

INSIDER TRADING, STOCHASTIC LIQUIDITY, AND EQUILIBRIUM PRICES

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October 15, 2012

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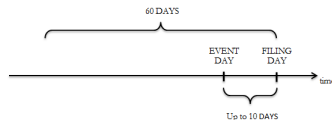
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DO MEASURES OF STOCK LIQUIDITY REVEAL THE PRESENCE OF INFORMED TRADERS?

- Microstructure literature predicts that measures of trading liquidity (bid/ask spread and price impact) should be informative about the presence of adverse selection (Glosten and Milgrom (1985), Kyle (1985), Easley and O'Hara (1987))
- For example, Kyle (1985) proposes seminal model of insider trading:
 - Insider knows terminal value of the firm that will be revealed to all at T .
 - market maker sets price equal to expected value given total order flow which is the sum of uninformed noise trader demand and insider trades.
 - ⇒ Insider trades proportionally to difference between private valuation and price, and inversely related to time and price impact.
 - ⇒ In equilibrium, price responds to order flow linearly.
 - ⇒ In cross-section, Kyle's λ , which can be estimated from a regression of price changes on order flow, should be higher for stocks with more informed trading (relative to liquidity/noise trading)
- Several empirical measures of liquidity (inventory and adverse selection costs) proposed in the literature. (Glosten (1987); Glosten and Harris (1988); Stoll (1989); Hasbrouck (1991); Amihud (2002), Goyenko et al. (2009))
- Question: how well do these measures perform at picking up the presence of informed trading?

EMPIRICAL MOTIVATION

In recent paper *'Do prices reveal the presence of informed trading?'*, we hand-collect data on informed trades from Schedule 13-D filings – Rule 13d-1(a) of the 1934 Securities Exchange Act that requires (Item 5(c)) the filer to "... describe any transactions in the class of securities reported on that were effected during past 60 days..."



- Event day – beneficial ownership of Schedule 13-D filer crosses 5% threshold
- Filing day – Schedule 13-D filing (electronically) submitted to the SEC

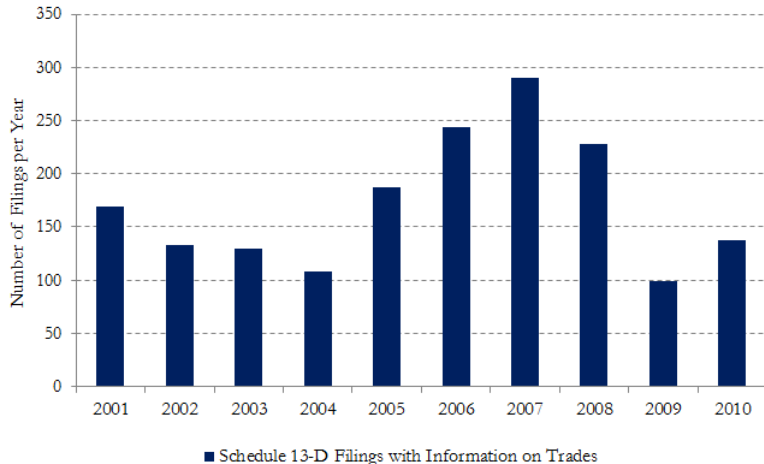
Document that Trades executed by Schedule 13-D filers are informed:

- Announcement returns
- Profits of Schedule 13-D filers
- Performance of trading strategies that replicate informed transactions

Find that standard liquidity measures **do not** reveal the presence of informed traders:

- For example, Kyle's λ is lower when informed investors trade aggressively
- Both high frequency and low frequency measures.

