Diagnostic Testing with Maple T.A.

André Heck and Leendert van Gastel

Universiteit van Amsterdam, AMSTEL Institute,
Kruislaan 404, 1098 SM Amsterdam, The Netherlands

Abstract. At the University of Amsterdam we use the Maple T.A. system for diagnostic testing of incoming mathematics and science students. Purpose of the tests is to identify students at risk of failing their mathematics and science courses in the first year and to provide students insight in their starting level of mathematical knowledge and skills. In this paper we describe our experiences with Maple T.A.

1 Background

In the Netherlands, the transition from secondary school to higher education is currently the centre of interest. Universities struggle with heterogeneous groups of students that have a disjointed body of knowledge and skills, and that are not well-prepared for the more abstract level of studies at university. Technical universities experience that incoming students lack basic mathematical knowledge and skills. They sound the alarm in national newspapers and they send a letter of advice to the Minister of Education. In reaction to the complaints from higher education and in view of the upcoming change of the examination programme, the Minister of Education questions one of the latest reforms in the education system, viz., the 'Studiehuis' (study house), in which pupils are expected to learn how to learn, are self-responsible for their learning, and work independently.

With regards to the transition from school mathematics to university mathematics, the current complaints of educators at Dutch universities about the level of mathematical knowledge and skills of incoming students are much the same as those mentioned ten years ago by the London Mathematics Society in the report *Tackling the Mathematics Problem* [1]:

1. Students lack fluency and reliability in numerical and algebraic manipulation and simplification;
2. There is a marked decline in students' analytical powers when faced with simple two-step or multi-step problems;
3. Most students entering higher education no longer understand that mathematics is a precise discipline in which exact, reliable calculation, logical exposition and proof play essential roles.

Similar trends have been found across many European countries and many approaches to deal with the declining standard of mathematical knowledge and skills of new entrants to mathematics, science, economy, and engineering department have been proposed and implemented (see e.g. [2, 3]). The following

two recommendations are made in [4]: (i) use diagnostic tests to measure the mathematical competence of students upon arrival at university and (ii) provide prompt and effective support to students whose mathematical background is found wanting by the tests.

At the University of Amsterdam, an approach is being developed to cope with the mathematics problem at the threshold of the university. The approach is based on the trust that incoming students are in principle quite capable of overcoming the initial difficulties and can attain the desired level in a rather short term. Because of the heterogeneity, it is very important that both students and teachers are informed about the individual level and progress of each student. So initial and continuous assessments are an important part of the approach. For mastering the mathematical abilities, many exercise questions are envisaged together with a condensed mathematics text. A trajectory will be designed that is based on these assessments and the mastery learning of the mathematical abilities by doing many exercises. In 2005 we tried out the following four-step approach with all hundred students entering the study of chemistry, physics, astronomy, mathematics, and bio-exact sciences.

Step 1. Immediately upon arrival at the Faculty of Science, all freshmen take the one-hour diagnostic test that can be found in the appendix.

Step 2. During the first four weeks of the Calculus 1 course that all students take, they have weekly a session of two hours to practise basic mathematics.

Step 3. In the fifth week of the study year we let the students make the second diagnostic mathematics test. This digital test is made similar to the first test by randomisation of each question. Through this pretest-posttest design, students and staff can see the progress made in the meantime.

Step 4. Tutors take care of remedial teaching of mathematics using the recently published exercise book [5] and freshmen work in small self-help groups on mathematics problems under guidance of a student tutor.

An important requirement for the approach is that not much staff will be available in the long term. There are no structural funds available for this goal. So the challenge is to design a trajectory that involves only a modest staff deployment. For this reason, a pilot is undertaken with automated assessment in mathematics using the commercial package Maple T.A. [6]. This system uses the computer algebra package Maple [7] to generate and mark thousands of mathematical problem exercises from generic templates and it can deliver these exercises in various forms to students, ranging from diagnostic assessments, self-tests, and practice sessions to placement tests and summative assessments.

The development of this approach is part of two projects with other higher education institutions in the Netherlands. The project Web-spijkeren [8] has as a goal to develop didactical scenarios and assessment instrumentation for freshmen with heterogeneous mathematics levels. The project MathMatch [9] develops course materials and a database with mathematical questions for mathematics, science, and technology studies.

Our work is still in its early stages. This paper describes the instrumentation used and the results of the first try-out.

