When Capital is a Funding Source: The XVA Anticipated BSDEs

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18th Winter school on Mathematical Finance
Lunteren, The Netherlands, 23 January 2019

Based on

Crépey, S., R. Élie, W. Sabbagh, and S. Song (2018). When capital is a funding source: The XVA Anticipated BSDEs. Working paper available at https://math.maths.univ-evry.fr/crepey.

Albanese, C., S. Caenazzo, and S. Crépey (2017). Credit, funding, margin, and capital valuation adjustments for bilateral portfolios. *Probability, Uncertainty and Quantitative Risk 2*(7), 26 pages.

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Stress Test RISK Management and Financial Steering

Outline

1. A General ABSDE Well-Posedness Result

2. The XVA equations are well posed, including in the realistic case where capital can be used for funding variation margin

3. XVA nested Monte Carlo computational strategies

A General ABSDE Result

- Anticipated BSDEs "of the McKean type" in the line of

  Peng, S. and Z. Yang (2009). Anticipated backward stochastic differential equations (ABSDEs), or BSDEs "of the McKean type" *The Annals of Probability 37*(3), 877–902.

- In the XVA context, see also Agarwal, Marco, Gobet, López-Salas, Noubiagain, and Zhou (2018) in relation with their modeling of the so-called initial margin.

- Standard weak martingale representation setup, driven by
  - a d variate Brownian motion B
  - a compensated integer valued random measure $M(dt, de) = j(dt, de) - \eta(t, e)\pi(de)dt$, where the σ finite measure π integrates $1 \wedge |e|^2$ and η is bounded.

Given a positive integer l, we introduce:

- $\mathcal{S}_2^l$, the space of $\mathbb{R}^l$ valued adapted càdlàg processes Y such that

$$\|Y\|^2_{\mathcal{S}_2^l} = \mathbb{E}\big[\sup_{0\le t\le T} |Y_t|^2\big] < +\infty;$$

- $\mathcal{H}_2^l$, the space of $\mathbb{R}^{l\otimes d}$ valued predictable processes Z such that

$$\|Z\|^2_{\mathcal{H}_2^l} = \mathbb{E}\big[\int_0^T |Z_t|^2 dt\big] < +\infty;$$

- $\widehat{\mathcal{H}}_2^l$, the space of l variate predictable random functions U such that

$$\|U\|^2_{\widehat{\mathcal{H}}_2^l} = \mathbb{E}\Big[\int_0^T \int_E |U_t(e)|^2 \eta(t,e)\pi(de)dt\Big] = \mathbb{E}\Big[\int_0^T |U_t|_t^2 dt\Big] < \infty.$$

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- We assume that every $(\mathbb{F}, \mathbb{P})$ square integrable martingale null at time 0 has a representation of the form

$$\int_0^t Z_s dB_s + \int_0^t \int_E U_s(e) M(ds, de)\,, \;\; 0 \leq t \leq T,$$

for suitable integrands $Z \in \mathcal{H}_2^1$ and $U \in \widehat{\mathcal{H}}_2^1$.

Let there be given a map ρ from $\mathcal{S}_2^l \times \mathcal{H}_2^l \times \widehat{\mathcal{H}}_2^l$ into the space of $\mathbb{F}$ predictable processes satisfying the following:

Assumption 1

There exists a constant c_ρ such that, for any $t \in [0, T]$ and (Y, Z, U), (Y', Z', U') in $\mathcal{S}_2^l \times \mathcal{H}_2^l \times \widehat{\mathcal{H}}_2^l$,

$$|\rho_t(Y, Z, U) - \rho_t(Y', Z', U')|^2 \leq$$

$$c_\rho^2 \mathbb{E}_t \Big[\sup_{t \leq s \leq T} |Y_s - Y'_s|^2 + \int_t^T \big(|Z_s - Z'_s|^2 + |U_s - U'_s|_s^2 \big) ds \Big]. \blacksquare$$

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Example 1

Conditional tail expectation ($\sim$ expected shortfall) of a random loss ℓ

$$\mathbb{ES}_t^{\alpha}[\ell] = \mathbb{E}_t[\ell | \ell > \mathbb{V}a\mathbb{R}_t^{\alpha}(\ell)] = (1-\alpha)^{-1}\mathbb{E}_t[\ell \mathbb{1}_{\{\ell > \mathbb{V}a\mathbb{R}_t^{\alpha}(\ell)\}}].$$