TIME DISTRIBUTION OF EVENTS

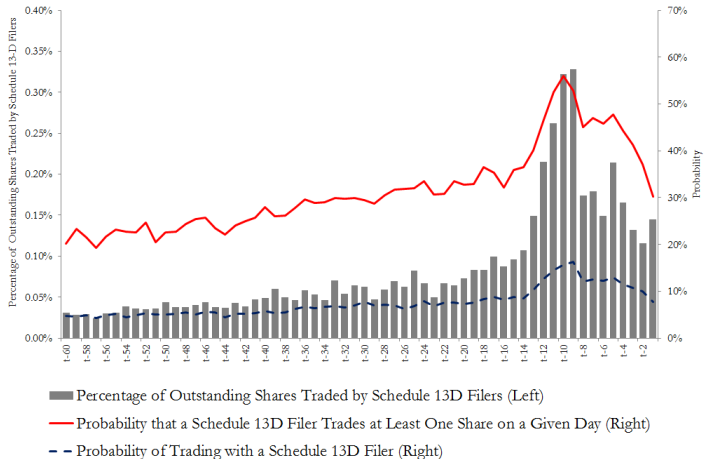


TRADING STRATEGY OF INFORMED TRADERS - SUMMARY STATS

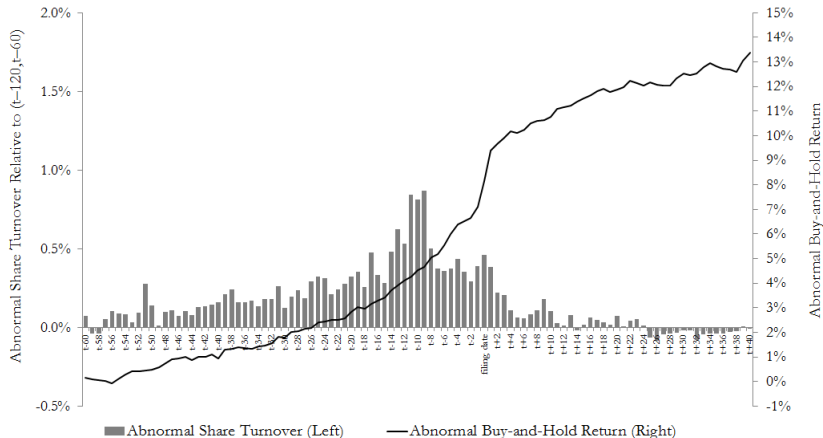
	Mean			Median		
	Full Sample (1)	Before (2)	After (3)	Full Sample (4)	Before (5)	After (6)
Stock ownership on the filing date	7.68%			6.20%		
Number of trading days	15.1	13.1	3.7	13	11	3
% of trading days with informed trades	34.8%	34.7%	57.1%	30.0%	29.4%	57.2%
Informed volume	1,304,126	948,175	535,561	393,387	252,201	144,038
Informed volume per trading day	132,194	102,165	195,784	30,191	21,667	45,102
Informed volume (m\$)	25.6	17.9	11.3	3.3	2.2	1.3
Informed volume per trading day (m\$)	3.2	2.6	4.6	0.3	0.2	0.5
Change in stock ownership	3.8%	2.5%	1.8%	2.8%	1.8%	0.9%
Change in ownership per trading day	0.5%	0.3%	0.8%	0.2%	0.1%	0.3%
Informed turnover (%)	29.9%	26.1%	36.2%	22.6%	19.4%	27.8%

- When informed investors act, their trades constitute around 25% of daily volume
- Informed do not trade every day

TRADING STRATEGY OF INFORMED TRADERS - FILING DATE



BUY-AND-HOLD ABNORMAL RETURN - FILING DATE



Two month excess return is around 10%

PROFITS FROM INFORMED TRADING

Quantile (1)	Market CAP (\$)	Trading Profit (\$)		Total Profit (\$)		Value Created (\$)	
	Median (2)	Mean (3)	Median (4)	Mean (5)	Median (6)	Mean (7)	Median (8)
Q1 - low	23,659,879	50,877	9,810	100,295	52,559	1,270,457	844,707
Q2	63,368,069	90,877	26,025	197,147	59,418	1,819,414	1,828,604
Q3	151,542,849	308,514	65,642	369,945	291,283	6,626,805	5,969,746
Q4	404,095,821	568,519	124,864	1,133,935	307,943	19,634,771	8,424,417
Q5 - high	1,818,551,960	1,434,720	586,761	2,982,360	1,501,983	46,216,662	28,635,234