2 Overview of Maple T.A. with Blackboard Support

In this section we detail the computer algebra based assessment system Maple T.A. for Blackboard [6]. This system can be shortly described as a Blackboard-integrated system for creating tests, assignments and exercises, allowing automatic assessment of student responses and the creation of model solutions via the computer algebra package Maple. The integration with Blackboard concerns the user administration (authentication and grade book).

While some systems for computer algebra based testing and assessment already exist or are under development (for example, AiM [10], Stack [11], and WebALT [12]), the main reasons for us to choose the commercial system Maple T.A. for Blackboard are that it

– is developed far enough to have a rich set of question and assignment types;
– is based on two components, viz., Maple [7] and EDU Campus [13], which have been proved successful in educational practice;
– can be used with large groups of students without a high load on the computer environment;
– is integrated with the virtual learning environment currently in use at our university;
– offers licensed use of a familiar computer algebra system in an assessment environment.

In the following subsections we discuss the process of creating randomised questions, marking answers, and providing model solutions in Maple T.A., and the built-in facilities for students and teachers.

2.1 Authoring Questions

Initially, it is necessary to create a bank of questions that can be used in assignments. Maple T.A. questions are stored internally in Brownstone's EDU code, which is a scripting language. There exist four ways of authoring questions:

1. editing a plain-text script file in EDU code;
2. via the Question Bank Editor;
3. using the LaTeX to EDU conversion facility;
4. by way of a Maple 10 document.

Each method has its pros and cons. The first method of coding in EDU format can be done both on-line and off-line and provides the author maximum flexibility, but it is undocumented and very error prone. Use of the question bank editor can only be done while being connected to the internet. It makes heavy use of caching intermediate work and therefore the danger of loosing work and item banks is rather large. The Maple 10 document does not support all question types. We prefer the LaTeX authoring mode because it supports advanced question types, it allows off-line editing, and it supports re-use of many course materials already written in this typesetting language. It lowers the learning

curve of many a university teacher who wants to use the assessment system. One only has to convert via a web-facility the LaTeX code into EDU format. It is a pity that this is not made available as an off-line facility to come along with the Maple T.A. software.

Many of the question types available in Maple T.A. will be familiar to users of existing computer-aided assessment systems. Common examples of closed question types are multiple choice, multiple selection, true/false, ordering, clickable image, and matching questions. For these types answers are predefined, and can be marked automatically without difficulty. Common examples of open question types are fill-in-the-blanks (text or numerical value), essay, and graphical sketch problems. For these free-response questions it is more difficult to mark automatically.

But specific to Maple T.A. is the availability of mathematical free-response question types: (restricted) formula, multi-formula, and Maple-graded questions. The latter type is the one that we will discuss more deeply in this paper because, we believe that free-response questions are indispensable in evaluating mathematical knowledge and skills. However, there is still a purpose for multiple choice questions. For example, in Questions 22 and 25 of the pretest, the distractors make students think about the subject. Besides, a larger variety of question types in mathematical tests is attractive in itself. It is also a common misbelief to think that only standard mathematical questions leading to a clear and unique answer can be automatically assessed and that higher mathematical skills are out of reach in this approach. We refer to [14] for examples in which a computer algebra system is used to verify mathematical properties of given answers (out of an infinite number of possible, correct answers) and to assess advanced mathematical skills.

Henceforth we will use the following typical example to illustrate issues of generating randomised questions and of marking and commenting on students' responses. It is the LaTeX code for a randomised version of Question 5 in the pretest.

```
\begin{question}{Maple}
\name{powers} \type{formula}
\qutext{Simplify as much as possible: $$\frac{a^{\var{alpha}}
  (\var{npow3}b)^{\frac{1}{4}}c^{\frac{\var{gamma1}}{2}}}
  {\sqrt{a b c^{\var{gamma2}}}\sqrt{\var{n}}}}\,.$$
  assuming that $a$, $b$, $c$ are positive numbers.}
\maple*{expr := $RESPONSE;
  symexpr := select(has, expr, {a,b,c});
  numexpr := eval(expr,{a=1,b=1,c=1});
  symtrue := select(has, $r, {a,b,c});
  with(StringTools, CountCharacterOccurrences);
  if simplify(symexpr-symtrue, symbolic)=0 and
      CountCharacterOccurrences("$RESPONSE", "a")=1 and
      CountCharacterOccurrences("$RESPONSE", "b")=1 and
      CountCharacterOccurrences("$RESPONSE", "c")=1
  then
      if numexpr=sqrt($n) or numexpr=($n)^(1/2)
```

```
    then grade := 1.0
    else grade := 0.5
    end if;
  else grade := 0.0
  end if;
  grade;}
\code{$n = switch(rint(3),2,3,5);
  $npow3  = int($n^3);
  $alpha  = range(2,5);
  $gamma1 = range(1,3,2);
  $gamma2 = range(5,9,2);
  $r = maple("simplify(a^($alpha)*(($npow3)*b)^(1/4)
    *c^(($gamma1)/2)/sqrt(sqrt($n)*a*b*c^($gamma2)), symbolic)");
  $dr = maple("printf(MathML:-ExportPresentation($r))");}
\comment{The correct answer is \var{dr}.\newline
  You get this answer by rewriting the denominator as product
  of powers with rational exponents, followed by simplification.}
\end{question}
```

