Electricity Demand Response and Responsiveness Incentives

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Agenda

Demand Response (DR)

- Contract between a consumer and a producer (or a retailer)
- The consumer is paid to reduce consumption a certain number of days choosen by the producer.
- The number of days of price events is determined at inception.
- The days are choosen dynamically (price event day) and the customer is informed the day before.

Forms of DR

- Domestic: lower price on non–event price day (10 c/kWh vs normal tariff of 15 c/kWh) higher price during price event days (67 c/kWh). Around 30 price–event days per year.
- Rebate: consumer's receives money for the consumption they saved compared to a baseline. Used in industry. Potential baseline manipulation.

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Figure: Camden Yards baseball park

Enerwise was fined a $780.000 penalty by the Federal Energy Regulation Commission 143 FERC 61218 as of June 7th, 2013 for manipulation of a demand response program.

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DR contract fails to enhance responsiveness, i.e. achieving a regular consumption reduction.

Responsiveness & moral hazard

- Moral hazard problem in Principal–Agent contractual relation: the form of the contract modifies the behaviour of the consumer.
- Insurer (Principal) / insured (Agent) ; land owner (Principal) / farmer (Agent)
- In domestic DR, the consumer is the Agent. The consumer reaps the benefit of present decrease in price and re–evaluate the constraints of consumption reduction during price events.

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Question

What is the optimal contract?

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- We show that the optimal contract has a rebate form... which is baseline–proofness.
- Using LCL data, we illustrate the potential benefits from responsiveness incentives.

Non-exhaustive literature

- Laffont and Martimort, *The Theory of Incentives*, Princeton, 2002.
- Holmström and Milgrom, *Econometrica*, 1987.
- Sannikov *Rev. Econ. Stud.*, 2008.
- Cvitanič & Zang, Contrat Theory in Continuous-time models, Springer, 2013.
- Cvitanič, Possamaï & Touzi, Dynamic programming approach to Principal-Agent problems, *Finance & Stochastics*, 2018.

Model

The consumer (The Agent)

Dynamics of the consumption on a price event of duration T

$$X_T^{a,b} = X_0 + \int_0^T (-\sum_{i=1}^N a_i(s))ds + \int_0^T \sum_{i=1}^N \sigma_i \sqrt{b_i(s)} dW_s^i$$

a_i and b_i efforts to reduce average consumption and volatility of usage i
Consumer's criterion:

$$V^A(\xi) := \sup_{\nu:=(a,b)} J_{\mathrm{A}}(\xi,\nu) := \mathbb{E}^\nu \left[U_A\Big(\xi + \int_0^T (f(X_s^\nu) - c(\nu_s))\, ds\Big)\right]$$

with $U_A(x) = -e^{-r\,x}$, $f(x) = \kappa x$, constant marginal value κ

$$c(a,b) := \underbrace{\frac{1}{2}\sum_{i=1}^N \frac{{a_i}^2}{\mu_i}}_{c_1(a)} + \underbrace{\frac{1}{2}\sum_{i=1}^N \frac{\sigma_i({b_i}^{-1}-1)}{\lambda_i}}_{c_2(b)},\ 0 \le a_i,\ \ 0 < b_i \le 1.$$

The producer (The Principal)

$$J_{\mathrm{P}}(\xi,\nu) := \mathbb{E}^{\nu}\Big[U\Big(-\xi - \int_0^T g(X_s)ds - \frac{h}{2}\langle X\rangle_T\Big)\Big] \text{with } U(x) = -e^{-p\,x}$$

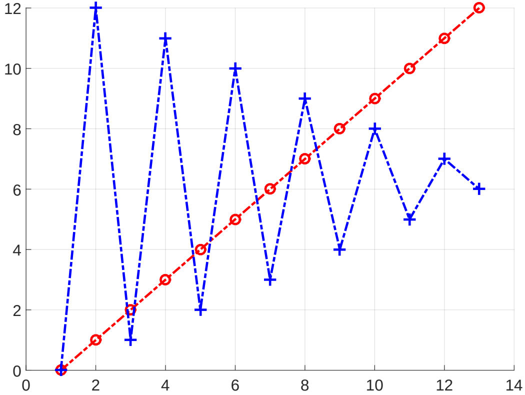
- $g(x) = \theta x$ generation cost function with constant marginal cost θ
- h direct cost of volatility