- For example, an activist that targets a \$400 million market cap company expects to benefit \$1.13 million.
- The average stake of such activist is 7.42% of shares outstanding, or about \$30 million.
- Thus, the expected event-period return is 5.8%, which is equivalent to 40.6% annualized return.

DO INSIDERS HAVE PRICE IMPACT?

Panel A

	(fday-60,fday-1) (1)	(fday-420,fday-361) (2)	difference (3)	t-stat (4)
<i>eret</i>	0.0011	0.0002	0.0009***	4.08
<i>to</i>	0.0103	0.0072	0.0030***	9.06

Panel B

	days with informed trading (1)	days with no informed trading (2)	difference (3)	t-stat (4)
<i>eret</i>	0.0062	-0.0004	0.0067***	4.92
<i>to</i>	0.0218	0.0085	0.0132***	12.36

LIQUIDITY MEASURES

- High-Frequency Measures

$$\hat{\lambda}_{it} \quad r_{itn} = \delta_{it} + \lambda_{it} \sum_k \text{sign}(dvol_{itnk}) \sqrt{|dvol_{itnk}|} + \varepsilon_{itn}$$

$$(Effective\ Spread) \quad \text{espread}_{itk} = 2|\ln(P_{itk}) - \ln(M_{itk})|$$

$$(Realized\ Spread) \quad \text{rspread}_{itk} = 2q_{itk}(\ln(P_{itk}) - \ln(M_{itk+5}))$$

$$(Price\ Impact) \quad \text{pimpact}_{itk} = 2q_{itk}(\ln(M_{itk+5}) - \ln(M_{itk}))$$

where P is transaction price and M is midpoint and Stock i 's liquidity measure on day t is the dollar-volume-weighted average over all trades.

- Low-Frequency Measures

$$(Amihud's\ Illiquidity) \quad \text{Illiquidity}_{it} = 1000|r_{it}|/Volume_{it}$$

$$(Amivest\ Liquidity) \quad \text{Liquidity}_{it} = 0.001 Volume_{it}/|r_{it}|$$

$$(Bid - Ask\ Spread) \quad \text{baspread}_{it} = \frac{\text{ask}_{it} - \text{bid}_{it}}{0.5(\text{ask}_{it} + \text{bid}_{it})}$$

ARE STOCKS “LESS LIQUID” WHEN INFORMED TRADE?

	(fday-60,fday-1) (1)	(fday-420,fday-361) (2)	difference (3)	t-stat (4)
Low Frequency Measures				
<i>illiquidity</i>	0.4438	0.4794	-0.0357**	-2.29
<i>liquidity</i>	26.7813	19.1090	7.6723**	7.99
<i>baspread</i>	0.0132	0.0149	-0.0017***	-3.13
<i>pin</i>	0.4298	0.5000	-0.0703***	-10.85
<i>psgamma</i>	-0.0025	-0.0012	-0.0013	-0.54
<i>zero</i>	0.0671	0.0659	0.0012	0.51
<i>zero2</i>	0.0590	0.0553	0.0037*	1.83
High Frequency Measures				
$\lambda * 10^6$	16.3110	18.0540	-1.7430*	-1.68
<i>espread</i>	0.0133	0.0148	-0.0015**	-2.48
<i>rspread</i>	0.0057	0.0064	-0.0008***	-2.77
<i>pim pact</i>	0.0077	0.0083	-0.0006	-1.03

