INSIGHT.DATA.CLARITY.

Presented by Jean-Philippe Bouchaud

January 21, 2019

Market Impact: A (Short) Review

Many thanks to Z. Eisler, I. Mastromatteo, B. Toth, M. Benzaquen, J. Bonart, F. Bucci, J. Donier, M. Gould

Price Impact

What is price impact?

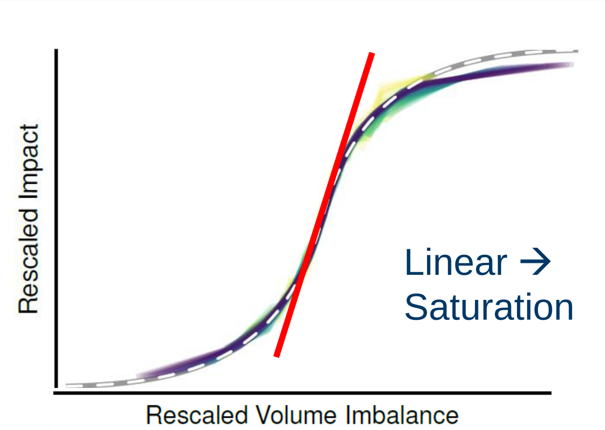
- Price impact = correlation between an arriving order and the subsequent price change
- Buy/sell trades push the price up/down – on average

Two schools of thought:

- A. Market impact is information revelation (people trade because they know where the price is going (??))
- B. Market impact is a 'mechanical' statistical effect (like the response of a physical object) ✓
- This is highly relevant:
  - Allows information (but also noise!) to be included in prices
  - Induces extra execution costs – large but often overlooked
  - Makes marked-to-market valuation over-optimistic
  - Can lead to crashes – the impact of a trade can trigger other trades

Price Impact: some empirical findings

- **Price impact of <u>single</u> market orders:** clearly positive and decaying but:
  - > Strongly non universal (tick size dependent)
  - > Apparent saturation for large MO volume q (but q usually smaller than volume at best v_b)
  - > Long-range (power-law) autocorrelation of the sign of MOs (more below)
- **Impact of the aggregate order imbalance:** nice scaling for different window sizes N



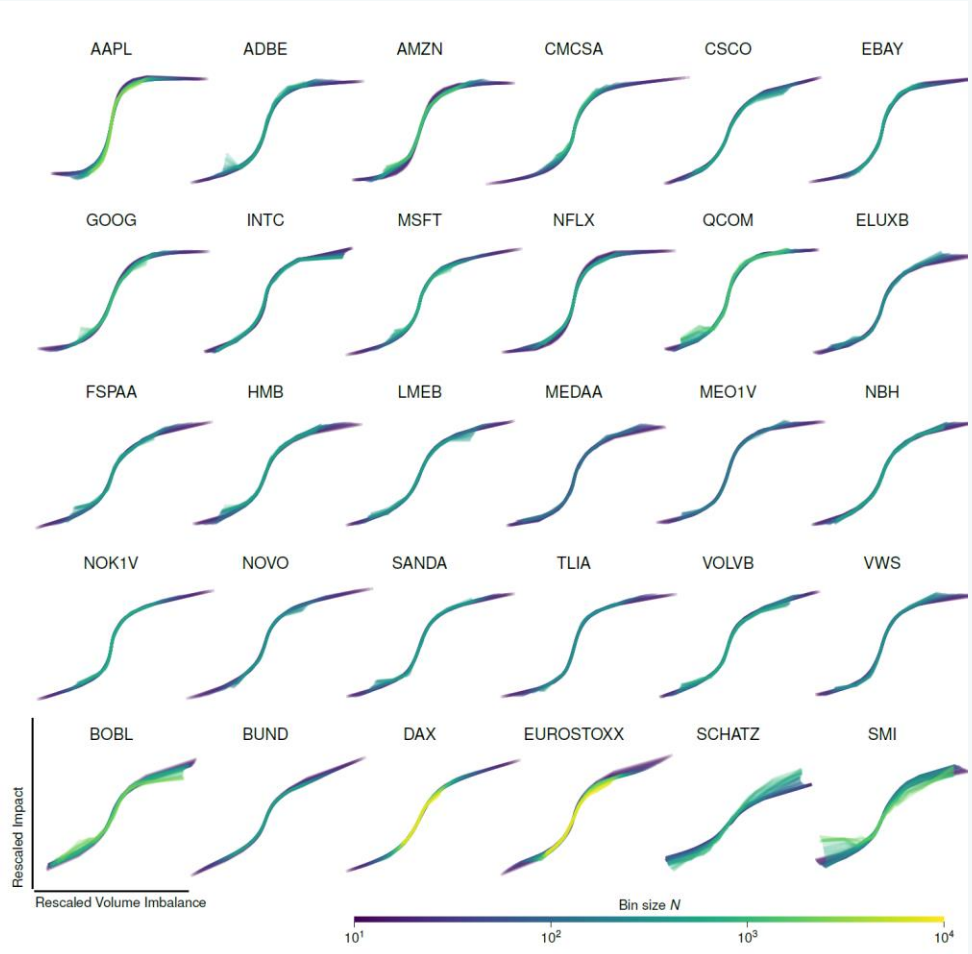
$$\mathcal{R}_N(\mathcal{Q}) \approx R_N \mathscr{F}\left(\frac{\mathcal{Q}}{Q_N}\right)$$

$$Q_N \approx Q_1 N^{\xi}$$

$$R_N \approx R_1 N^{\psi}$$

F. Patzelt, JPB, Phys. Rev. E 2018

Aggregate Impact for a variety of assets



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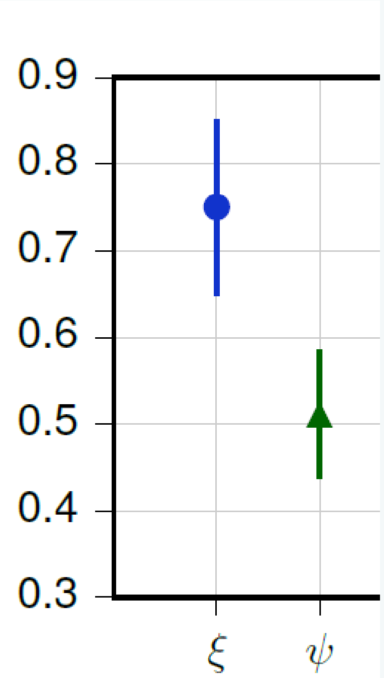