Producer's objective:

$$V^{\mathrm{sb}} := \sup_{\xi} J_{\mathrm{P}}(\xi,\nu^{\star}(\xi)).$$

together with the participation constraint of the consumer

$$V_A(\xi) \geq R_0 =: -e^{-rL_0}.$$



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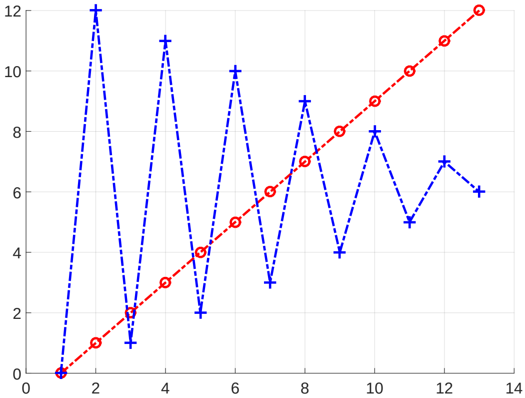
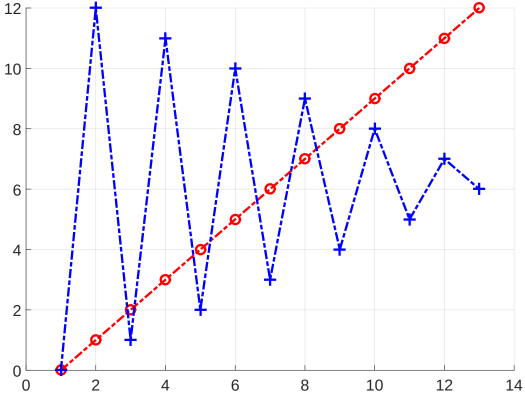
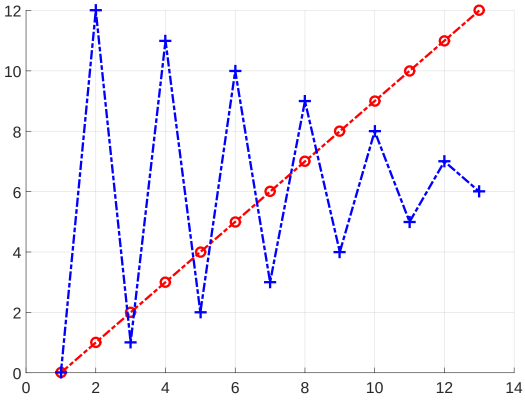


Total consumption X = Total consumption X



Total consumption X = Total consumption X

$\langle X \rangle = 1^2 + 1^2 + ... + 1^2 = 12$



Total consumption X = Total consumption X

$\langle X\rangle = 1^2 + 1^2 + ... + 1^2 = 12$ $\quad$ $\langle X\rangle = 12^2 + 11^2 + 10^1.. + 1^2 = 650$

Remarks

- Timing: first, the producer proposes a paying rule, knowing L_0; then, the consumer accepts or reject the contract: if he accepts, the price event happens later; the producer measures $X_\cdot$ and pays or charges the consumer.
- The producer does not observe the efforts a and b on usages. She only observes the consumption X.
- The problem is non–Markovian. The contract is written on the observation of the whole path of the consumption on $[0, T]$.
- This problem is designated as the second–best.

First–best

$$V^{\mathrm{FB}} := \sup_{\xi,\nu} \left\{ J_{\mathrm{P}}(\xi,\nu) \quad : J_{\mathrm{A}}(\xi,\nu) \geq R_0 \right\}$$

Consumer's Hamiltonian

$$H(z,\gamma) := H_{\mathrm{m}}(z) + H_{\mathrm{v}}(\gamma),\ \ z,\gamma \in \mathbb{R},$$

where

$$H_{\mathrm{m}}(z) := -\inf_{a \geq 0} \left\{ a \cdot \mathbf{1} z + c_1(a) \right\}, \quad H_{\mathrm{v}}(\gamma) := -\frac{1}{2} \inf_{b \in (0,1]} \left\{ c_2(b) - \gamma |\sigma(b)|^2 \right\},$$

which admits the minimizer

$$\widehat{a}_j(z) := \mu_j z^-, \quad \widehat{b}_j(\gamma) := 1 \wedge \left(\lambda_j \gamma^-\right)^{-\frac{1}{2}}.$$

