Determinants of expected inflation in affine term structure models

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What is market implied expected inflation?

From the bond market we can determine the break-even inflation for a certain maturity:

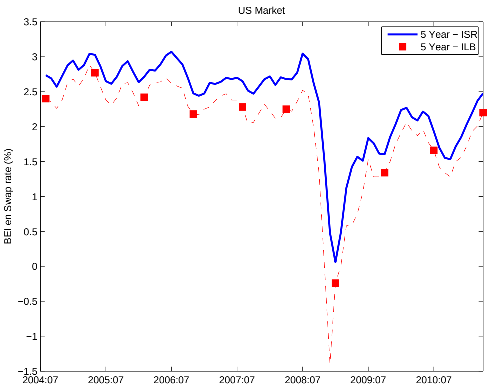
- BEI = Nominal interest rate - real interest rate

There are two derivatives that can be used for identification: inflation-linked bonds and inflation swap rates.

Inflation swap rate: The inflation swaps are fairly liquid in both US and UK markets. (See e.g. Fleckenstein, Longstaff, and Lustig (2010))
Evidence suggests TIPS suffer from illiquidity effects. (DAmico, Kim, and Wei (2010) and Haubrich, Pennachi, and Ritchken (2012))

Inflation swap rates

This figure denotes the US inflation swap rate with a maturity of 5 year and the TIPS-implied BEI inflation.



Decomposing break-even inflation

The break-even inflation can be decomposed in the following terms (D'Amico, Kim, and Wei (2011)):

- BEI = Expected inflation + inflation risk premium + convexity term (Jensen's term).

These terms are not directly observable in the market. Consequently, we need a structural model to determine market implied expected inflation.

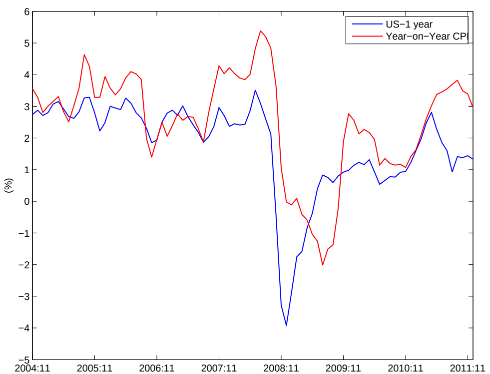
Objective

What are the dynamics of the expected inflation implied by our affine term structure model?

- Typically these models use CPI inflation time series (Ang, Bekaert and Wei (2008)).
- Survey forecasts (See e.g. Haubrich et al. (2012))
- Market expectations of inflation might be driven by market factors as well.

Important component: CPI inflation

The US realized year on year inflation and inflation swap rate combined in one graph.



Introduction: An affine term structure model

Assume nominal bond prices with maturity n, $P_t^N(n)$, are exponentially affine functions of two latent state variables

$$P_t^N(n) = \exp(A_n^N + B_n'^N X_t), \tag{1}$$

For the unobserved state variables assume a VAR model using one time lag

$$X_t = \Phi_0 + \Phi_1 X_{t-1} + \Sigma \epsilon_t, \tag{2}$$

Nominal pricing kernel

To derive nominal bond prices in a no arbitrage framework, we need a pricing kernel M_t^N such that

$$P_t^N(n) = E_t\left[M_{t+1}^N P_{t+1}^N(n-1)\right]. \tag{3}$$

We assume the standard affine pricing kernel

$$M_{t+1}^N = \exp\left(-r_t^N - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\epsilon_{t+1}\right), \tag{4}$$

where λ denotes the price of risk.

Real pricing kernel

Since we need to price real bond prices, we derive the real pricing kernel M_t^R. The relation between the nominal and real pricing kernel is

$$M_t^R = M_t^N \pi_t, \tag{5}$$

where π_t denotes inflation based on changes in price levels. We assume inflation is a function of inflation indicators

$$\pi_{t+1} \equiv \frac{I_{t+1}}{I_t} = \exp(\rho' X_{t+1}^{EC}) \tag{6}$$

The implied real short rate

In line with the affine bond prices, the short rate is defined as

$$r_t^N = \delta_0^N + \delta_1'^N X_t, \tag{7}$$

Due to the relation of real pricing kernel and our previous assumptions, the real short rate is

$$r_t^R = r_t^N - \overbrace{\rho'(\Phi_0 + \Phi_1 X_t)}^{\text{Expected inflation}} - \overbrace{\rho'\Sigma\lambda_t}^{\text{Inflation risk premium}} - \overbrace{\frac{1}{2}\rho'\Sigma\Sigma'\rho}^{\text{Convex term}} . \tag{8}$$

Function for the coefficients of the yields

By substitution of the pricing kernel and the affine yields in the following equation,

$$P_t^N(n) = E_t\left[M_{t+1}^N P_{t+1}^N(n-1)\right], \tag{9}$$

we can derive recursive coefficients for the state variables.

$$Y_t^N(n) = \bar{A}_n^N + \bar{B}_n'^N X_t, \tag{10}$$

Issues

Estimation of no arbitrage term structure models is not directly straightforward:

- Typical estimation procedure: Numerical optimization of the Log Likelihood function.
- A common observation: Unit root problems of the factors cause flat likelihoods.
- Consequently, local optima are derived.
- Example: Hamilton and Wu (2012) with results Ang and Piazessi (2003).