F. Patzelt, JPB, Phys. Rev. E 2018

Impact of Metaorders

- Impact of single orders or series of anonymous orders can be measured using public data, but is of limited use to answer the truly relevant information for trading “metaorders”

Metaorders:

- For a liquid small tick stock the instantaneous volume at best is approx. 10^{-5} of market cap., while the total daily traded volume is 500 times larger.
- Most of the available volume is “latent”, only progressively revealed during the day
- Large trades must be sliced/diced and executed incrementally using both MO and LOs
  - What is the impact **I(Q)** of a metaorder of size Q and sign ε?
  - $I(Q) := E[\,(\Delta p/p)\,.\,\varepsilon \mid Q\,]$ average relative price change between the beginning and the end of the execution of the metaorder, with the correct sign (and *not* $E\,[\,|\Delta p/p|\,]$)

Sqrt-Impact of Metaorders

A metaorder of size Q has a sqrt price impact:

$$I(Q) = Y\sigma_T\sqrt{\frac{Q}{V_T}}$$

where:

Q is the volume of the metaorder

σ_T is the volatility of the market

V_T is the total volume traded in the market

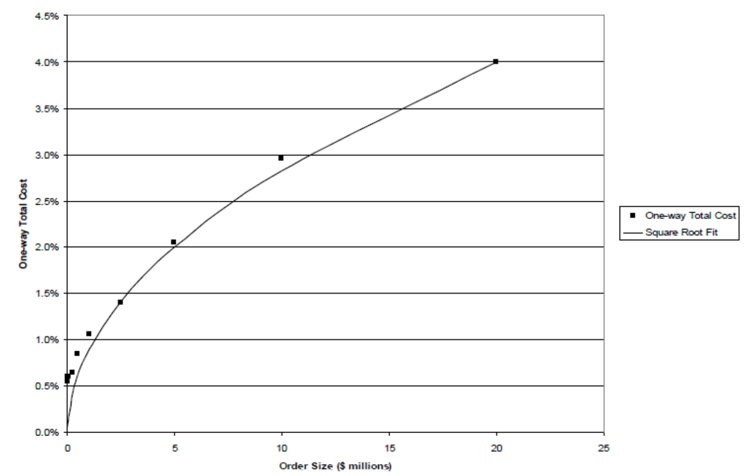
(Y of order 1)

Important notes:

- Impact is usually small compared to vol itself
- Requires a lot of averaging to be seen
- Beware of conditioning artefacts

A universal empirical result?

Independently but consistently reported by many groups since the mid-eighties (Loeb 83 (!), BARRA 95, Almgren 05, Engle, Kissel, JPM, DB, LH, CFM, Ancerno data, AQR)



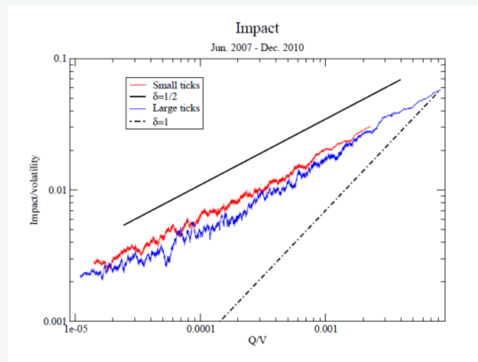
US Stocks, Loeb

Impact of Metaorders

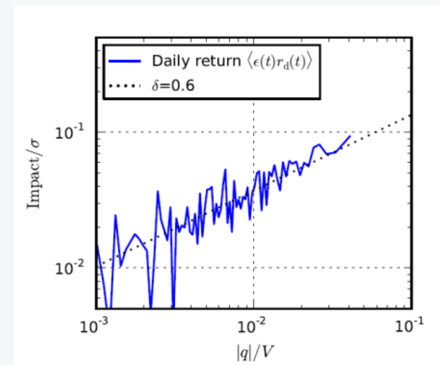
A universal empirical result? (CFM data)

$$I(Q) = Y \sigma_T \sqrt{\frac{Q}{V_T}}$$

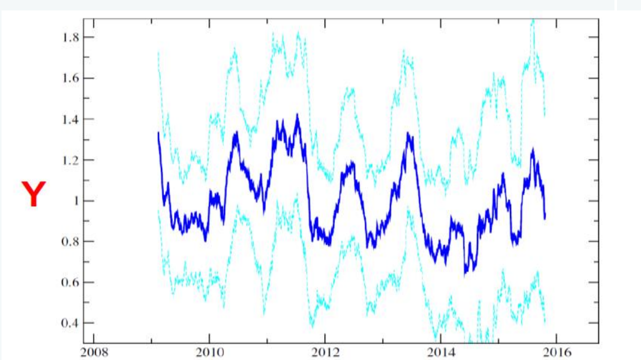
Futures



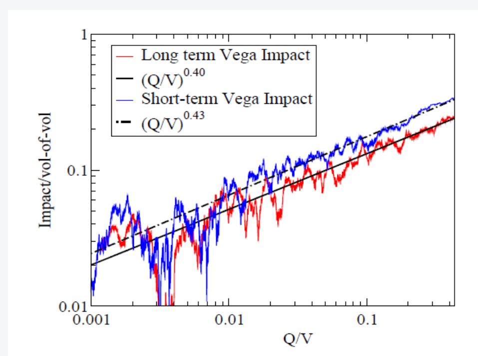
Intl stocks



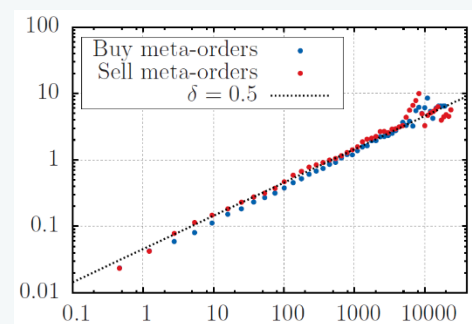
Remarkable stability of Y



US stock implied vol



Bitcoin!

