling*. In this practical investigation task you are going to work with the *modelling environment* of Coach.

You will see that dealing with such a computer model looks like working with recursive formulas and closed formulas. Quite often, models have to do with differences, difference quotients, and (numerical) differentiation.

## **The Report**

You must write a report about the progress of your research. It should not only contain the results of your work; also write down your planning and reasoning. Because we are interested in how easy or difficult you find this task and how you experience working with Coach, we are also curious about “mistakes” or digressions that you have made along the way. Besides, from mistakes you learn much, if not most. And no one reaches his or her goal without digressions! Show us how far you got or why you could not get further.

The report must be well organised and neatly written. So, please use a text editor. If you have problems with formulas, fill them in by pencil. If you are not good in drawing with the computer: drawings by hand are also accepted.

*You can copy and paste tables, diagrams, and text from Coach into a Word document. You are advised to start Word, in conjunction with Coach. Then you can copy intermediate results like graphs, tables, and whatever you wish into a Word document. You can decide later what you will use of this in the final report.*

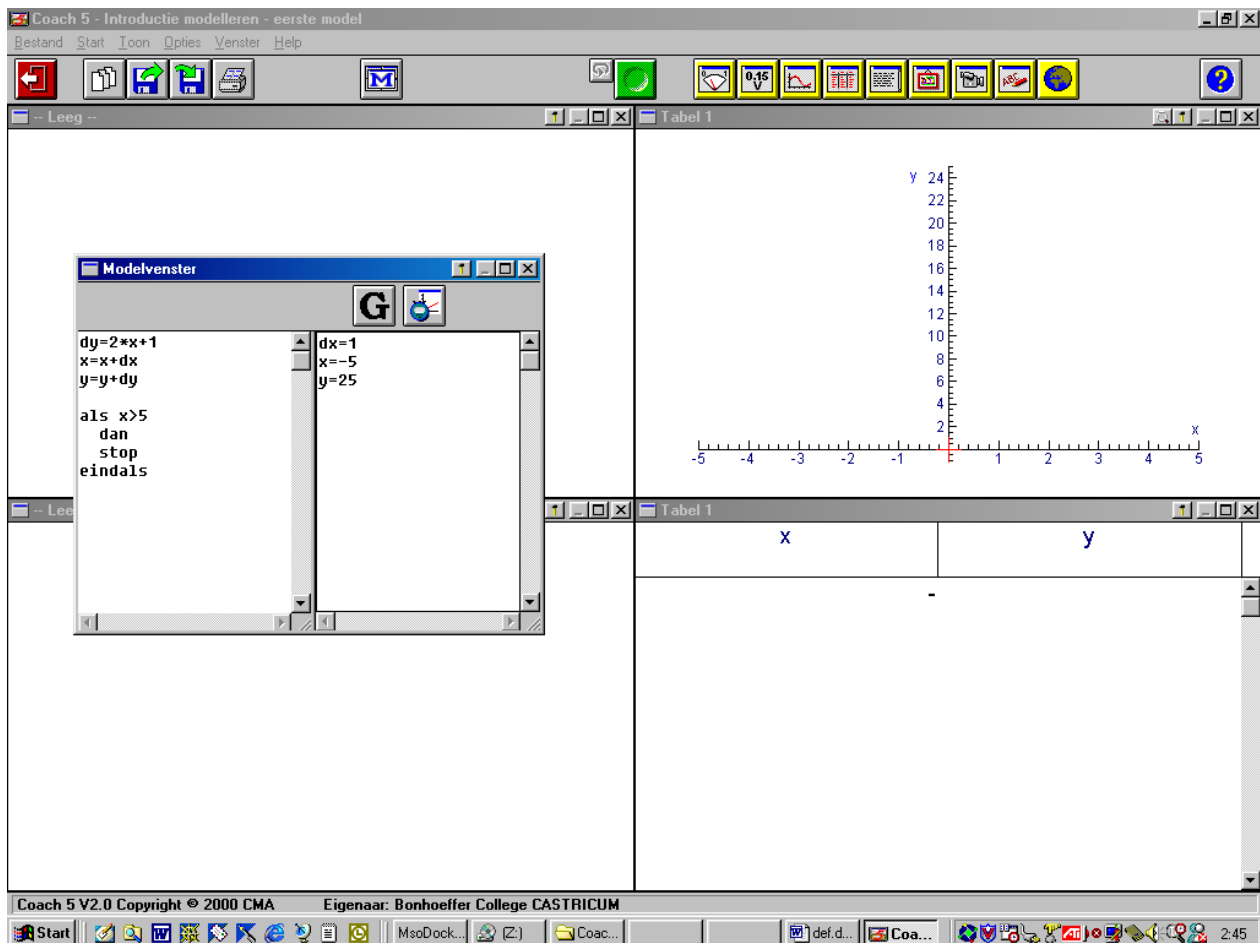
## **Contents of the Report**

- Introduction (title, author(s), date, abstract, table of contents, ...)
- Report of the various parts of the research in such a way that the reader not only sees the final results, but that he or she can also understand how the research work progressed.  
It may be interesting to report how things should *not* be done. Maybe you find a nice result or method that someone else may not have found or thought of, or that could be used in other research work. Or write down suggestions for improving the investigation task and/or Coach.
- Conclusions or answers to the research questions (if possible)
- Final conclusion about the tasks and the usability of Coach.
- A lab journal that contains, among other, things the time you spent on various parts of the work and the division of work.
- Please return the Coach-CD and a diskette that contains your Coach results and your electronic report.

## Getting Started with Modelling

In Coach you can make all kinds of *projects*. A project consists of one or more *activities*. Our project is on a diskette. To get there, you must do the following:

- Start the Coach 5 Home Version: Modelling.
- Press the button: 'Choose Project' (second button from the left in the toolbar). Only a list of the projects that reside on the CD-ROM is displayed. But we must get to the diskette.
- Arbitrarily select a project, for example: 'Introduction to modelling'.