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State-space notation

Define the following state space model:

$$\begin{bmatrix} Y_t^1 \\ Y_t^2 \end{bmatrix} = \begin{bmatrix} \bar{A}^1 \\ \bar{A}^2 \end{bmatrix} + \begin{bmatrix} \bar{B}'^1 \\ \bar{B}'^2 \end{bmatrix} X_t + \begin{bmatrix} 0 \\ \Omega \end{bmatrix} \eta_t, \tag{11}$$

To derive the Latent state variables assume yields measured without error (Y_t^1) and invert those. (See e.g. Chen and Scott (1993))

First step: Reduced form parameters

Use OLS to determine the coefficients of the measurement equations

$$X_t^{EC} = \Phi_0^* + \Phi_{1EC}^* X_t^{EC} + \epsilon_{ECt}^* \tag{12}$$

$$Y_t^1 = A_1^* + \Phi_{11}^* Y_{t-1}^1 + \epsilon_{1t}^* \tag{13}$$

$$Y_t^2 = A_2^* + \Phi_{21}^* Y_t^1 + \Phi_{2EC}^* X_t^{EC} + \epsilon_{2t}^*. \tag{14}$$

The first equation is a VAR model of the observable state variables. The second equation is a VAR model of the unobservable state variables, the underlying process is implied by State equation. The third equation is a regression of the measurement equation.

Reduced form parameters

These OLS coefficients match the structural model parameters as follows:

$$\begin{aligned}
\Phi^*_{1EC} &= \Phi_1^{EC} \\
A_1^* &= A_1 - B_{1L}\Phi_1^L B_{1L}^{-1} A_1 \\
\Phi^*_{11} &= B_{1L}\Phi^*_{11} B_{1L}^{-1} \\
A_2^* &= A_2 - B_{2L} B_{1L}^{-1} A_1 \\
\Phi^*_{21} &= B_{2L} B_{1L}^{-1} \\
\Phi^*_{2EC} &= B_{2EC}
\end{aligned}$$

Second step Chi-squared

To determine the optimal structural model parameters, we employ a Chi-squared minimization

$$\min_{\theta} T\left[\hat{\pi} - g(\theta)\right]' \hat{R}\left[\hat{\pi} - g(\theta)\right], \tag{15}$$

where $\hat{\pi}$ is the vector of reduced form estimates, $\hat{R}$ denotes the consistent estimate of the information matrix based, and the function $g(\theta)$ transforms the structural parameters θ in the reduced form.

Hamilton and Wu (2012) show that this minimization yields asymptotically variances of the parameters equivalent to the MLE.

Estimates of our model

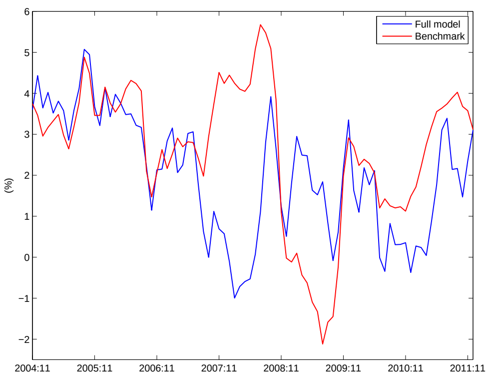
These results are based on a 3 latent factors while not taking into account the inflation risk premium.

Table: The decomposition of market implied expected inflation

	US		UK		
	Mean	Stdev	Mean	Stdev	
CPI	1.06	0.03	1.34	0.03	
Commodity market inflation	-0.11	0.00	-0.09	0.00	
House price index	0.13	0.01	0.08	0.01	
Volatility asset markets	-0.01		0.00	-0.03	0.00

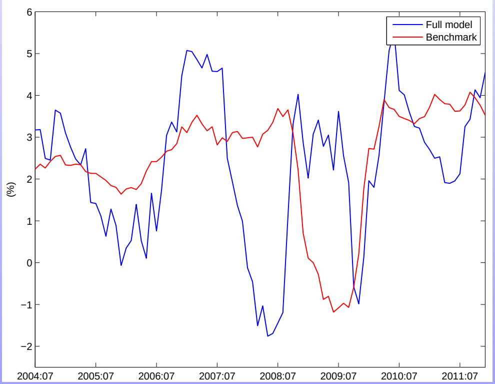
Implied expected inflation US

The benchmark model (only CPI) versus our full model: The suggested factors improves our fit.



Implied expected inflation UK

The benchmark model (only CPI) versus our full model The suggested factors improves our fit.



Summary and conclusions

Summarizing:

- Small sample bias in OLS estimates.
- CPI captures most of the market expectations.

Further research

- Bayesian analysis