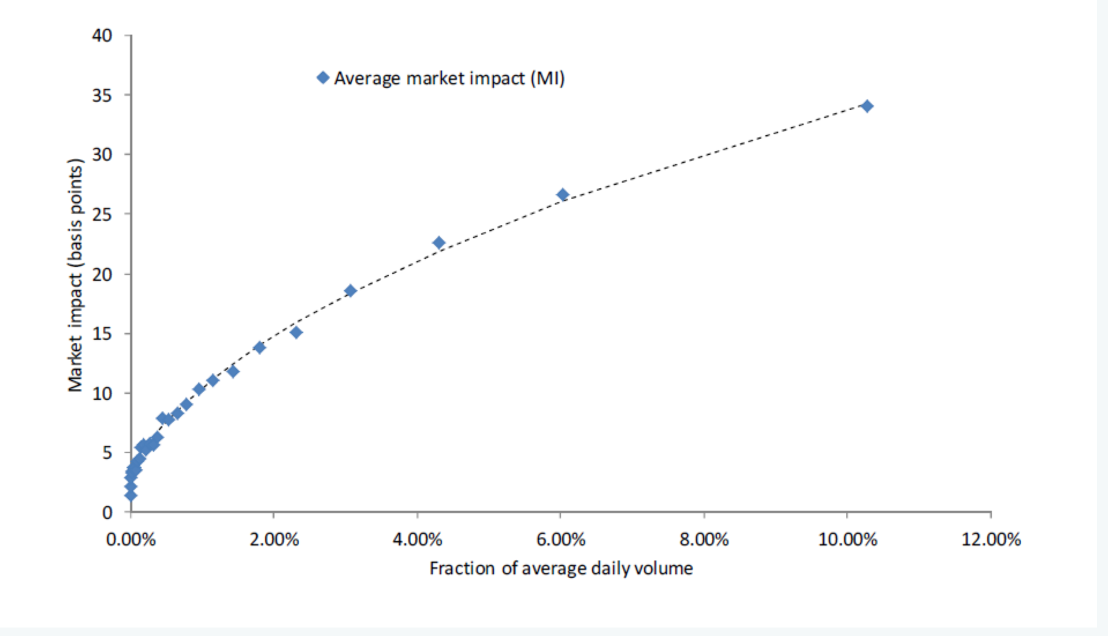


Impact of Metaorders

A universal empirical result? (AQR, 2018)

$$I(Q) = Y\sigma_T\sqrt{\frac{Q}{V_T}}$$

Intl stocks



The Square-Root Impact Law

$$I(Q) = Y\sigma_T \sqrt{\frac{Q}{V_T}}$$

- Kyle: impact is linear in Q
- Remarkable stability of results:
  - Style of trading, strategies, markets, period (1980 – 2018), tick sizes, treatment of data etc.
  - Hints that microstructure and HFT effects are not relevant, only “macro-liquidity”
  - Impact is, to first approximation, *independent* of the execution time of the metaorder!
- A genuine “physical law” of financial markets?
- Understanding why is important both conceptually and for applications

A first approach: the “Propagator Model” (2004)

- Assumption: each MO has a time-decaying impact described by a bare “propagator” G(t)
- Impact of different MOs add linearly + noise

$$m_t = m_{t_0} + \sum_{t_0 \le n < t} G(t-n)\varepsilon_n + \sum_{t_0 \le n < t} \xi_n$$

- The MO signs ε are long-range correlated $\mathbb{E}[\varepsilon_{t+\ell}|\varepsilon_t = 1] = C(\ell) \sim \frac{c_\infty}{\ell^\gamma}$ $\gamma \sim 0.5$

Note: trade signs are *uncorrelated* in Kyle !

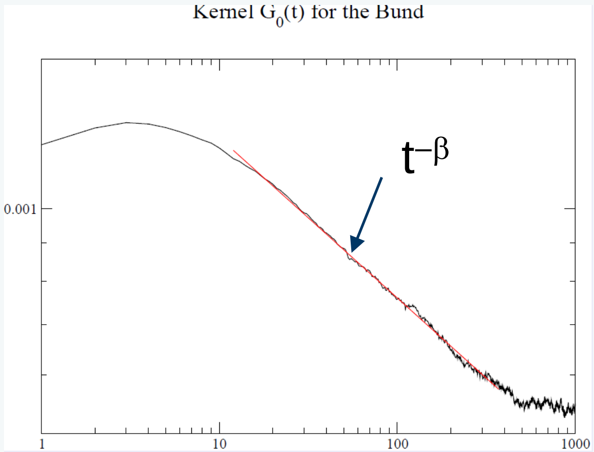
- The decaying impact must be such that the resulting price dynamics is diffusive

$$G(\ell) \approx_{\ell \gg 1} \frac{\Gamma_\infty}{\ell^\beta} \qquad \beta = \beta_c := (1-\gamma)/2$$

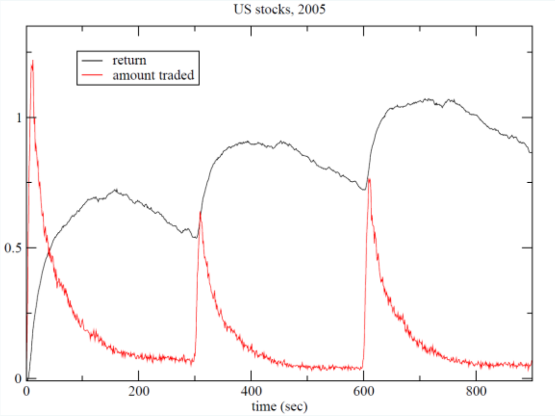
Propagator model: data

- ▶ The propagator model accounts well for impact decay
- ▶ Model calibrated on all trades or on CFM's lead to similar propagators

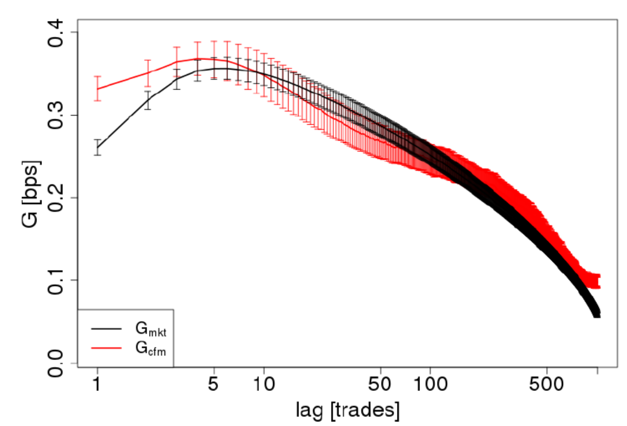
All trades (Bund)



Impact decay (front loaded Trades, 2005)



All trades vs. CFM trades (US stocks)