- Standard square integrability and Lipschitz assumptions on an (I variate) BSDE terminal condition ξ and coefficient f, except
  - f "'depends on ϱ" and is only monotone in y

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We consider the following l variate ABSDE:

$$\begin{cases} Y_T = \xi \text{ and, for } t \leq T, \\ -dY_t = f\left(t, Y_t, Z_t, U_t, \rho_t(Y, Z, U)\right) dt - Z_t dB_t - \int_E U_t(e) M(dt, de), \end{cases} \quad (1)$$

to be solved in (Y, Z, U) in $\mathcal{S}_2^l \times \mathcal{H}_2^l \times \widehat{\mathcal{H}}_2^l$.

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Theorem 1

The ABSDE (1) *has a unique solution* (Y, Z, U) *in* $\mathcal{S}_2^l \times \mathcal{H}_2^l \times \widehat{\mathcal{H}}_2^l$, *which is the limit in* $\mathcal{S}_2^l \times \mathcal{H}_2^l \times \widehat{\mathcal{H}}_2^l$, *with a geometrical convergence rate, of the Picard iteration defined by* $(Y^{(0)}, Z^{(0)}, U^{(0)}) = (0, 0, 0)$ *and, for* $n \geq 1$,

$$\begin{cases} Y_T^{(n)} = \xi \text{ and, for } t \leq T \\ -\,dY_t^{(n)} = f\left(t, Y_t^{(n)}, Z_t^{(n)}, U_t^{(n)}, \rho_t(Y^{(n-1)}, Z^{(n-1)}, U^{(n-1)})\right) dt \\ \qquad\qquad - Z_t^{(n)} dB_t - \int_E U_t^{(n)}(e) M(dt, de). \end{cases}$$

Proof. We extend the arguments in Peng and Yang (2009) to

- jumps
  - because of the counterparty defaults,
- monotone coefficient
  - as we assume interest rates only bounded from below,
- a more general anticipated dependence of the coefficient
  - on the martingale part of the value process. ∎.

A Special Case

Assume that

$$\rho(Y, Z, U) = \bar{\rho}\Big(Y, \int_0^{\cdot} Z_s dB_s + \int_0^{\cdot}\int_E U_s(e) M(ds, de)\Big)$$
$$f(t, y, z, u, \varrho) = \bar{f}(t, y, \varrho),$$

where $\bar{\rho}$ and $\bar{f}$ satisfy the obviously amended forms of our previous assumptions on ρ and f.

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Then, denoting by $m(S)$ the $(\mathbb{F}, \mathbb{P})$ canonical Doob–Meyer local martingale component of an $(\mathbb{F}, \mathbb{P})$ special semimartingale S;

1. The ABSDE (1) for (Y, Z, U) in $\mathcal{S}_2^l \times \mathcal{H}_2^l \times \widehat{\mathcal{H}}_2^l$ is equivalent, via the martingale representation property, to the following equation to be solved for a (special) semimartingale Y in $\mathcal{S}_2^l$ with $m(Y)$ in $\mathcal{S}_2^l$:

$$Y_t = \mathbb{E}_t\Big[\xi + \int_t^T \bar{f}\Big(s, Y_s, \bar{\rho}_s\big(Y, m(Y)\big)\Big) ds\Big], \quad t \leq T, \qquad (2)$$

2 which is in turn equivalent to the following system of equations for a (special) semimartingale Y in $\mathcal{S}_2^l$ and a martingale $N(= m(Y))$ in $\mathcal{S}_2^l$:

$$
\begin{aligned}
&N_0 = Y_0 \text{ and, for } t \in (0, T],\\
&dN_t = dY_t - \bar{f}\,(t, Y_t, \bar{\rho}_t(Y, N))\,dt\\
&Y_t = \mathbb{E}_t\Big[\xi + \int_t^T \bar{f}\big(s, Y_s, \bar{\rho}_s(Y, N)\big)\,ds\Big];
\end{aligned}
$$