Optimal contract [Cvitanic, Possamaï & Touzi (2018)]

- The optimal contract is of the form:

$$Y^{Y_0,Z,\Gamma} := Y_0 + \int_0^t Z_s dX_s + \frac{1}{2}\int_0^t (\Gamma_s + rZ_s{}^2) d\langle X\rangle_s - \int_0^t \big(H(Z_s,\Gamma_s) + f(X_s)\big) ds,$$

  where Z_s and Γ_s are payment rates for efforts on the average consumption and on volatility, and $H(Z_s,\Gamma_s) + f(X_s)$ is the natural benefit the consumer gets when receiving incentives Z_s, Γ_s.
- Whatever the processes Z and Γ, one has $V^A(Y^{Y_0,Z,\Gamma}) = U_A(Y_0)$.

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- Whatever the processes Z and Γ, one has $V^A(Y^{Y_0,Z,\Gamma}) = U_A(Y_0)$.

- Whatever the payment rates Z and Γ, the Agent will receive his required reservation utility.
- Thus, the Principal can use the payment rates to solve his own optimisation problem, using standard stochastic control methods.

Optimal contract

First-best

The optimal first-best contract is given by:

$$\xi_{\mathrm{fb}} = L_0 - \kappa X_0 T + \int_0^T c(\nu_t)dt + \int_0^T \pi^{\mathrm{e}}_{\mathrm{fb}}(X_0 - X_t)\,dt - \frac{1}{2}\int_0^T \pi^{\mathrm{v}}_{\mathrm{fb}} d\langle X\rangle_t,$$

where

$$\pi^{\mathrm{e}}_{\mathrm{fb}} := \frac{r}{r+p}\kappa + \frac{p}{r+p}\theta, \qquad \pi^{\mathrm{v}}_{\mathrm{fb}} := \frac{p}{r+p}h,$$

and the optimal efforts are:

$$a_{\mathrm{fb}}(t) := \mu\delta^{-}(T-t), \quad b_{\mathrm{fb}}(t) := 1 \wedge \Big(\lambda(h + \rho\,\delta^2(T-t)^2)\Big)^{-\frac{1}{2}},$$

with $\delta := \kappa - \theta$.

- $\delta > 0 \Rightarrow$ off–peak hours; $\delta < 0 \Rightarrow$ peak hours.
- Price of energy: constant convex combination of marginal cost and value
- Price of volatility: constant risk–sharing of the direct cost of volatility
- The contract has a rebate form.

Second–best

The second–best optimal contract is given by $\xi_{\text{sb}} = \xi^{\text{f}}_{\text{sb}} + \xi^{\text{v}}_{\text{sb}}$ where

$$\xi^{\text{f}}_{\text{sb}} := L_0 - \kappa T X_0 - \int_0^T H(z_{\text{sb}}, \gamma_{\text{sb}})(t)dt$$

$$\xi^{\text{v}}_{\text{sb}} := \int_0^T \pi^{\text{e}}_{\text{sb}}(t)\big(X_0 - X_t\big)dt - \frac{1}{2}\int_0^T \pi^{\text{v}}_{\text{sb}}(t)d\langle X\rangle_t,$$

and

$$\pi^{\text{e}}_{\text{sb}}(t) := \kappa + z'_{\text{sb}}(t), \qquad \pi^{\text{v}}_{\text{sb}}(t) := h + p\big(z_{\text{sb}}(t) - \delta(T-t)\big)^2,$$

where z_{sb} is a deterministic function of time, solution of a scalar optimisation problem for each time. The optimal efforts are

$$a_{\text{sb}}(t) := \mu z_{\text{sb}}(t)^-, \quad b_{\text{sb}}(t) := 1 \wedge \Big(\lambda\gamma_{\text{sb}}(t)^-\Big)^{-\frac{1}{2}},$$

$$\gamma_{\text{sb}}(t) := -h - r z_{\text{sb}}(t)^2 - p(z_{\text{sb}}(t) - \delta(T-t))^2.$$

- Prices of energy and volatilty are now non–constant deterministic function of time.
- Price of volatility is always greater than the first–best price.
- In peak–hours, the price of energy is also greater than the first–best.
- The contract has a rebate form where the initial consumption level is the baseline.
- Baseline–proofness: Whatever the initial condition (baseline), the consumer gets no more than L_0.

