

NWO/STAR/PWN

**20th Winter School on
Mathematical Finance**

Special topics:

Functional convex ordering
of stochastic processes

Optimal transport in finance

January 23–25, 2023

**Conference Hotel Kontakt der Kontinenten,
Soesterberg**

Sponsored by NWO, STAR, PWN, and FWO

Winter School on Mathematical Finance

In recent years, the mathematical theory associated with financial risk management and the pricing of contingent claims has been a highly active field of research. The area has established itself as one of the most vigorously growing branches of applied mathematics. Model-based analysis of contracts and portfolios has become a standard in the finance industry, and the number of academic institutions offering curricula in financial mathematics has increased rapidly. In this context, the winter school on Mathematical Finance that will take place on January 23–25, 2023 in Soesterberg aims at providing a meeting place for participants both from industry and from academia and will be organized in *a safe, Coronaproof environment*. The program provides ample opportunity for discussion.

The special topics of the 20th winter school are *Functional convex ordering of stochastic processes*, and *Optimal transport in finance*. These are the subjects of minicourses that will be taught by two distinguished speakers: Professors Jan Obłój (University of Oxford) and Gilles Pagès (Sorbonne Université). Additionally there will be three one-hour special invited lectures by Professors José Manuel Corcuera (Universitat de Barcelona), Christoph Reisinger (University of Oxford), and Luitgard Veraart (London School of Economics and Political Science). Thirty-minute lectures on recent research work in the Netherlands will be presented by Mike Derksen (Deep Blue Capital), Jian He (ING, University of Amsterdam), Matteo Michiellon (ABN AMRO, University of Amsterdam), and Stan Olijslagers (CPB, Netherlands Bureau for Economic Policy Analysis).

Auspices, sponsoring and grants

The Winter School takes place under the auspices of the mathematics cluster STAR and of PWN. The stochastics groups of the mathematics departments of the universities in the Netherlands cooperate in STAR. PWN (Platform Wiskunde Nederland) is a national organization that aims to strengthen the position of mathematics in the Netherlands in all its aspects. The winter school is supported financially by STAR, PWN, by the Netherlands Organization for Scientific Research (NWO) and by the Research Foundation - Flanders (FWO). Administrative assistance is provided by the Korteweg–De Vries Institute for Mathematics of the University of Amsterdam.

The FWO research network *Modelling and Simulation with applications in Finance, Insurance and Economics* has made available a limited number of grants for young researchers (PhD students and postdocs) associated to the network to be used as a waiver of the registration fee for the winter school. For those researchers the grants completely cover the registration fee. For other young researchers a limited number of grants of € 300 each is available as a reduction on the registration fee for the winter school. Priority will be given to grant applications from PhD students whose supervisor is a member of the network, but others are invited to apply as well. Applications for the grant can be sent by email to both Michel Vellekoop and Peter Spreij (make sure both are addressed, spreij@uva.nl and m.h.vellekoop@uva.nl). Applications are required to contain a brief motivation why the grant should be beneficial for the research of the applicant, a brief motivation why the applicant has a specific need for the grant, a (link to) a CV of the applicant and the name of her/his principal supervisor. Applications should be submitted before the deadline, November 20, 2022.

Organizers

The winter school is organized by:

Michel Vellekoop (Faculty Economics and Business, University of Amsterdam; e-mail m.h.vellekoop@uva.nl)

Peter Spreij (Korteweg–De Vries Institute for Mathematics, University of Amsterdam and IMAPP, Radboud University; e-mail spreij@uva.nl).

Program outline

The program starts with registration and coffee on Monday, January 23, from 10:30 to 11:30, and ends on Wednesday, January 25, at 16:00. The following events are planned:

Minicourses

Jan Obłój

Optimal transport techniques in finance

Gilles Pagès

Functional convex ordering of stochastic processes : a constructive approach with applications to finance

Special invited lectures

José Manuel Corcuera

Path-dependent Kyle equilibrium model

Christoph Reisinger

title

Luitgard Veraart

Systemic Risk in Markets with Multiple Central Counterparties

Short contributions

Mike Derksen

Stochastic price formation in call auctions

Jian He

A Bayesian filter based dimension reduction approach for the pricing grid

Matteo Michiellon

Implied risk-neutral default probabilities via conic finance

Stan Olijslagers

Discounting the Future: on Climate Change, Ambiguity Aversion and Epstein-Zin preferences

Schedule of lectures

	Monday January 23	Tuesday January 25	Wednesday January 26
Pages 09:00 - 10:00		Pagès	Pagès
10:30 - 11:30		Pagès	Pagès
11:30 - 12:30	Pagès	Oblój	Oblój
14:00 - 15:00			Oblój
15:00 - 16:00	Oblój	Oblój	Reisinger
16:00 - 17:00	Veraart	Corcuera	
17:30 - 18:00	Derksen	He	
18:00 - 18:30	Olijslagers	Michielon	

Venue

The winter school will take place in Conference Hotel Kontakt der Kontinenten, Amersfoortsestraat 20, 3769 AS Soesterberg, phone: +31 (0) 346 35 17 55. Located in the heart of the country, Kontakt der Kontinenten is one of the top accommodations in the Netherlands in terms of attractiveness of surroundings. Access by car or by public transportation is easy. By train, one should reach the city of Amersfoort, and then proceed from the station by bus (about 15 minutes, Bus 34 or Bus 56). See <https://9292.nl/en> for planning public transportation, use **Kontakt der Kontinenten** as the destination. Typically, from Schiphol (Amsterdam Airport) it takes 70–80 minutes. The bus stop is in front of the hotel (see directions on the webpage below). If you come by car, it is best to use a modern navigator. It is also possible to take a taxi from the taxi stand at railway station Amersfoort. For further details, please see <https://www.kontaktderkontinenten.nl/en/english/> or a Google map (search for **Kontakt der Kontinenten**).