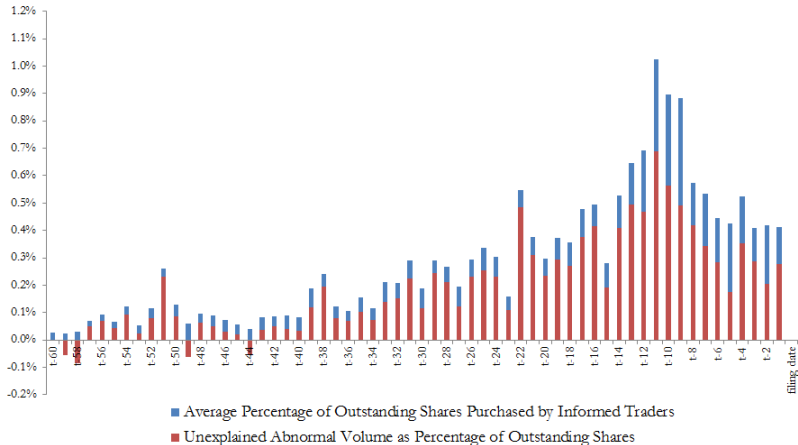
ARE STOCKS “LESS LIQUID” WHEN INFORMED TRADE?

	days with informed trading (1)	days with no informed trading (2)	difference (3)	t-stat (4)
Low Frequency Measures				
<i>illiquidity</i>	0.2575	0.4808	-0.2233***	-11.42
<i>liquidity</i>	36.6643	24.8088	11.8555***	8.59
<i>baspread</i>	0.0109	0.0126	-0.0017***	-5.89
<i>zero</i>	0.0637	0.0632	0.0005	0.14
<i>zero2</i>	0.0635	0.0552	0.0083**	2.26
High Frequency Measures				
$\lambda * 10^6$	12.3510	16.8494	-4.4985***	-5.36
<i>espread</i>	0.0109	0.0125	-0.0016***	-2.89
<i>rspread</i>	0.0046	0.0054	-0.0007**	-2.62
<i>pim pact</i>	0.0062	0.0072	-0.0011*	-1.94

SUMMARY OF EMPIRICAL PAPER

- Schedule 13-D filers have valuable information when they purchase shares of targeted companies
 - Thus, the information asymmetry is high when Schedule 13-D filers purchase shares
 - We find that excess return and turnover are higher when insiders trade, which seems to indicate that they have price impact
 - However, we find that measures of information asymmetry and liquidity indicate that stocks are more liquid when informed trades take place
 - This evidence seems at odds with predictions of theoretical and empirical literature (e.g., simple Kyle (1985) model and derived empirical price impact measures, Glosten and Harris (1988), Hasbrouck (1991), Amihud (2002), Easley- O'hara (1996))
- ⇒ The endogeneity issue seems more problematic than the literature may have previously recognized.

ABNORMAL SHARE TURNOVER - REVISITED



THEORETICAL CONTRIBUTION

- We extend Kyle's (insider trading) model to allow for general stochastic changes in volatility of uninformed order flow.
- Main results:
 - Equilibrium price displays (**endogenous**) stochastic volatility if noise trader vol is predictable.
 - Price impact (Kyle's λ) is stochastic: lower (higher) when noise trading volatility is higher (lower).
 - Price impact (Kyle's λ) is submartingale: execution costs are expected to deteriorate over time.
 - Informed trade more aggressively when noise trading volatility is higher and when measured price impact is lower.
 - More information makes its way into prices when noise trading volatility is higher.
 - Adverse selection execution costs for uninformed noise traders can be higher when noise trading is higher (and λ is lower).

INSIDER

- We follow Back (1992) and develop a continuous time version of Kyle (1985)
- Risk-neutral insider's maximization problem:

$$\max_{\theta_t} \mathbb{E} \left[\int_0^T (v - P_t) \theta_t dt \right] \quad (1)$$

- As in Kyle, we assume there is an insider trading in the stock with perfect knowledge of the terminal value v
- It is optimal for the insider to follow absolutely continuous trading strategy (Back (1992)).
- Related work: Back and Pedersen (1998), Admati Pfleiderer (1988) and others...