From the above code listing it is clear that Maple is not only used to check the student's response, but also to calculate the answer itself, to provide feedback, and last but not least to create randomised questions. In other words, the teacher does not calculate the correct answer, mark a student's answer or provide feedback: these are done by the computer algebra system.

In the first five lines of the \code part, several random variables are defined in the EDU scripting language to create a mathematical formula

$$\frac{a^{\alpha}(n^3 b)^{\frac{1}{4}} c^{\frac{1}{2}\gamma_1}}{\sqrt{\sqrt{n}abc^{\gamma_2}}} \ ,$$

where

$$n \in \{2,3,5\}, \quad \alpha \in \{2,3,4,5\}, \quad \gamma_1 \in \{1,3\}, \quad \gamma_2 \in \{5,7,9\} \,.$$

This means that we have in fact created here seventy-two exercises of similar type. This randomisation of questions makes it possible to generate many tests with questions of similar types, which is ideal for pretest and posttest designs. But also in self-assessment it is very useful that students can redo questions of similar type until they have reached the requested level of understanding.

The dollar variables defined in the \code part can be used in other parts of the question. For example, when the question gets instantiated, then the \var{alpha} in the question text (the argument of \qutext) is replaced by the randomly generated value of the parameter $alpha. In the last two lines of the \code fragment, Maple is used to compute the correct answer ($r) and a MathML expression ($dr) for pretty display in the feedback, using standard mathematical notation.

2.2 Marking and Feedback

A computer algebra system is needed to verify automatically whether the answer given by a student to a free-response mathematical question is algebraically equivalent to the correct answer as predefined by the author (in most cases a teacher). The simplest and much used Maple T.A. code for checking an answer has the following form:

$$\texttt{evalb(simplify(\$RESPONSE - } \textit{correct\_answer} \texttt{) = 0);}$$

In other words, we subtract the author's answer from the student's answer, simplify the intermediate expression, and test whether it is zero. The `maple*` code in our sample question is more complicated for two reasons:

1. We do not simply accept any answer that is algebraically equivalent with the predefined answer as fully correct.
2. We give partial credit when the symbolic part of the answer is correct, but the numerical part is wrong.

Thus, we split the student's answer in a numerical and symbolic part and we do a simple check on the number of times that the symbols a, b, and c appear in order to identify 'wrong' symbolic parts of the answer like $c^{\frac{1}{2}\gamma_1} \cdot c^{-\frac{1}{2}\gamma_2}$. If the symbolic part of the question is correct, but a wrong numerical factor is provided in the answer, the student gets half of the possible score.

It is our experience that the more precise you want to be about the acceptable form of an answer to a free-response question, the more difficult and error prone the coding of a question becomes. Working for one hour on the creation of a question is then not exceptional, even for an experienced author. Also some experience and a painstaking accuracy are needed for selecting suitable ranges of random variables that lead to questions of equal difficulty and not to impossible or trivial problems.