J. Kockelkoren, B. Toth, Z. Eisler, JPB

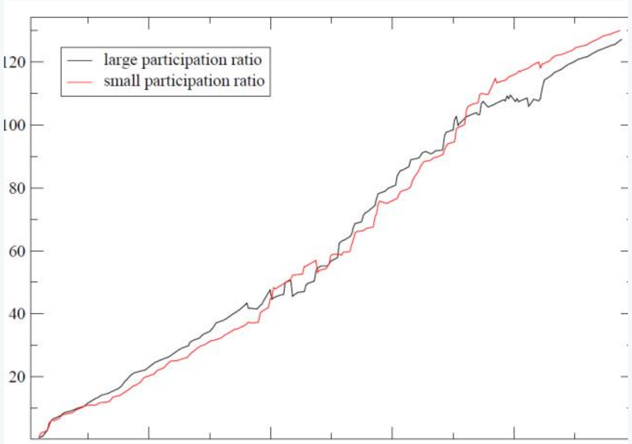
Propagator model: metaorders and aggregate impact

Impact of metaorders within the propagator model: qualitatively ok, but not accurate

$$\frac{\Gamma_\infty}{(1-\beta)} f^\beta \left(\frac{Q}{v}\right)^{1-\beta}$$

f: participation ratio

$$\beta = \beta_c := (1-\gamma)/2$$



Issues:

- $\gamma = 0.5 \rightarrow \beta = 0.25$
- $\beta = 0.5 \rightarrow \gamma = 0$??
- Dependence on f is empirically weaker than $f^{1/2}$
- Square-root law is *independent* of f!

Aggregate Impact within the propagator model

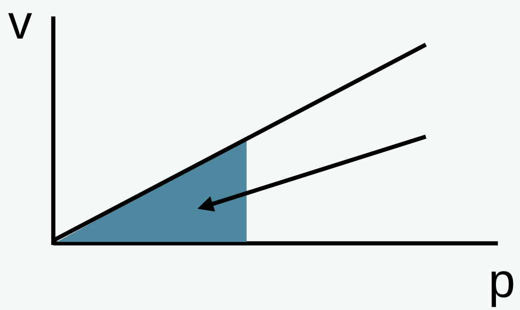
$$\mathcal{R}_N(Q) \approx R_N \mathscr{F}\left(\frac{Q}{Q_N}\right)$$ but with $\xi=1$ (0.75) and $\psi=1-\beta$ (0.5)

$$Q_N \approx Q_1 N^\xi$$
$$R_N \approx R_1 N^\psi$$

Sqrt Impact: a locally linear supply/demand?

Intuition

- Impact is limited by the volume on the other side
- Assume by fiat volume of opposite sellers is linear in price
- More resistance (less impact) as the price increases

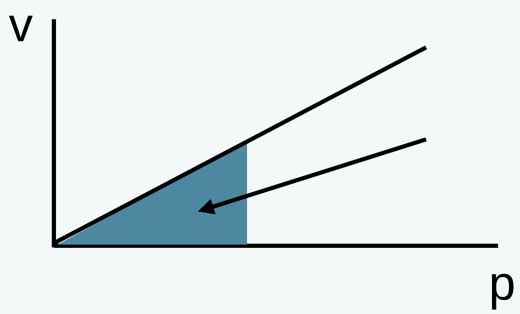


$Q = pv/2 \sim p^2 \rightarrow$ Sqrt impact!

A dynamical theory of liquidity

But WHY should the liquidity profile be linear and vanish around the current price?

- Our theory*: a purely statistical effect, even with “zero-intelligence” trades: provided the price makes a random walk, and for a generic order flow, the probability to have an unexecuted order close to the current price is indeed linearly small



$Q = pv/2 \sim p^2 \rightarrow$ Sqrt impact!

*B. Toth, et al. PRX (2011), I. Mastromatteo et al., PRL (2014, J. Donier et al., Quant. Fin (2015)

A dynamical theory of liquidity

- A mathematical model for the latent order book

$$\partial_t \varphi_{\mathrm{b}} = D\partial_{xx}\varphi_{\mathrm{b}} - \nu\varphi_{\mathrm{b}} + \lambda\Theta(x_t - x) - R_{\mathrm{ab}}(x)$$
$$\partial_t \varphi_{\mathrm{a}} = D\partial_{xx}\varphi_{\mathrm{a}} - \nu\varphi_{\mathrm{a}} + \lambda\Theta(x - x_t) - R_{\mathrm{ab}}(x)$$

- Orders on the bid size (b) and ask side (a) are:

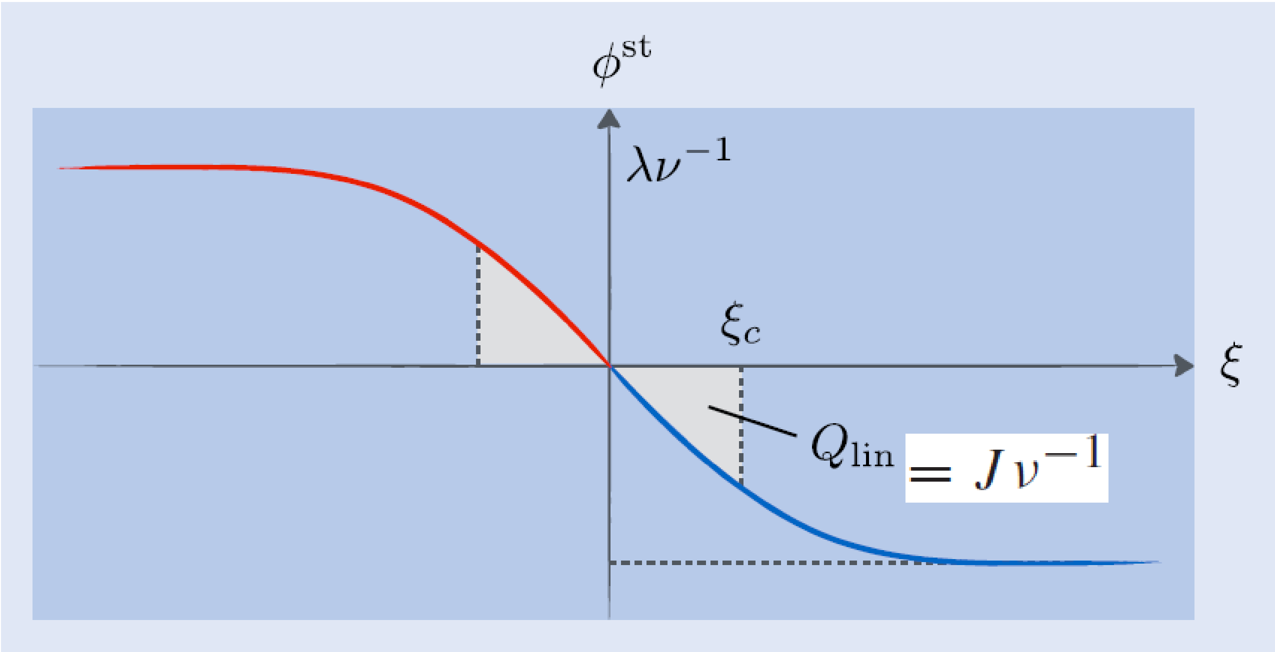
1) Deposited with rate λ
2) Cancelled with rate ν
3) Randomly modified with a diffusion rate D
4) Executed when they meet at price x_t with rate R_{ab}