MARKET MAKER

- The market maker is also risk-neutral, but does not observe the terminal value. Instead, he has a prior that the value v is normally distributed $N(\mu_0, \Sigma_0)$
- The market maker only observes the total order flow:

$$dY_t = \underbrace{\theta_t dt}_{\text{informed order flow}} + \underbrace{\sigma_t dZ_t}_{\text{uninformed order flow}} \quad (2)$$

- where σ_t is the stochastic volatility of the uninformed order flow:

$$d\sigma_t = m(t, \{\sigma\}_{s \leq t})dt + \nu(t, \{\sigma\}_{s \leq t})dM_t$$

and M_t is orthogonal (possibly discontinuous) martingale.

- Since the market maker is risk-neutral, equilibrium imposes that

$$P_t = \mathbb{E}[v | \{Y_u\}_{u \leq t}, \{\sigma_u\}_{u \leq t}] \quad (3)$$

- We assume that the market maker and the informed investor observe σ_t .

PREVIEW OF RESULTS

- This may seem like a trivial extension of the Kyle (1985) model, as one might conjecture that one can simply 'paste' together Kyle economies with different noise-trading volatilities But, not so!
- The insider will optimally choose to **trade less in the lower liquidity states** than he would were these to last forever, because he anticipates the future opportunity to trade more when liquidity is better and he can reap a larger profit
- Of course, in a rational expectations' equilibrium, the market maker foresees this, and adjusts prices accordingly. Therefore, **if noise trader volatility is predictable**, price dynamics are more complex than in the standard Kyle model:
 - Price displays stochastic volatility
 - Price impact measures are time varying and not necessarily related to informativeness of order flow.

SOLVING FOR EQUILIBRIUM

- First, we conjecture a trading rule followed by the insider:

$$\theta_t = \beta(t, \sigma_t, \Sigma_t)(v - P_t)$$

- Second, we derive the dynamics of the stock price consistent with the market maker's filtering rule, conditional on a conjectured trading rule followed by the insider

$$dP_t = \lambda(t, \sigma_t, \Sigma_t)dY_t$$

- Then we solve the insider's optimal portfolio choice problem, given the assumed dynamics of the equilibrium price
- Finally, we show that the conjectured rule by the market maker is indeed consistent with the insider's optimal choice

GENERAL FEATURES OF EQUILIBRIUM

- Price impact is stochastic:

$$\lambda_t = \sqrt{\frac{\Sigma_t}{G_t}} \quad (4)$$

- where Σ_t is remaining amount of private information

$$\Sigma_t = \mathbb{E} \left[(v - P_t)^2 \mid \{Y_u\}_{u \leq t}, \{S_u\}_{u \leq t} \right] \quad (5)$$

- and G_t is remaining amount of uninformed order flow variance, solves BSDE:

$$\sqrt{G_t} = \mathbb{E}_t \left[\int_t^T \frac{\sigma_u^2}{2\sqrt{G_u}} du \right] \quad (6)$$

- Optimal strategy of insider is:

$$\theta_t = \frac{1}{\lambda_t} \frac{\sigma_t^2}{G_t} (v - P_t) \quad (7)$$

⇒ Insider trades more aggressively when

- the ratio of private information (σ_t) to 'equilibrium-expected' noise trading volatility (G_t) is higher, and
- when price impact λ_t is lower.