The feedback to the student is in our sample question just the correct answer, together with a short description of how the problem can be solved. There exist a few options in Maple T.A. to give feedback that intelligently depends on the answer given by the student. Multiple choice and multiple selection questions allow a different comment for each option. In a free-response mathematical question a given answer can be compared with some wrong answers that the teacher expects, possibly based on his or her experience in class, and feedback can provided accordingly. A simple example of tailored feedback in a Maple-graded question is the following question about integration, in which differentiation or omission of the integration constant are likely mistakes:

```
\begin{question}{Maple}
\name{integration} \type{formula}
\qutext{Compute the anti-derivative of $x^{\var{n}}$.}
\maple*{type(simplify($RESPONSE-$a2), {constant,name});}
\code{$n  = range(2,7);
  $a1 = maple("diff(x^($n),x)");
  $a2 = maple("integrate(x^($n),x)");}
```

```
\comment{${switch(indexof($RESPONSE, dummy, $a1, $a2),
  "If marked as incorrect, check your answer by differentiation.",
  "You did differentiation, not integration!",
  "Good, but think of integration constant next time.")}}
\end{question}
```

2.3 Facilities for Students and Teachers

Generating exercises and automatically assessing students responses in a mathematical way is important in a computer algebra based assessment system, but there is more needed for successful implementation of computer-aided assessment. As important are issues like the way students can enter mathematical expressions and see what formulas they actually type, the delivery process of tests and assessments, the administration of students results, the monitoring of students' work, and the integration with a virtual learning environment. In our case, the learning environment is Blackboard 6 and we use Maple T.A. through its Blackboard building block.

From students' point of view, a Maple T.A.-based assignment is just as like any other assignment in Blackboard: (s)he clicks on a hyperlink and the assignment appears in a separate window. One or more questions are presented on a single page, one page after another, and the student can try to answer the question(s) in any order. The student enters a response to a question, presses the button to go to the next exercise or selects the button to jump to another question. When (s)he has finished work or time is up for the assignment, the student pressed the 'grade' button. The student is warned about questions that (s)he may have forgotten to answer and is given the opportunity to repair this and submit the work again, or to force immediate marking. If the teacher has not decided differently, the student can immediately know the (provisional) mark obtained by looking it up in the Blackboard grade book and (s)he can immediately obtain via the 'Maple T.A. Tools' a detailed report, in which responses are compared with the teacher's answer and in which, at the discretion of the teacher, model solutions of the problems are presented.

The above process of doing a Maple T.A. assignment within the Blackboard environment runs smoothly and needs not much training beforehand, except possibly the way of entering mathematical expressions and the process of submitting work. Maple T.A. provides two ways to enter mathematical formulas, viz., the 'Maple syntax' mode and the 'Symbol' mode. The Maple syntax for entering mathematical expressions is only natural for students after some Maple experience through lab work; they are helped enough by the possibility to preview the formula entered in standard mathematical notation or by a reference chart to quickly look up input details. For those who lack Maple experience, like our students at the beginning of their study, the 'Symbol' mode offers a way out or a helping hand. In this mode, a student uses the built-in 2d-math editor, which is also present in Maple 10, to enter formulas in a manner that relates directly to standard mathematical notation and that makes use of templates for common formatting. Implicit multiplication ensures that `3x` and `x y` are considered the

same as `3*x` and `x*y`, respectively. Common commands are automatically formatted: for example, `^` automatically moves the cursor to superscript position, and `/` creates a division bar and moves the cursor to the denominator. Expression palettes with fill-in-the-blank templates for common mathematical objects are available and can be defined by authors. The equation editor, which is a Java applet that is automatically downloaded to the student's computer the first time it is requested, enables a student to enter responses easily and see whether they look the same way as (s)he would write them down.

All administration is done via a web interface. Using this interface, an instructor can design assignments, generate aggregate statistics and grade reports of students' results, and monitor the progress made by individual students. A teacher has at his or her disposal a variety of assignments types with different policies to choose from: anonymous practice (no need for a login, no recording of results), homework or quiz (login needed, recording of results), a proctored exam (authorisation needed), mastery session (progress control), and study session (flash card learning). Questions can be drawn from various question banks and may be tested beforehand. Note that editing of questions can only be done separately, with the help of the question bank editor or by editing a LaTeX source file of a question bank and re-converting it into EDU format. Questions in an assignment may be randomly selected from a pool of questions and the selected questions may be scrambled at the instructor's will. The teacher can also set several policies for an assignment such as visibility on the web, duration and schedule of the assignment, passing score, availability of hints and links to additional notes, the level and timing of feedback, and additional requirements for doing the assignment.