3 The Picard iteration of Theorem 1 for (Y, Z, U) is equivalent to the following Picard iteration for $(Y, N = m(Y))$ in (3): $Y^{(0)} = N^{(0)} = 0$ and, for $n \geq 1$,

$$Y_t^{(n)} = \mathbb{E}_t\Big[\xi + \int_t^T \bar{f}\big(s, Y_s^{(n)}, \bar{\rho}_s(Y^{(n-1)}, N^{(n-1)})\big)ds\Big], \;\; 0 \leq t \leq T,$$

$N_0^{(n)} = Y_0^{(n)}$ and, for $t \in (0, T]$,

$$dN_t^{(n)} = dY_t^{(n)} - \bar{f}\big(t, Y_t^{(n)}, \bar{\rho}_t(Y^{(n-1)}, N^{(n-1)})\big)dt.$$

Outline

- XVAs: Pricing add-ons (or rebates) with respect to the counterparty-risk-free value of financial derivatives, meant to account for counterparty risk and its capital and funding implications.
- VA stands for valuation adjustment and X is a catch-all letter to be replaced by C for credit, D for debt, F for funding, M for margin, and K for capital.

The main XVA protagonists.

MtM	Mark-to-market
CVA	Credit valuation adjustment
FVA	Funding valuation adjustment
CA	Contra-assets valuation
CR	Capital at risk
KVA	Capital valuation adjustment

- Consider a bank engaged into bilateral trading with a client, with final maturity of the portfolio T.
- Let MtM (mark-to-market) denote the counterparty risk free value of the client portfolio of the bank, counted positively when positive to the bank
- Let R_c denote the recovery rate of the client in case it defaults at time τ_c.
- Let VM denote the variation margin from the client to the bank
    - collateral guarantee tracking the mark-to-market of the portfolio, assumed remunerated at the risk-free rate by the receiving party
- Let λ denote the funding spread of the bank

CVA and FVA in the base case without capital at risk

- Pricing stochastic basis $(\mathbb{F}, \mathbb{P})$ with risk-neutral discount factor β
- Pricing cash flows by risk-neutral discounted expectation yields, for $0 \leq t \leq T$:

$$\mathrm{CVA}_t = \mathbb{E}_t\Big[\mathbb{1}_{\{t<\tau_c\leq T\}}\beta_t^{-1}\beta_{\tau_c} \times (1-R_c)\big(\mathrm{MtM}_{\tau_c} - \mathrm{VM}_{\tau_c}\big)^+\Big],$$

$$\mathrm{FVA}_t = \mathbb{E}_t\int_t^{\tau_c\wedge T}\beta_t^{-1}\beta_s \times \lambda_s\Big(\mathrm{MtM}_s - \mathrm{VM}_s - \mathrm{CVA}_s - \mathrm{FVA}_s\Big)^+ ds.$$

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- These equations can be readily extended to several clients, initial margin, positive liquidation times, centrally cleared derivatives, etc.

EPE

- Assuming deterministic interest rates, the time 0 CVA can be rewritten as

$$\mathrm{CVA}_0 = (1 - R_c) \int_0^T \beta_t \mathrm{EPE}(t) \mathbb{P}(\tau_c \in dt), \tag{3}$$

for an expected positive exposure (EPE) defined as

$$\mathrm{EPE}(t) = \mathbb{E}\big((\mathrm{MtM}_s - \mathrm{VM}_s)^+ | s = \tau_c\big)_{|\tau_c = t}$$

Wrong-Way Risk

- The exposure based CVA formula (3) is popular with practitioners as it decouples the credit and the market sides of the problem.
- But it is specific to the valuation time 0 and it can hardly be exploited rigorously beyond the simplistic setup where the default of the client is independent of the corresponding counterparty exposure
  - whereas wrong-way risk, i.e. the risk of adverse dependence between the credit risk of the counterparty and the underlying market exposure, is a key CVA feature.
- Analogous stylized FVA formulas integrated with respect to the credit (CDS curve) of the bank itself, with similar limitations.

- The capital at risk (CR) of the bank is dynamically modeled as the conditional expected shortfall (economic capital $\mathbb{ES}_t$), at some quantile level a ($= 97.5\%$), of the one-year-ahead trading loss L of the bank, i.e., also accounting for discounting:

$$\mathrm{CR}_t = \mathbb{ES}_t^a\Big(\int_t^{t+1} \beta_t^{-1}\beta_s dL_s\Big).$$

Proposition 1

Assuming a constant hurdle rate h, the amount needed by the bank to remunerate its shareholders for their capital at risk in the future is

$$\mathrm{KVA}_t = h\mathbb{E}_t \int_t^T e^{-\int_t^s (r_u+h)du} \mathrm{CR}_s ds\,, \quad t \in [0, T]. \tag{4}$$

Proof.