GENERAL FEATURES OF EQUILIBRIUM

- Stock price displays time-varying volatility:

$$dP_t = \frac{(v - P_t)}{G_t} \sigma_t^2 dt + \sqrt{\frac{\Sigma_t}{G_t}} \sigma_t dZ_t \quad (8)$$

- Note, that information asymmetry is necessary for price process to be non-constant.
- G_t is the crucial quantity to characterize equilibrium. Its BSDE solution satisfies:

$$G_t \leq \mathbb{E}\left[\int_t^T \sigma_s^2 ds\right]$$

- If $\underline{\sigma} \leq \sigma_t \leq \bar{\sigma}$ then we can show (using a BSDE comparison theorem) that:
 There exists a maximal bounded solution to the recursive equation for G with:

$$\underline{\sigma}^2 (T - t) \leq G_t \leq \bar{\sigma}^2 (T - t) \quad (9)$$

- For several special cases we can construct an explicit solution to this BSDE:
 - σ_t deterministic.
 - σ_t general martingale.
 - $\log \sigma_t$ Ornstein-Uhlenbeck process.
 - σ_t continuous time Markov Chain.

GENERAL FEATURES OF EQUILIBRIUM

- $\lim_{t \rightarrow T} P_t = v$ 'Stochastic bridge' property of price in insider's filtration.
- Market depth ($1/\lambda_t$) is martingale.
- Price impact (λ_t) is a submartingale (liquidity is expected to deteriorate over time).
- $d\Sigma_t = -dP_t^2$ (information gets into prices faster when stock price variance is high, which occurs when noise trader volatility is high).
- Total profits of the insider are equal to $\sqrt{\Sigma_0 G_0}$.
- Realized execution costs of uninformed can be computed pathwise as

$$\int_0^T (P_{t+dt} - P_t) \sigma_t dz_t = \int_0^T \lambda_t \sigma_t^2 dt$$

- Unconditionally, expected execution costs of uninformed equal insider's profits.

GENERAL MARTINGALE DYNAMICS

- We assume that uninformed order flow volatility is unpredictable (a martingale):

$$\frac{d\sigma_t}{\sigma_t} = \nu(t, \sigma^t) dM_t, \quad (10)$$

- We can solve for $G(t) = \sigma_t^2(T - t)$,
- Then market depth is a martingale:

$$\frac{1}{\lambda_t} = \frac{\sigma_t}{\sigma_v},$$

where $\sigma_v^2 = \frac{\Sigma_0}{T}$ is the annualized initial private information variance level.

- The trading strategy of the insiders is $\theta_t = \frac{\sigma_t}{\sigma_v(T-t)}(v - P_t)$
- Equilibrium price dynamics are identical to the original Kyle (1985) model:

$$dP_t = \frac{(v - P_t)}{T - t} dt + \sigma_v dZ_t. \quad (11)$$

IMPLICATIONS OF MARTINGALE DYNAMICS

This example shows we can extend Kyle's equilibrium by simply 'plugging-in' stochastic noise trading volatility:

- Market depth varies linearly to noise trading volatility,
 - Insider's strategy is more aggressive when noise trading volatility increases,
 - both effects offset perfectly so as to leave prices unchanged (relative to Kyle):
 - Prices display constant volatility.
 - private information gets into prices linearly and independently of the rate of noise trading volatility (as in Kyle).
- ⇒ In this model empirical measures of price impact will be time varying (and increasing over time on average), but do not reflect any variation in asymmetric information of trades.

CONSTANT EXPECTED GROWTH RATE

- We assume that uninformed order flow volatility follows a geometric Brownian Motion:

$$\frac{d\sigma_t}{\sigma_t} = mdt + \nu dW_t, \quad (12)$$

- We can solve for $G(t) = \sigma_t^2 B_t$ where $B_t = \frac{e^{2m(T-t)} - 1}{2m}$,
- Then market depth is $\frac{1}{\lambda_t} = e^{-mt} \sigma_t \sqrt{\frac{B_0}{\Sigma_0}}$
- Equilibrium price dynamics follow a one-factor Markov non-homogenous bridge process:

$$dP_t = \frac{(v - P_t)}{B_t} dt + e^{mt} \sqrt{\frac{\Sigma_0}{B_0}} dZ_t. \quad (13)$$