- A drift term towards the price x_t can be added without changing the main result

A dynamical theory of liquidity: stationary profile

Why should the liquidity profile be linear and vanish around the current price?

$$\phi(x,t) = \varphi_{\mathrm{b}}(x,t) - \varphi_{\mathrm{a}}(x,t)$$



$$J := \partial_\xi \phi^{\mathrm{st}}(\xi)\big|_{\xi=0} \qquad \xi_{\mathrm{c}} = \sqrt{D\nu^{-1}}$$

A dynamical theory of liquidity: impact of a metaorder

$$\phi(x,t) = (\mathcal{G}_\nu \star \phi_0)(x,t) + \int \mathrm{d}y \int_0^\infty \mathrm{d}\tau \, \mathcal{G}_\nu(x-y, t-\tau) s(y,\tau)$$

with:

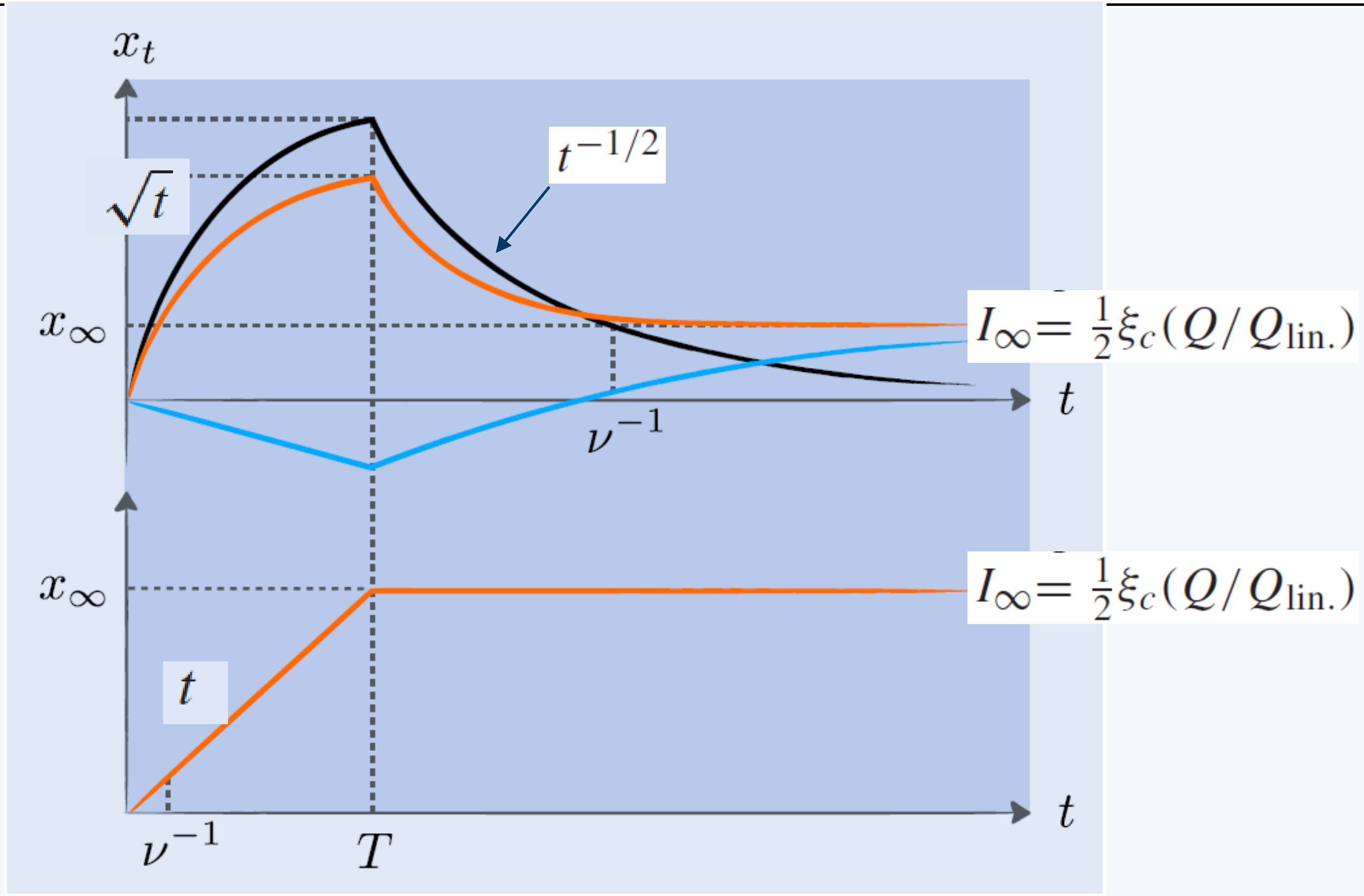
$$\mathcal{G}_\nu(x,t) = e^{-\nu t}\mathcal{G}(x,t) \qquad \mathcal{G}(x,t) = \Theta(t)\frac{e^{-\frac{x^2}{4Dt}}}{\sqrt{4\pi Dt}}$$

and a “source” term with a metaorder intensity m:

$$s(x,t) = m_t \delta(x - x_t) \cdot \mathbb{1}_{[0,T]} + \lambda \,\mathrm{sign}(x_t - x)$$

$$\rightarrow \quad y_t = \frac{1}{\mathcal{L}} \int_0^t \frac{\mathrm{d}s \, m_s}{\sqrt{4\pi D(t-s)}} e^{-\frac{(y_t - y_s)^2}{4D(t-s)}}$$