- Press the button: 'Open activity' (second button from the left in the toolbar). Now you see the activities of the selected project. (Not the correct one for us.)
- Press in the dialog window the button: Browse.
- Select disk drive A:
- Choose: 'our first model'.



Your screen should look like above.

You see a model window, a coordinate system, and an empty table.

The model window is split into two parts. On the left-hand side stands the actual mathematical model; on the right-side side are the initial values of variables.

You see:  $x$  starts from  $-5$ ,  $y$  from  $25$ . Furthermore,  $dx$  equals  $1$ :  $x$  increases with step size  $1$ .

On the left-hand side you see how first  $dy$  is computed and then the  $x$ - and  $y$ -values are updated. Hereafter it starts all over again.

The computer model is executed until  $x > 5$ .

## **Some Practice First**

*Sometimes an answer is requested. In your report you must at least pay attention to this.*

## **Executing the Model**

- Press the green 'Start' button. It becomes red. The computer model is executed. The graph appears and the table is filled.
- Write down what you think that will happen if you take  $dx = 0.5$ .
- Take  $dx = 0.5$  and execute the model. Compare the result with your prediction.
- Notice that the new graph does not fit in the diagram window.  
Right-click in the diagram window. A tool menu appears.  
Choose: 'Zoom to fit'.
- "Play" with some more initial choices of  $dx$ ,  $x$ , and  $y$ .  
Hints:
  1. You can interrupt a computation at any moment by pressing the red button.
  2. You can close and reopen the model window by pressing the M-button.
  3. You can also monitor the execution of the model. Right-click in the model window and select the menu item 'Monitor'. The toolbar of the model window changes: a green button and a slider bar appear. The green button is for starting the execution; the slider bar controls the pace at which execution takes place.
  4. When the diagram window becomes too messy: right-click in the diagram window and select 'Delete background graphs > Previous runs'.

## **The Diagram Window**

- The toolbar of the diagram window has five buttons. Explore the meaning of these buttons.
- Right-click in the diagram window.  
Investigate the purpose of 'Scan' and of 'Diagram style'.
- Right-click in the diagram window.  
Select 'Analyse > Function-fit'.  
Select from the 'Function Type' scroll-menu various types.  
Do *not* press the OK button, but use the 'Auto-fit' button to let Coach compute the graph that fits the model data best for the selected function type. Which formula fits you graph best?  
Press 'OK'.
- Right-click in the diagram window and select 'Simulate'.  
Find out what you can simulate.
- Press the 'Exit activity' button (utmost left), do *not* save. All remains the same.

