Mathematical Behavioural Finance
A Mini Course

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Overview of This Course

- Chapter 1: Introduction

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- Chapter 2: Portfolio Choice under RDUT - Quantile Formulation

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- Chapter 4: Portfolio Choice under CPT

Chapter 1: Introduction

Section 1

Expected Utility Theory

Evaluation of Future Cash Flow

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- How to compare random variables?
- *Expected value* or *mean* $E[\tilde{X}]$: $110 \times 60\% + 90 \times 40\% = 102$

St Petersburg Paradox

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- The **expected** payoff

$$E[\tilde{X}] = \frac{1}{2} \times 2 + \frac{1}{4} \times 4 + \frac{1}{8} \times 8 + \cdots + \frac{1}{2^n} \times 2^n + \cdots = +\infty!$$

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- "Few of us would pay even 25 ducats to enter such a game" (R. Martin 2004, *The Stanford Encyclopedia of Philosophy*)

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- ... or 4 ducats

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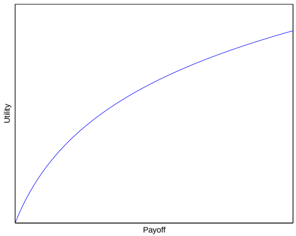

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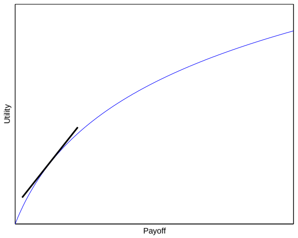

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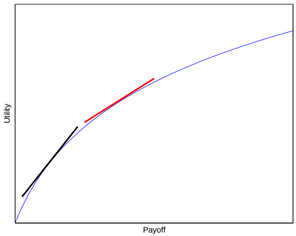

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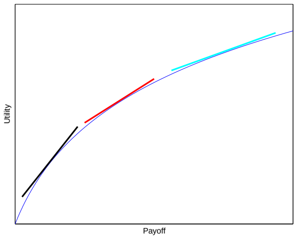

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- Behaviour of a **rational** agent necessarily coincides with that of an agent who values uncertain payoffs using expected **concave** utility

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- *Neoclassical economics*

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- *Reaganomics*: "Only by reducing the growth of government, can we increase the growth of the economy"

Section 2

Expected Utility Theory Challenged

Paradoxes/Puzzles with EUT

EUT is systematically violated via experimental work, and challenged by many paradoxes and puzzles

- Allais paradox: Allais (1953)
- Ellesberg paradox: Ellesberg (1961)
- Friedman and Savage puzzle: Friedman and Savage (1948)
- Equity premium puzzle: Mehra and Prescott (1985)
- Risk-free rate puzzle: Weil (1989)

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- Merton Miller: "If you transfer a dollar from your right pocket to your left pocket, you are no wealthier. Franco (Modigliani) and I proved that rigorously"

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- I paid immediately ... **filled with gratitude and joy**

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  - D: lose \$7,500 for sure

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- "$B + C > A + D$"

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  - F: 75% chance to lose $7,500, 25% chance to gain $2,500

Decisions Depend on Frames (Cont'd)

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  - E: 75% chance to lose \$7,600, 25% chance to gain \$2,400
  - F: 75% chance to lose \$7,500, 25% chance to gain \$2,500
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Decisions Depend on Frames (Cont'd)

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  - F: 75% chance to lose \$7,500, 25% chance to gain \$2,500
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- However: $B + C = E$, $A + D = F$!

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- "$F = E + \$100 > E$"
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- *Frame dependence*: frames are not transparent, but opaque

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- B: Earn \$100,000/year while all your colleagues earn at *most* \$50,000/year
- B was more popular
- *Reference point*: what matters is **deviation** of wealth from certain benchmark, not wealth itself

Risk Aversion vs. Risk Seeking

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- **Risk averse on gains, risk seeking on losses**

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- *Loss aversion*: pain from a loss is more than joy from a gain of the same magnitude

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- *Probability weighting (distortion)*: People tend to exaggerate, **intentionally or unintentionally**, small probabilities of **both** winning big and losing big

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- *Risk-free rate puzzle* (Weil 1989): observed risk-free rate is too low to be explainable by classical CCAPM

Economic Data 1889–1978 (Mehra and Prescott 1985)

	Consumption growth		riskless return		equity premium		S&P 500 return	
Periods	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
1889–1978	**1.83**	**3.57**	**0.80**	**5.67**	**6.18**	**16.67**	**6.98**	**16.54**
1889–1898	2.30	4.90	5.80	3.23	1.78	11.57	7.58	10.02
1899–1908	2.55	5.31	2.62	2.59	5.08	16.86	7.71	17.21
1909–1918	0.44	3.07	-1.63	9.02	1.49	9.18	-0.14	12.81
1919–1928	3.00	3.97	4.30	6.61	14.64	15.94	18.94	16.18
1929–1938	-0.25	5.28	2.39	6.50	0.18	31.63	2.56	27.90
1939–1948	2.19	2.52	-5.82	4.05	8.89	14.23	3.07	14.67
1949–1958	1.48	1.00	-0.81	1.89	18.30	13.20	17.49	13.08
1959–1968	2.37	1.00	1.07	0.64	4.50	10.17	5.58	10.59
1969–1978	2.41	1.40	-0.72	2.06	0.75	11.64	0.03	13.11