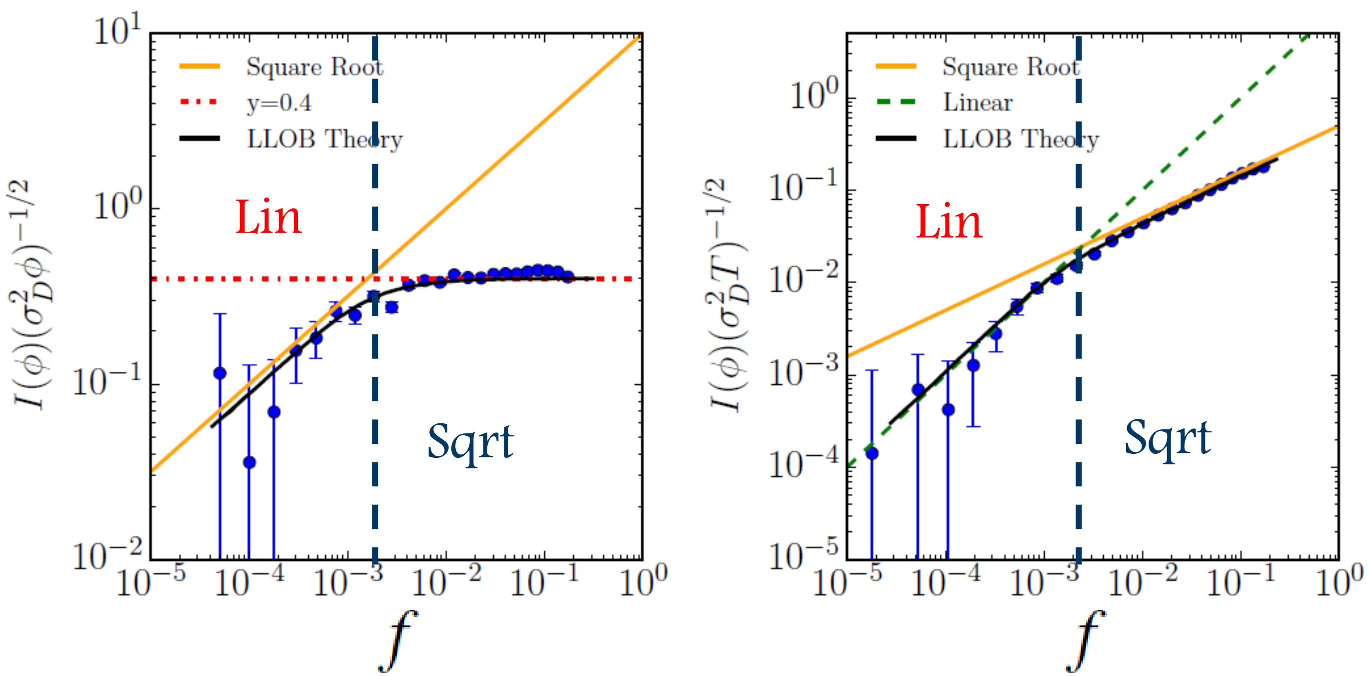
A dynamical theory of liquidity: impact of a metaorder



Permanent impact is *linear* in Q (Kyle on a macroscale – see Huberman/Stanzl, Rosenbaum)

A dynamical theory of liquidity: non-linear propagator

Ancerno data



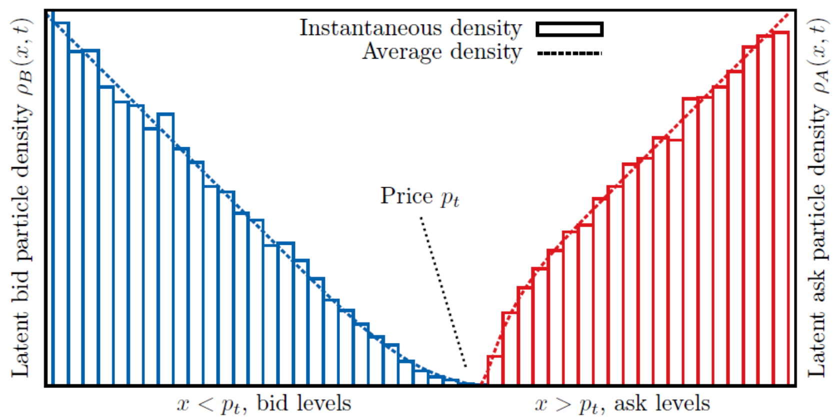
f: participation ratio (= m/J = Q/V) → Impact is independent on T in the sqrt region

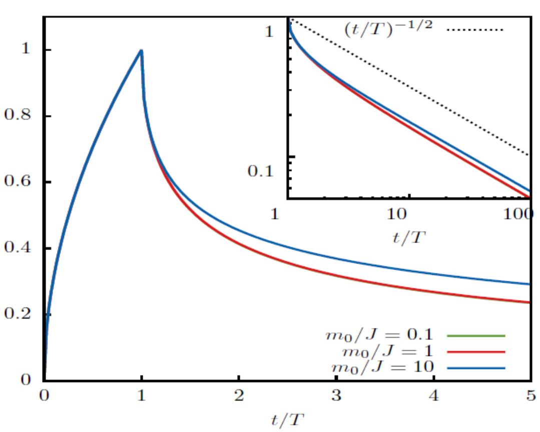
Note: the theoretical crossover should be f* ~ 1 ??

A *non-linear* propagator model

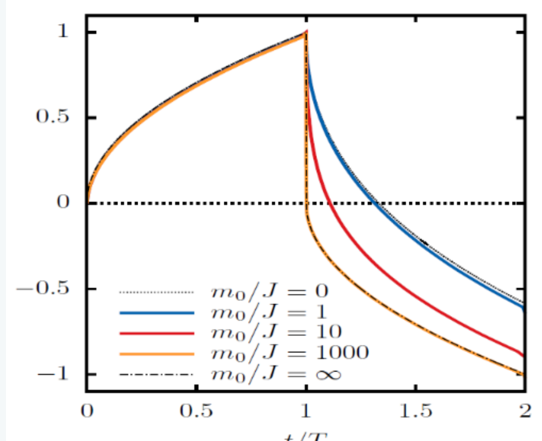
$$\partial_t \varphi_{\mathrm{b}} = D\partial_{xx}\varphi_{\mathrm{b}} - \nu\varphi_{\mathrm{b}} + \lambda\Theta(x_t - x) - R_{\mathrm{ab}}(x)$$
$$\partial_t \varphi_{\mathrm{a}} = D\partial_{xx}\varphi_{\mathrm{a}} - \nu\varphi_{\mathrm{a}} + \lambda\Theta(x - x_t) - R_{\mathrm{ab}}(x)$$

Note: single memory time scale := ν^{-1}





Decay of impact with $\beta = 1/2$ (?)



