Universitext

For further volumes:
http://www.springer.com/series/223

Giulia Di Nunno · Bernt Øksendal
Frank Proske

Malliavin Calculus for Lévy Processes with Applications to Finance

Springer

Giulia Di Nunno
Bernt Øksendal
Frank Proske
Department of Mathematics
University of Oslo
0316 Oslo
Blindern
Norway
giulian@math.uio.no
oksendal@math.uio.no
proske@math.uio.no

ISBN 978-3-540-78571-2 e-ISBN 978-3-540-78572-9
DOI 10.1007/978-3-540-78572-9
Springer Heidelberg Dordrecht London New York

Library of Congress Control Number: 2009934515

Mathematics Subject Classification (2000): 60H05, 60H07, 60H40, 91B28, 93E20, 60G51, 60G57

Cover design: WMXDesign GmbH, Heidelberg

The cover design is based on a graph provided by F.E. Benth, M. Groth, and O. Wallin

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

To Christian. To my parents.
G.D.N.

To Eva.
B.Ø.

To Simone, Paulina and Siegfried.
F.P.

Preface

There are already several excellent books on Malliavin calculus. However, most of them deal only with the theory of Malliavin calculus for Brownian motion, with [35] as an honorable exception. Moreover, most of them discuss only the application to regularity results for solutions of SDEs, as this was the original motivation when Paul Malliavin introduced the infinite-dimensional calculus in 1978 in [158]. In the recent years, Malliavin calculus has found many applications in stochastic control and within finance. At the same time, Lévy processes have become important in financial modeling. In view of this, we have seen the need for a book that deals with Malliavin calculus for Lévy processes in general, not just Brownian motion, and that presents some of the most important and recent applications to finance.

It is the purpose of this book to try to fill this need. In this monograph we present a general Malliavin calculus for Lévy processes, covering both the Brownian motion case and the pure jump martingale case via Poisson random measures, and also some combination of the two. We also present many of the recent applications to finance, including the following:

- The Clark–Ocone theorem and hedging formulae
- Minimal variance hedging in incomplete markets
- Sensitivity analysis results and efficient computation of the "greeks"
- Optimal portfolio with partial information
- Optimal portfolio in an anticipating environment
- Optimal consumption in a general information setting
- Insider trading

To be able to handle these applications, we develop a general theory of anticipative stochastic calculus for Lévy processes involving the Malliavin derivative, the Skorohod integral, the forward integral, which were originally introduced for the Brownian setting only. We dedicate some chapters to the generalization of our results to the white noise framework, which often turns out to be a suitable setting for the theory. Moreover, this enables us to prove results that are general enough for the financial applications, for example, the generalized Clark–Ocone theorem.

This book is based on a series of courses that we have given in different years and to different audiences. The first one was given at the Norwegian School of Economics and Business Administration (NHH) in Bergen in 1996, at that time about Brownian motion only. Other courses were held later, every time including more updated material. In particular, we mention the courses given at the Department of Mathematics and at the Center of Mathematics for Applications (CMA) at the University of Oslo and also the intensive or compact courses presented at the University of Ulm in July 2006, at the University of Cape Town in December 2006, at the Indian Institute of Science (IIS) in Bangalore in January 2007, and at the Nanyang Technological University in Singapore in January 2008.

At all these occasions we met engaged students and attentive readers. We thank all of them for their active participation to the classes and their feedback. Our work has benefitted from the collaboration and useful comments from many people, including Fred Espen Benth, Maximilian Josef Butz, Delphine David, Inga Baadshaug Eide, Xavier Gabaix, Martin Groth, Yaozhong Hu, Asma Khedher, Paul Kettler, An Ta Thi Kieu, Jørgen Sjaastad, Thilo Meyer-Brandis, Farai Julius Mhlanga, Yeliz Yolcu Okur, Olivier Menoukeu Pamen, Ulrich Rieder, Goncalo Reiss, Steffen Sjursen, Alexander Sokol, Agnès Sulem, Olli Wallin, Diane Wilcox, Frank Wittemann, Mihail Zervos, Tusheng Zhang, and Xunyu Zhou. We thank them all for their help. Our special thanks go to Paul Malliavin for the inspiration and continuous encouragement he has given us throughout the time we have worked on this book. We also acknowledge with gratitude the technical support with computers of the Drift-IT at the Department of Mathematics at the University of Oslo.

Oslo,
June 2008.

Giulia Di Nunno
Bernt Øksendal
Frank Proske

Contents