- A constant hurdle rate h means (as the KVA itself is loss-absorbing)

$$-d\mathrm{KVA}_t + r_t\mathrm{KVA}_t dt = h(\mathrm{CR}_t - \mathrm{KVA}_t)dt - d\mathrm{Mart}_t,$$

  i.e., setting $\bar{\beta}_t = e^{-\int_0^t (r_s+h)ds}$

$$-d(\bar{\beta}_t\mathrm{KVA}_t) = h\bar{\beta}_t\mathrm{CR}_t dt - \bar{\beta}_t d\mathrm{Mart}_t$$

- Added to a terminal condition $\mathrm{KVA}_T = 0$, this is equivalent to

$$\bar{\beta}_t\mathrm{KVA}_t = \mathbb{E}_t \int_t^T \bar{\beta}_s h\mathrm{CR}_s ds,$$

  which is (4). ∎

What is L?

- Assume market risk replicated by the bank but no XVA hedge
- Setting $Q = \mathrm{MtM} - \mathrm{VM}$ and $\mathrm{CA} = \mathrm{CVA} + \mathrm{FVA}$, we obtain

$$\begin{aligned} &L_0 = z \ (\sim \text{arbitrary, set to } \mathrm{CA}_0 \text{ henceforth) and, for } t \in (0, T], \\ &dL_t = d\mathrm{CA}_t - r_t \mathrm{CA}_t dt + (1 - R_c) Q^+_{\tau_c} \boldsymbol{\delta}_{\tau_c}(dt) \\ &\qquad + \lambda_t \big(\mathbb{1}_{\{t \le \tau_c\}} Q_t - \mathrm{CA}_t\big)^+ dt. \end{aligned}$$

→ L = martingale part of $(\mathrm{CA} + (1 - R_c) Q^+ \bullet \boldsymbol{\delta}_{\tau_c})$, where $\mathrm{CA} = \mathrm{CVA} + \mathrm{FVA}$

- $L = \mu + m(\mathrm{FVA})$, for the exogenous

$$\mu = m(\mathrm{CVA}) + m\big((1 - R_c) Q^+ \bullet \boldsymbol{\delta}\big).$$

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Using capital at risk as variation margin

- Accounting for the possibility for a bank to use capital at risk as variation margin (VM), the variation margin funding needs, i.e. the drift (modulo discounting) of the FVA BSDE, are diminished from $\left(\mathbb{1}_{\{t\leq\tau_c\}}Q_t - \mathrm{CA}_t\right)^+$ to

$$\left(\mathbb{1}_{\{t\leq\tau_c\}}Q_t - \mathrm{CA}_t - \mathrm{CR}_t\right)^+,$$

where

$$\mathrm{CR}_t = \mathbb{ES}_t^a\left(\int_t^{t+1} \beta_t^{-1}\beta_s dL_s\right),$$

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$(\mathrm{FVA}, \mathrm{KVA})$ ABSDEs

→ FVA anticipated BSDE of the form (2), with

$$\bar{\rho}(Y, N) = \mathbb{ES}_t \left[\int_t^{(t+1)\wedge T} \beta_t^{-1} \beta_s \left(d\mu_s + dN_s \right) \right], \quad t \in [0, T]$$

- And even $(\mathrm{FVA}, \mathrm{KVA})$ ABSDE system of such, as the KVA is actually part of capital at risk, hence the definition of CR_t needs in fact be refined into $\mathrm{CR}_t = \max \left(\mathbb{ES}_t^a \left(\int_t^{t+1} \beta_t^{-1} \beta_s dL_s \right), \mathrm{KVA}_t \right)$.

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Theorem 2

The XVA equations are well-posed, including in the realistic case where capital is fungible with variation margin. ∎

Proof. By application of Theorem 1 and observation 1. ∎

- Moreover, the reformulation of the Picard iteration made in the observations 2 and 3 opens the door to Monte Carlo approximation of the solution.

What is λ?