Theorem: No price manipulation

A *non-linear* propagator model

$$\partial_t \varphi_b = D\partial_{xx}\varphi_b - \nu\varphi_b + \lambda\Theta(x_t - x) - R_{ab}(x)$$
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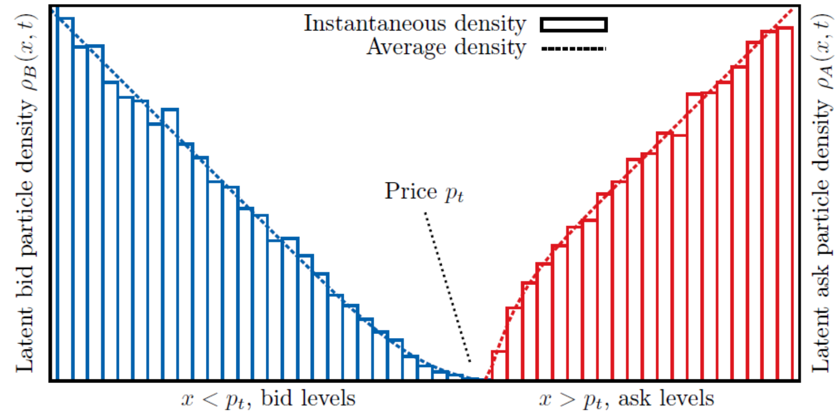

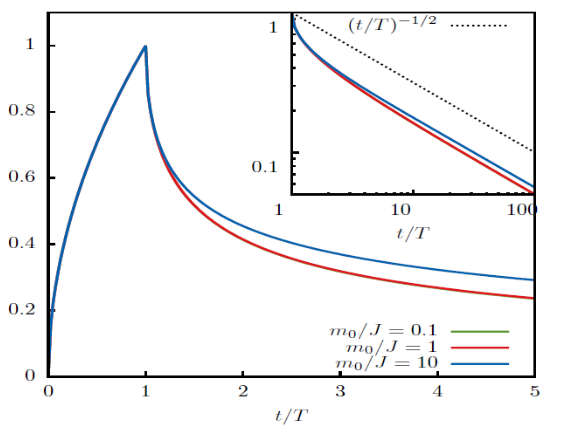

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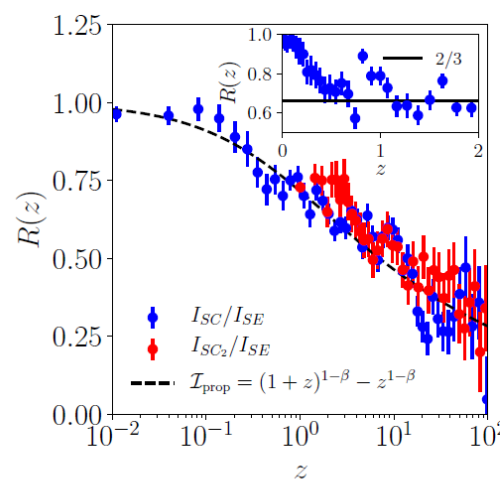

Remaining Issues/Loose ends

- In the linear regime, impact decay is too fast (β = ½ > β_c), which would lead to short term mean reversion (not observed in reality)
- The strict square-root impact is valid in the large participation ratio limit f > 1 (whereas most data is for Q/V ~ 0.1 – 10 %)
- Intuitively, the dynamics of liquidity is multiscale, from HFTs to slow trading

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- Intuitively, the dynamics of liquidity is multiscale, from HFTs to slow trading

- Generalized latent order book model: wide spectrum of time scales (for cancellation and/or order adjustments): M. Benzaquen, JPB (2017)
- This allows us to get $\beta < ½$ and escape the diffusivity paradox
- One gets a linear/non-linear crossover for a much smaller $\mathbf{f^* = J_s/J_f}$

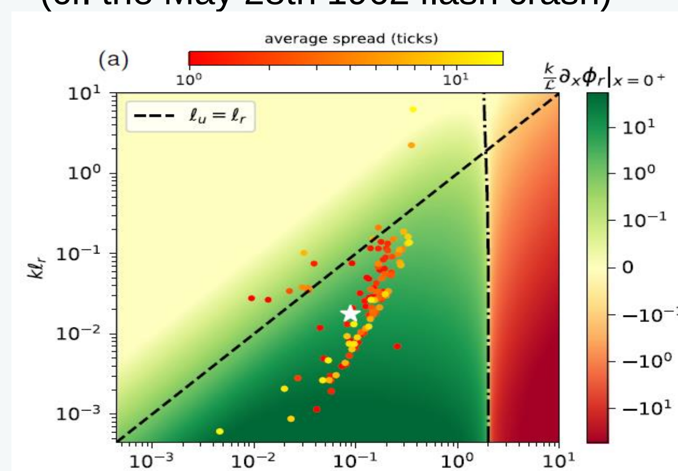
 (HFT contribute to most of the flow, but unable to resist large metaorders)
- Although we believe it to be the case, we have not been able to prove that any round trip has a positive average cost
- Many interesting loose ends from a mathematical point of view