IMPLICATIONS OF CONSTANT GROWTH RATE

- As soon as there is predictability in noise trader volatility, equilibrium prices change (relative to Kyle):
 - Price volatility increases (decreases) deterministically with time if noise trading volatility is expected to increase (decrease).
 - private information gets into prices slower (faster) if noise trading volatility is expected to increase (decrease).
- Interesting separation result obtains:
 - strategy of insider and price impact measure only depends on current level of noise trader volatility.
 - equilibrium is independent of uncertainty about future noise trading volatility level (ν).
 - As a result, equilibrium price volatility is deterministic
 - private information gets into prices at a deterministic rate, despite measures of price impact (and the strategy of the insider) being stochastic!

MEAN REVERSION

- We assume that uninformed order flow log-volatility follows an Ornstein-Uhlenbeck process:

$$\frac{d\sigma_t}{\sigma_t} = -\kappa \log \sigma_t dt + \nu dW_t. \quad (14)$$

- Series expansion solution for $G(t) = \sigma_t^2 A(T-t, x_t, \kappa)^2$, where

$$A(\tau, x, \kappa) = \sqrt{T-t} \left(1 + \sum_{i=1}^n (-k\tau)^i \left(\sum_{j=0}^i x^j \sum_{k=0}^{i-j} c_{ijk} t^k \right) + O(\kappa^{n+1}) \right), \quad (15)$$

where the c_{ijk} are positive constants that depend only on ν^2 .

- Market depth is stochastic and given by: $\lambda_t = \frac{\sqrt{\Sigma_t}}{\sigma_t A(T-t, x_t, \kappa)}$.
- The trading strategy of the insider is: $\theta_t = \frac{\sigma_t}{\sqrt{\Sigma_t} A(T-t, x_t, \kappa)} (\nu - P_t)$.
- private information enters prices at a stochastic rate: $\frac{d\Sigma_t}{\Sigma_t} = -\frac{1}{A(T-t, x_t, \kappa)^2} dt$.
- Stock price dynamics follow a three factor (P, x, Σ) Markov process with stochastic volatility given by:

$$dP_t = \frac{(\nu - P_t)}{A(T-t, x_t, \kappa)^2} dt + \frac{\sqrt{\Sigma_t}}{A(T-t, x_t, \kappa)} dZ_t. \quad (16)$$

IMPLICATIONS OF CONSTANT GROWTH RATE

- The first term in the series expansion of the $A(\tau, x, \kappa)$ function is instructive:

$$A(\tau, x, \kappa) = \sqrt{\tau} \left(1 - \frac{\kappa}{2} \tau \left(\frac{\nu^2 \tau}{6} + x \right) \right) + O(\kappa^2). \quad (17)$$

- With mean-reversion ($\kappa \neq 0$) uncertainty about future noise trading volatility (ν) does affect the trading strategy of the insider, and equilibrium prices.
- When $x = 0$ (where vol is expected to stay constant), the higher the mean-reversion strength κ the lower the A function. This implies that mean-reversion tends to lower the profit of the insider for a given expected path of noise trading volatility.
- If $\kappa > 0$ then A is decreasing in (log) noise-trading volatility (x_t) and in uncertainty about future noise trading volatility ν . This implies that stock price volatility is stochastic and positively correlated with noise-trading volatility.
- Equilibrium price follows a three-factor Bridge process with stochastic volatility.
- Private information gets incorporated into prices faster the higher the level of noise trading volatility, as the insider trades more aggressively in these states.
- Market depth also improves, but less than proportionally to volatility.