EUT Based Theories

- Recall EUT based formulae (single period)

$$\bar{r} - r_f \approx \alpha \mathbf{Cov}(\tilde{g}, \tilde{r}),$$
$$1 + r_f \approx \frac{1+\alpha\bar{g}}{\beta}$$

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  - So $\alpha \geq \frac{6.98\% - 0.80\%}{3.57\% \times 16.54\%} = 10.47$

Puzzles under EUT

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- Under EUT, a puzzle thus arises: the solution simultaneously requires a small relative risk aversion to account for the low risk-free rate and a large relative risk aversion to account for the high equity premium

Section 3

Alternative Theories for Risky Choice

Yaari's Dual Theory

Preference on random payoff $\tilde{X} \geq 0$ represented by (Yaari 1987)

$$V(\tilde{X}) = \int \tilde{X} d(w \circ \mathrm{P}) := \int_0^\infty w(\mathrm{P}(\tilde{X} > x))dx$$

where *probability weighting* $w : [0,1] \to [0,1]$, $\uparrow$, $w(0) = 0$, $w(1) = 1$

Risk Preference Reflected by Weighting

Assuming w is differentiable:
$V(\tilde{X}) = \int_0^\infty x d[-w(1 - F_{\tilde{X}}(x))] = \int_0^\infty x w'(1 - F_{\tilde{X}}(x)) dF_{\tilde{X}}(x)$
where $F_{\tilde{X}}$ is CDF of $\tilde{X}$

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- Risk seeking when $w(\cdot)$ is concave
- Simultaneous risk averse and risk seeking when $w(\cdot)$ is inverse-S shaped

Probability Weighting Functions

- Kahneman and Tversky (1992) weighting

$$w(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}},$$

- Tversky and Fox (1995) weighting

$$w(p) = \frac{\delta p^\gamma}{\delta p^\gamma + (1-p)^\gamma},$$

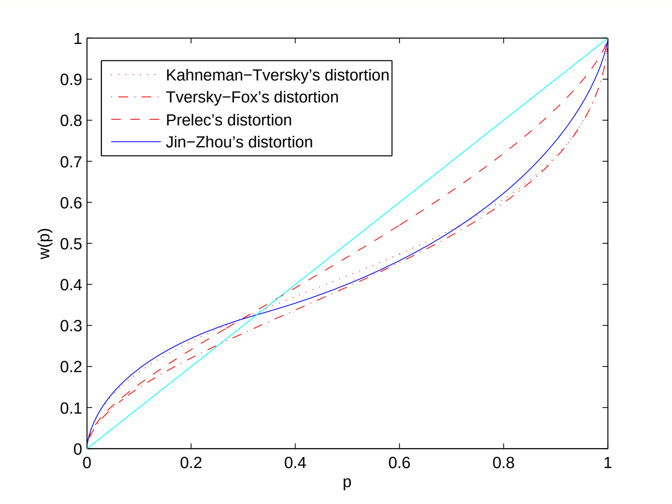
- Prelec (1998) weighting

$$w(p) = e^{-\delta(-\ln p)^\gamma}$$

- Jin and Zhou (2008) weighting

$$w(z) = \begin{cases} y_0^{b-a} k e^{a\mu + \frac{(a\sigma)^2}{2}} \Phi\left(\Phi^{-1}(z) - a\sigma\right) & z \leq 1 - z_0, \\ C + k e^{b\mu + \frac{(b\sigma)^2}{2}} \Phi\left(\Phi^{-1}(z) - b\sigma\right) & z \geq 1 - z_0 \end{cases}$$

Inverse-S Shaped Functions



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- Two components
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  - A (usually assumed) inverse-S shaped (probability) weighting function: individuals overweight tails

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- A dispositional factor and a situational factor to explain risky choices
  - *Dispositional factor* describes people's natural tendency to achieving security **and** exploiting potential
  - *Situational factor* describes people's responses to specific, immediate needs and opportunities

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  with $q_s, q_p > 0$ and $0 < \nu < 1$
- The nonlinear transformation z^{q_s+1} reflects the security and $1-(1-z)^{q_p+1}$ reflects the potential

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- A is the aspiration level, $0 < \alpha < 1$

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- Note: Tversky and Kahneman (1992) used discrete random variables

Section 4

Summary and Further Readings

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- Behavioural theories with new risk preferences have emerged

Further Readings

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