There are more options in Coach. Also, there are quite often many different roads leading to the same result. Maybe you find yourself some new ways: write them down in your report.

## The First Model

### Where Does the Model come from?

Measurements of a hanging chain with attached weights gave the following schematic result.

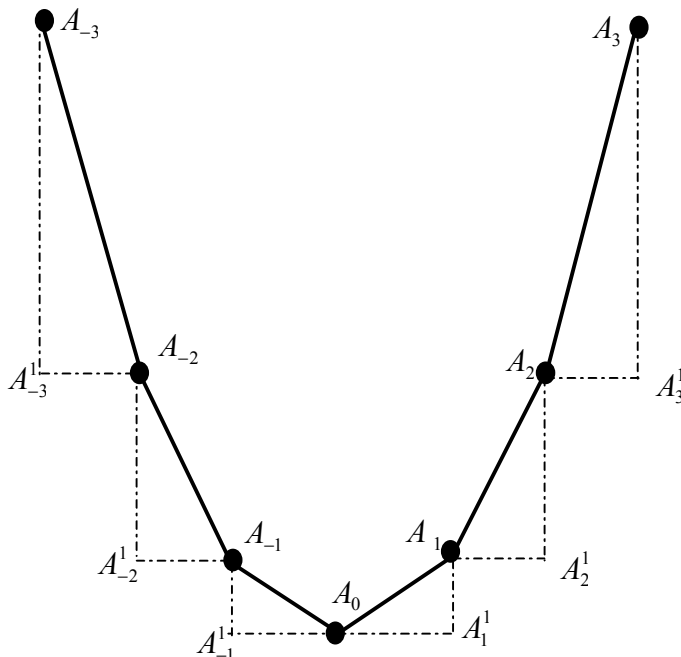
See the picture on the right.

On the chain hung symmetrically five equal weights. The two upper points represent the points of suspension. The other points are the points of application to which the weights had been attached. The horizontal distances between the weights were equal:

$$A_{-3}^1 A_{-2} = A_{-2}^1 A_{-1} = A_{-1}^1 A_0 = \dots = A_2 A_3^1 .$$

It turned out (approximately) that the slopes (= directional coefficients) of  $A_0 A_1$ ,  $A_1 A_2$  and  $A_2 A_3$  are in ratio 1:3:5.

A similar result holds for slopes on the left-hand side.



We start simple and choose in the first model  $A_{-3}^1 A_{-2} = A_{-2}^1 A_{-1} = A_{-1}^1 A_0 = \dots = A_2 A_3^1 = 1$ .

So:  $A_0 A_1^1 = 1$ .

### Task 1

- Verify that “our first model” matches the above.  
Try to get to the bottom of the model, so that you understand how it works.
- What relation is there between  $x$  and  $y$ ? (Give the formula)  
Can you prove this formula of the model?

*Hint 1: use a sum formula for an arithmetic sequence.*

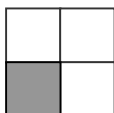
*Hint 2: compute the difference of the squares of  $n+1$  and  $n$ .*

*Hint 3: in the picture below you see that  $1^2 + 3 = 2^2$ .*

*Illustrate similarly that  $2^2 + 5 = 3^2$ .*

*Illustrate with a similar picture that for each  $n$  hold*

$$n^2 + (2n + 1) = (n + 1)^2 .$$



## Practising without a Closed Formula

You have seen that in this case there exists a simple closed formula that describes the relation between  $x$  and  $y$ , viz.,  $y = x^2$ . With this formula you can simplify the model enormously. But often it is not so simple or even impossible to find a closed formula. Often it is already difficult enough to find out how to proceed from say  $x = 5$  towards  $x = 6$ , or how to compute intermediate values.

This is why you may not use in the next tasks closed formulas like  $y = x^2$  *inside the specification of the model*. You may use them to find or prove regularities. After all, it is practising.