Second–best without responsiveness incentives

The second–best optimal contract without responsiveness incentives is given by $\xi_{\mathsf{sb}_m} = \xi^{\mathsf{f}}_{\mathsf{sb}_m} + \xi^{\mathsf{v}}_{\mathsf{sb}_m}$ where

$$\xi^{\mathsf{f}}_{\mathsf{sb}_m} = L_0 - \kappa T X_0 + \frac{1}{2}\int_0^T r z^2_{\mathsf{sb}_m}(t)|\sigma|^2 dt - \int_0^T H_{\mathrm{m}}(z_{\mathsf{sb}_m}(t))dt,$$

$$\xi^{\mathsf{v}}_{\mathsf{sb}_m} = \int_0^T \pi^{\mathsf{e}}_{\mathsf{sb}_m}(X_0 - X_t)\,dt,$$

where

$$\pi^{\mathsf{e}}_{\mathsf{sb}_m} := (1-\Lambda)\kappa + \Lambda\theta,$$

and the optimal payment rate is $z_{\mathsf{sb}_m}(t) = \Lambda\delta(T-t)$ with

$$\Lambda := \frac{p|\sigma|^2 + \bar{\mu}\mathbf{1}_{\{\delta<0\}}}{(p+r)|\sigma|^2 + \bar{\mu}\mathbf{1}_{\{\delta<0\}}}.$$

- No price of volatility.
- Prices of energy is a constant convex combination of marginal cost and value of energy.
- In peak–period, the price depend on the volatilities and the costs of efforts.
- The contract has a rebate form where the initial consumption level is the baseline.
- Baseline–proofness: Whatever the initial condition (baseline), the consumer gets no more than L_0.

Numerical illustration

Calibration

- Make extensive use of the Low Carbon London 2013 pricing trial.
- We interpret the LCL pricing trial as the implementation of the optimal contract with uncontrolled responsiveness and linear energy value.

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Parameters shopping list

$$T \quad h \quad \kappa \quad \theta \quad p \quad r \quad \sigma \quad \mu_i \quad \lambda_i$$

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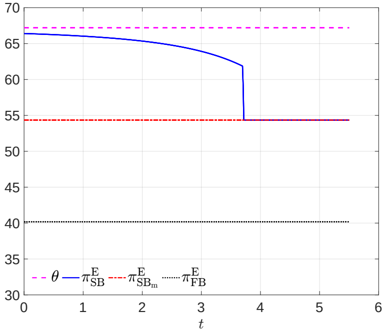
Parameters shopping list

$$T \quad h \quad \kappa \quad \theta \quad p \quad r \quad \sigma \quad \mu_i \quad \lambda_i$$

T	h	κ	θ	p	r	σ	μ	λ
5.5	$4.0\ 10^{-4}$	11.76	67.2	$0.6\ 10^{-2}$	$0.57\ 10^{-2}$	85	$9.3\ 10^{-5}$	$2.8\ 10^{-2}$

Table: Nominal values of the parameters. T in hours, κ and θ in p/kWh, σ in Watt.



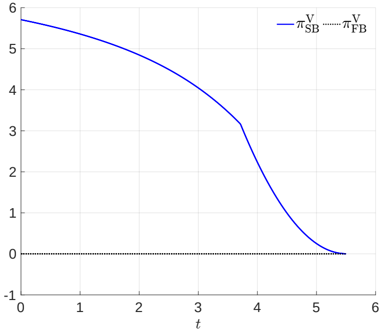


Figure: Prices for energy (p/kWh) and volatilty (p/kW^2).

Conservative estimate of the benefit from responsiveness incentives

	First–best	Second–best with responsiveness	Second–best without responsiveness
Cost of effort c_1	5.97	5.97	4.68
Cost of effort c_2	0.40	0.59	0
Total cost of effort	6.37	6.56	4.68
Producer's benefit	6.76	6.21	5.40
Average consumption reduction	52.15	45.17	40.00
Standard deviation consumption	46.49	39.61	85.06

Table: Costs in pence, consumption and standard deviation in Watt.

Perspectives

- Limited liability (no negative payments).
- Group of consumers with different energy valuation (adverse selection)
- Making a pricing trial with responsiveness incentives.

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