- "Credit and/or liquidity"
- But liquidity spreads are typically in the order of a handful of basis points while banks funding spreads can run into the hundreds of basis points
- If "credit mainly", it seems we forgot (at least) half of the story
- Banks are themselves risky and this is precisely the reason why we saw all these regulatory changes
- What then about DVA etc..?

Adding bank default

- In the realistic case of a defaultable bank, we need to account for the discrepancy between the so called clean pricing model used by the derivative traders of the bank, who ignore the default of the bank itself, and the pricing model of the XVA traders, that have the default of the bank in mind
- We also need to stop all equations 'before the bank default' in order to be aligned with the interest of bank shareholders, who have the decision power as long as the bank is nondefault

Unilateral versus bilateral XVAs

Theorem 3

Ultimately same equations as before after reduction of all XVA equations 'stopped before the bank default' to the clean pricing model (filtration and pricing measure).

- Hence the XVA equations are well posed and amenable to Monte Carlo simulation, including in the realistic case of a defaultable bank.

- Even though our setup includes the default of the bank itself, we end up with unilateral CVA, FVA and KVA (pre-default) formulas [and DVA is irrelevant] pricing the related cash flows until the final maturity T of the portfolio
  - As opposed to $\bar{\tau} = \tau \wedge T$
  - Under a reduced filtration (and possibly changed probability measure) ignoring the default of the bank itself, but without bank default intensity discounting.

- In particular, our approach is therefore naturally consistent with the regulatory requirement that the reserve capital

$$\mathrm{CA} = \mathrm{CVA} + \mathrm{FVA}$$

of a bank should not diminish simply because of a deterioration of the bank credit spread.

Outline

XVA Metrics for Bilateral Trade Portfolios

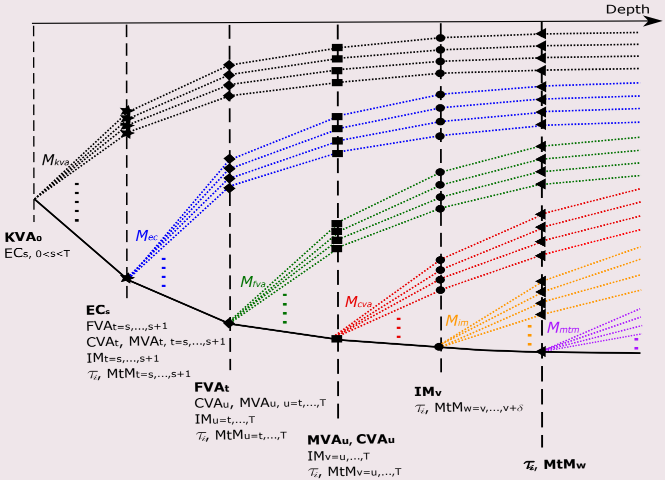
Assuming n netting sets (and one funding set):

- nonlinear CVA terminal payoffs, hence the CVA can only be computed at the level of each netting set
- semilinear FVA equation, hence, in principle, the FVA can only be computed at the level of the overall portfolio
- The KVA can only be computed at the level of the overall portfolio and relies on future conditional risk measures of the trading loss process of the bank, which itself involves future fluctuations of other XVAs, as these are part of the bank liabilities

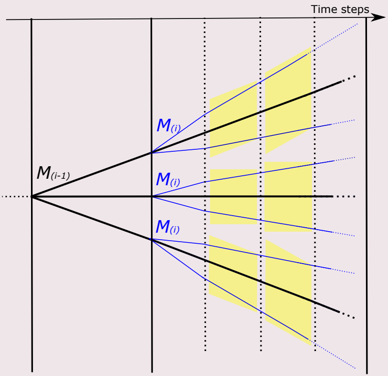
Algorithmic and computational challenges

- Heavy computations at the portfolio level
- Yet need accuracy so that (trade) incremental XVA computations are not in the numerical noise of the machinery

XVA dependence tree, from the most outer layer to the most inner one. The sub-tree rooted at the lowest node on each inner layer should be duplicated starting from each node above on the same layer.



FVA (inner) backwardations. The yellow pavings symbolize regressions. The fine blue paths denote inner resimulated paths.



The above XVA dependence tree yields the big picture in the most general situation and without any simplification. However:

- If the user is only interested in some of the XVA components, then only the sub-XVA tree corresponding to the most outer XVA of interest in the figure needs be processed computationally;
- If one or several layers can be computed by exact or approximate formulas instead of Monte Carlo simulation, then the corresponding layers drop out of the picture.