The Blackboard grade book and the Maple T.A. grade book are synchronised. When an instructor includes for the first time a Maple T.A. assignment in a Blackboard course, then a Maple T.A. class is automatically created for this course on the Maple T.A. server and the user accounts are forwarded from Blackboard to Maple T.A. In this way, all users' administration is taken care of. The instructor does not have to worry about this or bother an administrator to make student accounts available. In the Blackboard grade book, an instructor can view and modify students' grades, clear attempts, and view statistics for a particular grade book item. Via the Maple T.A. grade book a teacher can get detailed grade reports, obtain item statistics, have a look at work of individual students and comment on it, change marks, and authorise a student to start a fresh (proctored) test.

3 Our Experiences with Diagnostic Testing of Freshmen

In this section we discuss (i) the mathematical performance of the freshmen in the pretest and posttest taken in the first weeks of the study year, (ii) the students' opinions about the computer algebra based diagnostic testing, and (iii) our experiences with Maple T.A. for Blackboard.

3.1 Students' Results

The overall scores of the freshmen at in the pretest and posttest are shown in the two histograms of Fig. 1.
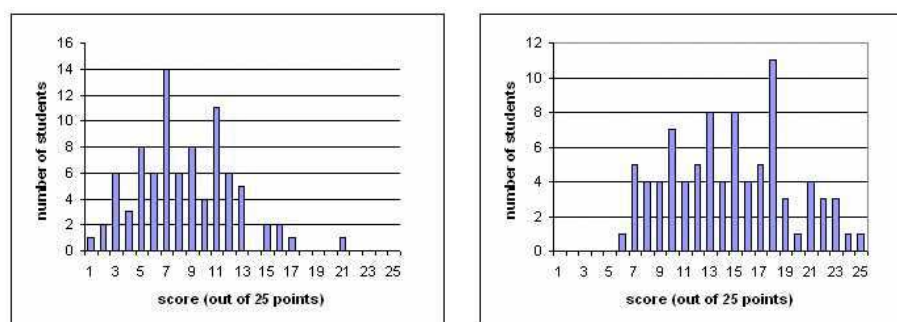
Fig. 1. Histogram of students' result in pretest (left) and posttest (right)

The progress of individual students between pretest and posttest is represented in the scatter diagram of Fig. 2: each dot represents the score of a student at the pretest and posttest.
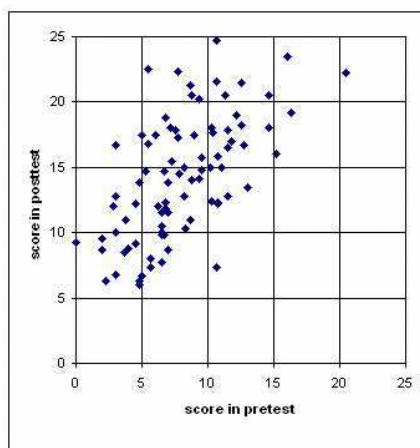
Fig. 2. Scatter diagram of students' scores in pretest and posttest

Needless to say that these low scores of beginning students worry university educators. Even though some students make remarkable progress in four weeks of remedial training, still only one-third of the student population entering the exact sciences programme has reached a desirable mathematical level.

3.2 Students' Opinions about the Diagnostic Testing

Just after the posttest, the students were asked to give their opinions by means of a survey. The response was sixty-five percent.

The students found that the goal of the diagnostic tests were sufficiently clear (90%). The chosen approach was appreciated by 86%. They confirmed that the tests helped to identify shortcomings in their mathematical abilities (69%), and also that the tests contributed to making progress with mathematics (68%). About 63% of the students found the tests stimulating to set to work. A minority (42%) said that they obtained more insight in the difference between school and university mathematics.

About the instrumentation, 61% of the students found that they could work well with the Maple T.A. software. This is remarkable, since there was no training beforehand. Students were permitted to use the graphing calculator and the standard formula chart. A small majority of the students (63%) thought that this influenced their score positively. It also means that a large number of students do not feel dependent on the graphing calculator and the formula chart, while teachers noticed during practice sessions that the students used them many times, even in case when it seems inappropriate.

At the end of the survey there was an open field for suggestions and remarks. About half of the students used this field. Nine students made it clear that they preferred a test on paper instead of the computer; an important reason they mentioned is that they could only fill in the end result and no intermediate steps. Maybe they felt disadvantaged by not been given clearly the opportunity to earn partial credit (although we did this in fact at a manual check of the grading). Six students made a positive comment about the usefulness of the approach. Two students made a remark about the planning of the pretest as it was planned just before the very first week of the start of first courses. This happened to be the period of the university introduction with all kind of activities and festivities. Arguably, a pretest was not appreciated at that moment.

