Stochastic grid bundling method for backward stochastic differential equations

Ki Wai Chau

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Abstract

In this work, we aim to apply stochastic grid bundling method (SGBM) to numerically solve backward stochastic differential equations.

The Stochastic Grid Bundling Method (SGBM) is a Monte Carlo based algorithm designed to solve backward dynamic programming problems, with applications in pricing Bermudan options in Jain and Oosterlee [2015]. This algorithm has been further extended computationally by the incorporation of GPU accelation in Leitao and Oosterlee [2015] and generalized to the computation of Credit Valuation Adjustment and Potential Future Exposure in De Graaf et al. [2014]. It takes advantage of both regress later technique and the adaptive local basis approach to give accurate results.

In usual regression methods for backward in time problems, the target function values at the end of a time interval is regressed on dependent variables measured at the beginning of the time interval, which creates a statistical error. The dependent variable is regressed here on a set of basis functions at the end of the interval instead in a regress-later method, and the conditional expectation across the interval is then computed exactly for each basis function. By removing the statistical error in the regression step, a regress-later method is capable of converging faster than a conventional method. Regress-later schemes have been further discussed in Glasserman and Yu [2004].

With the adaptive local basis approach, the regression basis is defined on a partition of the domain, and the exact partition depends on the simulated examples. Since the support is chosen so that they contain roughly the same number of samples and each local basis is non-overlapping, SGBM is easy to scale up and can facilitate the application of parallel computing to our algorithm.

We test the SGBM algorithm in a new problem setting such that we can take advantage of its nice properties and also get a better understanding of the underlying principles. The results has been summerized in Chau and Oosterlee [2018].

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