XVA Exposure Based Computational Approaches (cf. (3))

- First, compute the mark-to-market cube of the counterparty-risk free valuation of all contracts in any scenario and future time point.
- Then integrate in time the ensuing expected positive exposure (EPE) profile "against the client CDS curve" in order to obtain the CVA.
- A similar approach is applied to FVA (using the funding curve of the bank itself as exposure integration kernel) and other XVAs

- Mainstream in most banks
- However, an exposure-based approach is purely static, whereas a dynamic perspective is required for (even partial, but rigorous) XVA hedging purposes and for properly accounting for the feedback effects between different XVAs (e.g. from the CVA into the FVA).

- Moreover, an exposure-based XVA approach essentially assumes independence between the market and credit sides of the problem
- Beyond more or less elaborate patches such as the ones proposed in (Pykhtin 2012), (Hull and White 2012), (Li and Mercurio 2015), or (Iben Taarit 2017), it is hard to extend rigorously to wrong-way risk
    - Risk of adverse dependence between the credit risk of the counterparty (or bank itself) and the underlying market exposure

- Last but not least, an exposure-based XVA approach comes with little error control, at least whenever the exposure is computed by global regression
- This is due to the unconditional approximations involved in such global regressions.

Nested Monte Carlo

- An alternative is a nested Monte Carlo XVA computational approach, optimally implemented on GPUs.
- In an NMC perspective (see (Gordy and Juneja 2010) for a seminal reference), higher layers are launched first and trigger nested simulations on-the-fly whenever required in order to compute an item from a lower layer

XVA NMC Design Parameterization

- Assuming the same variance created through the different layers of the tree, the mean square error (MSE) of an $M_{(0)} \otimes M_{(1)} \otimes \ldots \otimes M_{(i)} = M_{(0)} \otimes M_{(0)} \otimes \ldots \otimes M_{(0)}$ NMC is the same as the one of an $M_{(0)} \otimes \sqrt{M_{(0)}} \otimes \ldots \otimes \sqrt{M_{(0)}}$ NMC
  - $O(M_{(0)}^{-\frac{1}{2}})$
  - Proof based on a uniform control of the moments of the error and regularity assumptions that are needed to justify the application of Taylor formula
  - cf. Gordy and Juneja (2010, Assumption 1), Abbas-Turki and Mikou (2015, Assumption 3.1), and Rainforth, Cornish, Yang, Warrington, and Wood (2017).

Semi-Nested MC Approach for the XVA ABSDEs

- Picard iteration (3)
- Nested Monte Carlo used for estimating the XVA metrics at outer simulation nodes
- Regression of the conditional expected shortfall risk measures

Stress Test

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Real portfolio study (Albanese, Caenazzo, and Crépey (2017))

- Representative banking portfolio with about 2,000 counterparties, 100,000 fixed income trades including swaps, swaptions, FX options, inflation swaps and CDS trades.
- $\text{VM} = 0$.

Representative banking portfolio XVA values.

XVA	$Value
CVA_0	242 M
$FVA_0^{(0)}$	126 M
FVA_0	62 M
KVA_0	275 M
FTDCVA	194 M
FTDDVA	166 M

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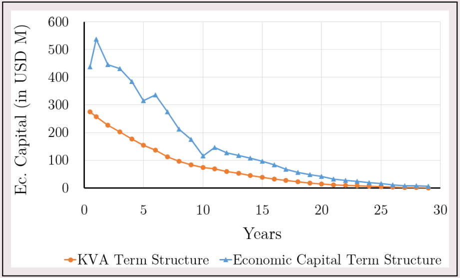
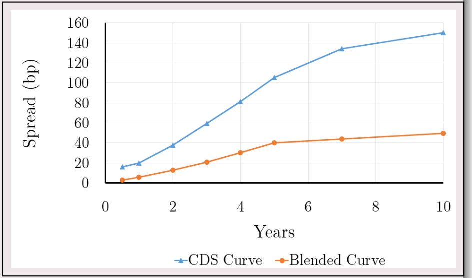
Left: Term structure of economic capital compared with the term structure of KVA.
Right:FVA blended funding curve computed from the ground up based on capital projections.





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Thanks for your attention!

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