3.3 Experiences with Maple T.A. for Blackboard

Luckily we had decided in advance that in the pretest students would use print-outs of the diagnostic test and that a student after solving a problem with pencil-and-paper would fill out the answer in the Maple T.A. test form. We planned this because we were not sure whether everything would run smoothly from technical and organisational points of view. Indeed we suffered from problems with students not having a computer account or not having a Blackboard account. Although we had anticipated this, creating accounts at the beginning of the diagnostic turned out to be too hectic a job. More important was that one-third of the students had unexpected login problems with the Maple T.A. assignment. These students, having access to the Blackboard course, should not have to login for the diagnostic test anyway! But unforeseen name conflicts between user accounts with special characters and the Maple T.A. administration of users caused these problems. Because one-third of the students were allowed

to hand in the pencil-and-paper work alone, one-third of the students decided to do the same. Consequently, only one-third of the students filled out the digital test form. But this percentage gave enough insight in the process of computer algebra based assessment and the points to take care of in the next diagnostic testing. By the way, knowing the technical problem was enough to fix it for the posttest by simply changing the settings of the Maple T.A. server.

Maple T.A. uses one file per student to keep the complete history of work. The instructor cannot delete irrelevant test data and consequently most of the aggregate statistics of the pretest and posttest generated by Maple T.A. was useless.

Afterwards the instructor had great difficulty in editing the diagnostic tests because many student were registered as still having active assignments. This seemed to be caused by the following submission process: many students submitted the diagnostic tests for marking and received warning about questions not answered or input problems. But many of them did not realise that they had to press the grade button again to force submission for marking. They thought that they has already submitted their work, saved and quit the test, and logged off the computer. The only way to get rid of such active users was to identify them manually and then force marking of their tests. Of course it is a wise design choice to ensure that an instructor cannot edit an assignment because some student may still be busy with the assignment, but there are circumstances in which an instructor knows better and wants to force removal or marking of active assignments without much effort.

We found serious bugs in the software, which luckily did not affect much the students' results. We noticed while we were checking the computer marking of given answers that adding extra spaces around the multiplication symbol caused that correct answers were marked as incorrect in Symbol entry mode. This kind of errors in marking could easily be identified by us and marks could easily be changed. Another problem that luckily did not affect students results was the following: in Symbol entry mode, the equation editor parses the given formula into an expression with valid Maple-syntax. Unfortunately, it also applies automatic simplification before checking the correctness of the answer. As a consequence of this, a student could answer for example to the first question of the pretest simply `1/(1/2+1/3+1/4)` and still get full marks because Maple already does the computation. Of course, no student came to this idea, but it is a serious weakness of the system. You do not want a marking procedure that depends on the selected input mode.

Despite the ten minutes introduction into the Maple T.A. students made foreseen errors in entering mathematical formulas. We noticed the use of uppercase characters instead of lowercase symbols and sometimes an algebraic operator was lacking, like in `xy` being entered instead of `x y` or `x*y`. Luckily, an instructor quickly notices these things when he or she scans the computer marking, but it remains annoying.

The above remarks should not give the impression of failure of Maple T.A. under practical circumstances with unexperienced students. It more reflects that

Maple T.A. is a young product, which is not yet over its teething troubles, but still needs more active development. Actually it worked very well in the posttest, when we knew that we should pay more attention to the submission process. In fact, the 'Symbol entry mode' allows usage of the assessment system with little introduction to the users.

We did the diagnostic tests with fifty simultaneous users of the Maple T.A. without any significant load on the server (dual processor machine at 3 GHz speed with Linux OS). We expect that we can use the testing and assessment system with large groups of students in the computer labs at one time, without running into serious problems. The connection between Blackboard and Maple T.A. system works well and makes life of an instructor easier.

4 Conclusion

The main question to address in the conclusion is whether computer algebra based diagnostic testing of mathematical competencies of freshmen meets its goals in a staff-extensive way. As can be seen from the substantial progress of the students, the trajectory does lead to an overall considerable amelioration of the mathematical abilities. The results of the pretest and posttest also indicate clearly that much attention must be paid to bringing freshmen as soon as possible to the required starting level of mathematical knowledge and skills in order to avoid study delay or early drop-out.