## Saving results

In the next task you can always start with our first model. You can simply adapt it. When you are finished, or want to continue another time, or want to store an intermediate result on a diskette, this goes as follows:

- Click in the activity toolbar: 'Save result as...' (fourth button from the left).
- Give a short description, for example 'Model 2'. (The first model is kept as it is.)
- Browse to A:
- Press 'OK'

**Save the Coach result of each task on the diskette.**

## Opening results

The next time you open the result file in the following way:

- Start Coach and select an arbitrary project if you start all over again.
- Press the button: 'Open result' (third button from the left in the toolbar).
- Browse to A:
- Select the desired result.

## Task 2

Adapt the model such that the same graph is drawn, but for a smaller value of  $dx$ : a model of a chain with more weights attached to it.

*Recall that you may not use the formula  $y = x^2$  in the model.*

It is a good idea to find out first whether the ratio 1:3:5:... remains valid for smaller step sizes than  $dx = 1$ . Because this seem the obvious thing to do first.

*Hint 1: use the graphing calculator (or Coach if you are familiar enough with the program) to investigate the function  $f(x) = x^2$ .*

*Hint2: explore in a geometrical way, just like you have done in task 1, hint 3.*

After you have found a result by trial and error, try to show that this is not a coincidence and “prove” it, just as you have done in task 1. Only then it becomes mathematics!

Write in your report how you have operated.

### Task 3

Make a model in case the coordinates of the points of suspension are (0, 10) and (10, 10), and the lowest point of the hanging chain has coordinates (5, 5).

Ensure that the model produces the same graph for various step sizes.

Also give the formula of the graph.

Again, write in your report how you have operated.

### Task 4

- Calculate with your pocket calculator the total length of the chain in our first model.
- Adapt the model so that it also computes the length of the chain.  
Call the length L and the increase per step dL.  
You can add the quantity L to the diagram or to the table via the 'Create/edit...!' menu-option.  
Look how x and y are presented at C1 and C2. Connect in the same way column C3 with L.
- Also get the length of the chain being computed in the model of task 2.  
For example,  $dx = 0.1$  will give you a good approximation of the length of a segment of the parabola.

### Task 5

In this task we challenge you to make a mathematical model of a chain that hangs under gravity, under its own weight. You can do this by looking at the situation drawn on page 5 for the case

$$A_{-3}A_{-2} = A_{-2}A_{-1} = A_{-1}A_0 = \dots = A_2A_3 = 1.$$

The “horizontal” distances are not equal anymore!

However, the ratio of the slopes remains the same.

- Take  $A_1^1A_1 = 0,5$  and compute the coordinates of  $A_{-3}$  till  $A_3$ .
- Make a “first model” of this.
- Make a model for  $\dots = A_{-3}A_{-2} = A_{-2}A_{-1} = A_{-1}A_0 = \dots = A_2A_3 = \dots = 0,1$ .
- Explain what the table that you have explored has to do with a chain that hangs under gravity.

## Appendix

### Installing Coach

- Insert the CD in the CD-ROM drive
- Go to the CD, for example via the desktop 'My Computer'.
- Double-click to open the CD.
- Double-click 'C5Start.exe'  
The installation program is started.
- Select 'English'
- Select 'Secondary School Education'
- Select 'Home Version'
- Select 'Coach 5'
- Select 'Coach 5 Full'
- Next
- Next
- Wait until you are asked a name and password. Fill in the name and the password exactly as it has been given to you
- After the screen with the installation program reappears:  
back, back, back, and exit.

In the 'Start' menu of your computer is now also present 'Coach 5 Home Version'.

You see that you can uninstall the program, whenever you wish.

*For this practical investigation task you need Modelling.*

### Mathematical Operations in Coach

Coach computes with angles in radians.

So, angles in degrees must be first converted into radians.

What do you want?	How you do this in Coach	What do you want?	How you do this in Coach
adding	$5+2$	$\sin(20^\circ)$	$\sin(20/\text{pi})$
subtracting	$5-2$	$\cos(20^\circ)$	$\cos(20/\text{pi})$
multiplying	$5*2$	$\tan(20^\circ)$	$\tan(20/\text{pi})$
dividing	$5/2$	$\sin^{-1}(0,5)$	$\arcsin(0.5)$
raising powers	$5^2$	$\cos^{-1}(0,5)$	$\arccos(0.5)$
$\sqrt{2}$	$\text{sqrt}(2)$	$\tan^{-1}(0,5)$	$\arctan(0.5)$
$\pi$	pi		