A TWO-STATE CONTINUOUS MARKOV CHAIN EXAMPLE

- Assume uninformed order flow volatility can take on two values $\sigma(0) < \sigma(1)$ where regime indicator $S_t \in [0, 1]$ follows:

$$dS_t = (1 - S_t)dN_0(t) - S_t dN_1(t), \quad (18)$$

where $N_i(t)$ is a standard Poisson counting process with jump intensity η_i respectively

- The solution is $G(t, s_t) = \mathbf{1}_{\{s_t=0\}} G^0(T-t) + \mathbf{1}_{\{s_t=1\}} G^1(T-t)$, where the deterministic functions G^0, G^1 satisfy the system of ODE (with boundary conditions $G^0(0) = G^1(0) = 0$):

$$G_\tau^0(\tau) = \sigma(0)^2 + 2\eta_0(\sqrt{G^1(\tau)G^0(\tau)} - G^0(\tau)) \quad (19)$$

$$G_\tau^1(\tau) = \sigma(1)^2 + 2\eta_1(\sqrt{G^1(\tau)G^0(\tau)} - G^1(\tau)) \quad (20)$$

- We compute execution costs of uninformed numerically in this case.
- Show that uninformed execution costs can be higher when noise trading volatility is higher (and Kyle lambda is actually lower).

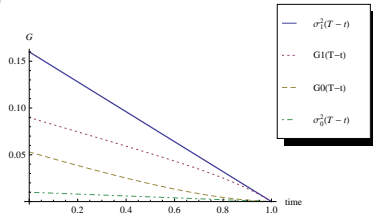


FIGURE: G function in high and low state

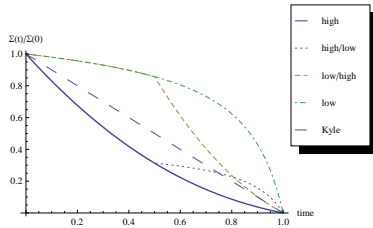


FIGURE: Four Private information paths

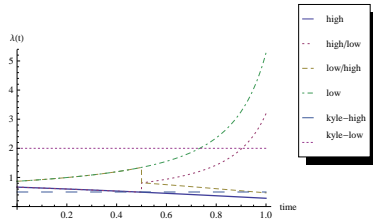


FIGURE: Four paths of price impact λ_t

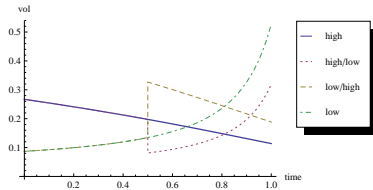


FIGURE: Four paths of Stock price volatility

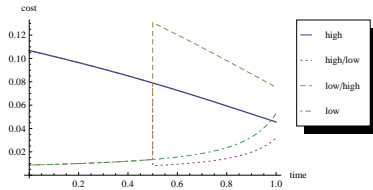


FIGURE: Four paths of uninformed traders execution costs

	Noise trading volatility paths			
	high	low	high/low	low/high
Execution costs $(\int_0^T \lambda_t \sigma_t^2 dt)$	0.078	0.017	0.054	0.057
Average price impact $(\int_0^T \lambda_t dt)$	0.487	1.740	1.023	0.853
Total 'number' of uninformed $(\int_0^T \sigma_t^2 dt)$	0.16	0.01	0.085	0.085
Normalized execution costs $(\frac{\int_0^T \lambda_t \sigma_t^2 dt}{\int_0^T \sigma_t^2 dt})$	0.487	1.740	0.636	0.671

- Average price-impact is not informative about execution costs to uninformed traders.
- Normalizing by 'abnormal' trading volume is crucial.
- Even so, average execution costs to uninformed are path-dependent.

CONCLUSION

- Recent Empirical paper finds that standard liquidity measures, including price impact, fail to reveal the presence of informed traders
- Propose extension of Kyle (1985) to allow for stochastic noise trading volatility. Seems more consistent with evidence:
 - Insider conditions his trading on 'liquidity' state.
 - Price impact measures are time-varying, and not necessarily higher when more private information flows into prices.
 - Execution costs can be higher when measured price impact is lower.
 - Generates stochastic price volatility.
- Future work:
 - Better measure of liquidity/adverse selection?
 - Absence of common knowledge about informed presence.
 - Model of activist insider trading. Why the 5% rule?