On the whole, the students appreciated that they were confronted with the mathematics abilities as desired by the universities, and were informed about their own level. Although the first assessment was even before the first course day, the students did not feel uncomfortable with the fact that they were assessed. As can be seen from the questionnaire, they understood well the purpose of the testing. We believe that for this the repeated information to students, tutor students and staff is very important.

The Maple T.A. system is able to generate and assess questions from the templates in the database. So the whole trajectory can be based on a single test template: it is used for the entry assessment, the formative exercises and the diagnosis tests. This is a very strong advantage of the approach with a tremendous potential for saving time of staff. However, this version of Maple T.A. showed some peculiarities in the submission process and in the automatic assessment by Maple of formula based items. These forced us to check manually the student results, so in this pilot the time efficiency requirement was not met. Nevertheless, we believe that once the peculiarities of the system are removed, also the condition of staff-extensiveness is satisfied.

Appendix: First Diagnostic Test

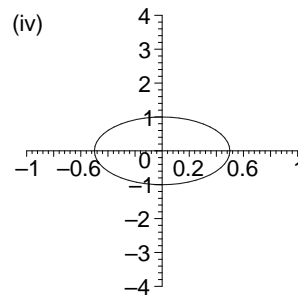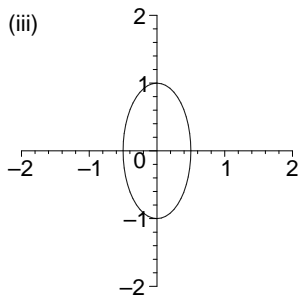1. Simplify as much as possible: $\dfrac{1}{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}}$.

2. Simplify as much as possible: $\left(\dfrac{6}{5}\right)^{-3} \times \left(\dfrac{3}{10}\right)^{2}$.

3. For kinetic energy E of a particle with mass m and velocity v holds: $E = \frac{1}{2}mv^2$. For the momentum p holds: $p = mv$. Express E in m and p, and simplify as much as possible.

4. The Gaussian Lens Formula is: $\dfrac{1}{f} = \dfrac{1}{b} + \dfrac{1}{v}$. Write f as a quotient of polynomials in b and v.

5. Simplify as much as possible: $\dfrac{a^2(8b)^{\frac{1}{4}}c^{\frac{1}{2}}}{\sqrt{abc^3\sqrt{2}}}$, assuming that a, b and c are positive numbers.

6. Expand brackets and simplify: $(2a + 3b)(3a - 2b)$.

7. Expand brackets and simplify: $(2x - y + z)^2 - z^2$.

8. Simplify as much as possible: $\dfrac{9r^2 - 4s^2}{3r + 2s}$.

9. Compute the exact value of the sum: $\displaystyle\sum_{k=-10}^{10} k$.

10. Given is a positive number a. Compute the exact value of the limit:
$\displaystyle\lim_{n\to\infty} \dfrac{n^2 - 2^n}{an^2 + e^n}$.

11. Given is the system of two equations in two unknowns x and y and two constants a and b: $\begin{cases} 2x + 3y = a \\ 4x + 6y = b \end{cases}$
Mark for each of the conclusions below whether it is true or false:

   (a) For some values of a and b the system has no solutions.
   (b) For some values of a and b the system has exactly one solution.
   (c) For some values of a and b the system has an infinite number of solutions.

12. Given is the point P with coordinates $(1, 2)$. We translate the coordinate system over the vector $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$ and leave the point P at its current position. What are the coordinates of P in the new coordinate system?

13. Which of the following graphs belong(s) to the curve described by the equation $x^2 + 4y^2 = 1$?

(iii)

(iv)
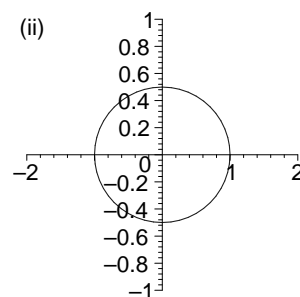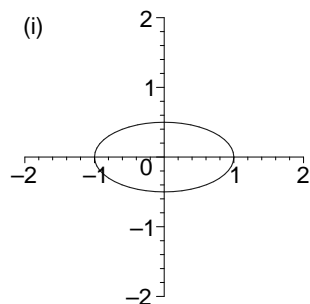
14. For $x > -2$ is the function $f(x) = \sqrt{1 + \dfrac{x}{2}}$. The formula of the inverse function is then: $f^{-1}(x) = \ldots$

15. We assume that the standard limit $\lim\limits_{x \to 0} \dfrac{\sin(x)}{x} = 1$ is known.

    Given is a number $a \neq 0$. Determine the exact value of the limit: $\lim\limits_{x \to 0} \dfrac{\sin(ax)}{\tan 2x}$.