Intrinsic Market Fragility

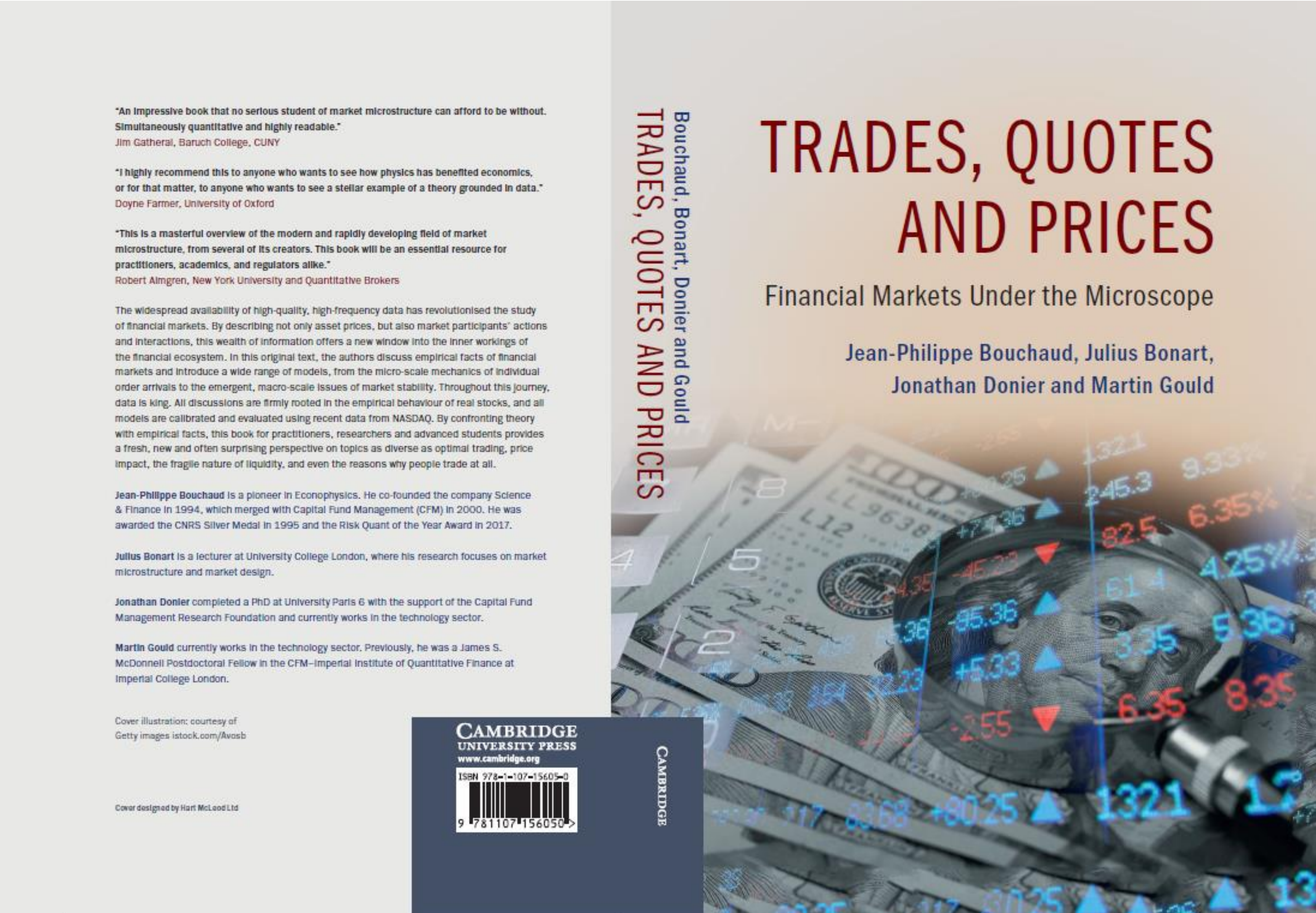
Broader Consequences for Market stability/fragility

- Liquidity at the best price is necessarily small (eaten by diffusive prices)
- This imposes splitting up metaorders and leads to an anomalously large impact for small trades
- Liquidity fluctuations are bound to play a crucial role: Micro-crises and jumps in prices without news, as indeed seen empirically ever since markets exist
- Volatility-liquidity feedback loop can become unstable due to lag in liquidity revelation → «flash crashes» (Dall'Amico, Fosset, Benzaquen, JPB 2018)

(cf. the May 28th 1962 flash crash)



Slower liquidity revelation →



"An impressive book that no serious student of market microstructure can afford to be without. Simultaneously quantitative and highly readable."
Jim Gatheral, Baruch College, CUNY

"I highly recommend this to anyone who wants to see how physics has benefited economics, or for that matter, to anyone who wants to see a stellar example of a theory grounded in data."
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"This is a masterful overview of the modern and rapidly developing field of market microstructure, from several of its creators. This book will be an essential resource for practitioners, academics, and regulators alike."
Robert Almgren, New York University and Quantitative Brokers

The widespread availability of high-quality, high-frequency data has revolutionised the study of financial markets. By describing not only asset prices, but also market participants' actions and interactions, this wealth of information offers a new window into the inner workings of the financial ecosystem. In this original text, the authors discuss empirical facts of financial markets and introduce a wide range of models, from the micro-scale mechanics of individual order arrivals to the emergent, macro-scale issues of market stability. Throughout this journey, data is king. All discussions are firmly rooted in the empirical behaviour of real stocks, and all models are calibrated and evaluated using recent data from NASDAQ. By confronting theory with empirical facts, this book for practitioners, researchers and advanced students provides a fresh, new and often surprising perspective on topics as diverse as optimal trading, price impact, the fragile nature of liquidity, and even the reasons why people trade at all.

Jean-Philippe Bouchaud is a pioneer in Econophysics. He co-founded the company Science & Finance in 1994, which merged with Capital Fund Management (CFM) in 2000. He was awarded the CNRS Silver Medal in 1995 and the Risk Quant of the Year Award in 2017.

Julius Bonart is a lecturer at University College London, where his research focuses on market microstructure and market design.

Jonathan Donier completed a PhD at University Paris 6 with the support of the Capital Fund Management Research Foundation and currently works in the technology sector.

Martin Gould currently works in the technology sector. Previously, he was a James S. McDonnell Postdoctoral Fellow in the CFM–Imperial Institute of Quantitative Finance at Imperial College London.

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TRADES, QUOTES AND PRICES

Financial Markets Under the Microscope

Jean-Philippe Bouchaud, Julius Bonart,
Jonathan Donier and Martin Gould

Price Impact: some initial remarks

Until the mid-90s, the lore was that the traded volume Q should be compared to MCap:

$$\Delta p/p = Q/MCap \sim 0.5 \text{ bp for } Q = 1\% \, V \quad (V = \text{Average Daily Volume})$$

Note: Trading Q=50% V in 1987 should have only moved the market by 0.1% (no feedback of Portfolio Insurance on prices…)

Kyle (1985) theory for impact: an insider hides in the flux of noise traders

$$\Delta p/p = \sigma N^{1/2} \; (Q/V) \sim 60 \text{ bp for } Q = 1\% \, V, \; \sigma = 2\%, \; N=1000 \text{ daily trades}$$

Note: linear, permanent impact

Empirically: the 'square-root' law (see later for more):

$$\Delta p/p = Y \, \sigma \, (Q/V)^{1/2} \sim 10 \text{ bp for } Q = 1\% \, V, \; \sigma = 2\%, \; Y=0.5 \quad ✓$$

Note: anomalous large impact for small Q/V!