16. Determine the quadratic function $f(x)$ with stationary point $(1, -4)$ and a zero in $x = 2$.

17. Compute the exact values of all solutions of the equation: $x^3 + 14x^2 = 72x$.

18. Compute the exact values of all real solutions of the equation:
    $e^{2x} - 2e^x - 3 = 0$.

19. Simplify as much as possible: $\dfrac{4^x - 1}{2^x - 1}$.

20. Differentiate with respect to t and simplify your answer as much as possible:
    $\ln(1 - t^2) + t^2$.

21. Differentiate with respect to x and simplify your answer as much as possible:
    $\dfrac{x^2 - 4}{x - 2}$.

22. Given are a function $f(x)$ with $f'(1) = 3.8$ and the function
    $g(x) = f(2x - 1)$. What is $g'(1)$.
    Select one of the following responses.
    ○ 1.9 ○ 2.8 ○ 3.8 ○ 4.8 ○ 7.6
    ○ For this you must know the formula of $f(x)$.

23. Below are drawn in random order the graphs of the function $f(x)$, its derivative $f'(x)$ and the second derivative $f''(x)$. Identify them.
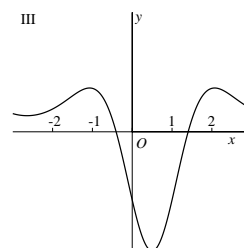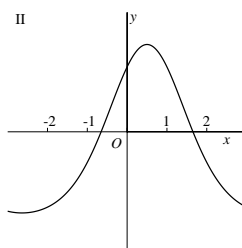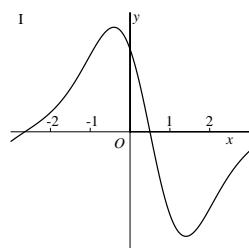
24. Compute the exact value of $\displaystyle\int_1^2 (x^3 + \sqrt{x})\, dx.$

25. Given is a function $f(t)$ with $\int_1^3 f(t)\,dt = 2.4$. What is $\int_3^5 f(t-2)\,dt$?

    Select one of the following responses.

    ○ 0.4   ○ 2.4   ○ 4.4   ○ For this you must know the formula of $f(t)$.

Acknowledgements

References

1. London Mathematical Society: Tackling the Mathematics Problem. Advisory report, 1995. `www.lms.ac.uk/policy`
2. SEFI Mathematics Working Group; Mustoe, L., and Lawson, D. (eds): Mathematics for the European Engineer — a Curriculum for the twenty-first Century. Report, 2002. `http://learn.lboro.ac.uk/mwg/core/latest/sefimarch2002.pdf`
3. Kent, P., and Noss, T.: Mathematics in the University Education of Engineers. A Report to The Ove Arup Foundation, 2003. `www.theovearupfoundation.org/arupfoundation/pages/download25.pdf`
4. Engineering Council: Measuring the Mathematics Problem. Report, 2000. `www.engc.org.uk/documents/Measuring_the_Maths_Problems.pdf`
5. van de Craats, J., and Bosch, J.: Basisboek Wiskunde (in Dutch, Basic Mathematics). Pearson Education Benelux, 2005. Most parts of the book are electronically available at `www.science.uva.nl/~craats`
6. Maple T.A.: Automated Online Testing & Assessment. Developer's website `www.maplesoft.com/products/mapleta`
7. Maple: `www.maplesoft.com/products/maple`
8. Web-spijkeren: SURF Project, information (in Dutch) electronically available at `www.web-spijkeren.nl`
9. MathMatch: Project of the Digital University, information (in Dutch) electronically available at `www.du.nl/mathmatch`
10. AIM: Assessment in Mathematics. `http://aiminfo.net`
11. Sangwin, C.J., and Grove, M.: STACK: addressing the needs of the "neglected learners". WebALT 2006 Proceedings.
12. WebALT, Web Advanced Technologies: `www.webalt.net`
13. Brownstone's EDU Campus: `www.brownstone.net/products/ecampus.asp`
14. Sangwin, C.J: Assessing mathematics automatically using computer algebra and internet. *Teaching Mathematics and it Applications* **23** (2004) 813–829.