Website

<https://staff.fnwi.uva.nl/p.j.c.spreij/winterschool/winterschool.html>

Abstracts

MINICOURSE I

Jan Obłój (University of Oxford)

Optimal transport techniques in finance TO BE CHANGED???

The recent decade has seen an explosion of interest in optimal transport (OT) techniques in the context of statistics, stochastic processes, optimization and beyond. These lecture series explore various case studies from these novel crossroads with the unifying theme of applications in quantitative finance. We will start with an introduction to the classical optimal transport with an emphasis on probabilistic methods. We will then consider the martingale version of the problem, explore its structure, and show how it translates into the robust pricing-hedging problem in finance. In this problem, one is given market prices of vanilla European call/put options and asks what are the no-arbitrage bounds on prices of other options. We will discuss many variants - motivated by the type of market information that is available in practice - and also touch on numerical methods. We will restrict ourselves to a discrete time setting but I will link these problems with the so-called Skorokhod embeddings in continuous time. Subsequently, we will turn to small model uncertainty and use OT methods to build small perturbations to a given model. I will show how this is analogous to OT applications in image classification. Fundamentally, we will see that given n data points, instead of perturbing these as points in a finite-dimensional space, it is more elegant to think of them via their empirical measure and do the perturbations in the infinite-dimensional space of measures. This will lead us to distributionally robust optimization and its sensitivity analysis. In particular, we will see how OT yields a non-parametric version of the classical Black-Scholes Vega.

MINICOURSE II

Gilles Pagès (Sorbonne Université)

Functional convex ordering of stochastic processes : a constructive approach with applications to finance

We will start by some background on convex (resp. monotone convex) ordering of \mathbb{R}^d -valued random vectors, namely random vectors (or probability distributions) satisfying

$$X \leq_{cv} Y \text{ if } \mathbb{E} f(X) \leq \mathbb{E} f(Y) \\ \text{for every convex (resp. non-decreasing convex) function } f : \mathbb{R} \rightarrow \mathbb{R}$$

provided both expectations have a sense. In particular we will recall classical results like Strassen and Kellerer's theorems ([Str65] and [Kel72]) which make the connection between convex ordering, martingales and peacocks (for p.c.o.c. itself acronym for the French "processus croissant pour l'ordre convexe") extensively investigated by Yor and co-authors.

We will first investigate both convex orderings of the marginal for Brownian diffusions with respect to their diffusion coefficients. We will show that these results also hold in the functional sense i.e. for functionals of their whole path. Then we will extend this

result to jump stochastic differential equations driven by Lévy processes but also Brownian stochastic integrals, etc., following [Pag16]. Some of these results are classical (see [Haj85] for monotone convex ordering of diffusions) or more recently known (see Rüschendorf and co-authors [BR06, BR07, BR08], Hobson [Hob98, Hob10] among others).

We apply these results to establish in local volatility models sensitivity results of path-dependent options with respect to their volatility. Doing so, we extend a result by [EKJPS98] and [BGW96] which produce upper- and lower-bounds based on a Black-Scholes formula for a vanilla option with convex payoff when the volatility function is itself bounded and bounded away from 0.

As a second step we investigate optimal stopping theory, replacing the path-dependent functional by the Snell envelope of a “vanilla” reward process written on a martingale diffusion, with an obvious connection with American options and again an application to their sensitivity to the volatility process (see [Pag16]).

The specificity of our approach is to be constructive in the sense that we first establish our results in discrete time (which has its own interests), typically for a discretization scheme of the underlying process and then rely on functional limit theorems “à la Jacod-Shiryaev” [JS03] to transfer the property to the continuous time model. When dealing with numerics in Finance, it usually produces *arbitrage free* approximating numerical methods (as far as volatility modeling is concerned). This can be seen as a paradigm.

In view of the importance taken by McKean-Vlasov equations (for mean-field games but also for Langevin algorithm) we will apply the above paradigm to McKean-Vlasov equations (see [LP20, LP22]) for both regular and monotone convex ordering.

A natural question on our way is to wonder whether it is possible to extend such an approach to non-Markovian dynamics. The answer is positive since our approach successfully applies to Volterra equations (see [JP22a], [JP22b]) with applications to rough stochastic volatility models.

References

- [BGW96] Yaacov Z. Bergman, Bruce D. Grundy, and Zvi Wiener. General properties of option prices. *The Journal of Finance*, 51(5):1573–1610, 1996.
- [BR06] Jan Bergenthum and Ludger Rüschendorf. Comparison of option prices in semimartingale models. *Finance Stoch.*, 10(2):222–249, 2006.
- [BR07] Jan Bergenthum and Ludger Rüschendorf. Comparison of semimartingales and Lévy processes. *Ann. Probab.*, 35(1):228–254, 2007.
- [BR08] Jan Bergenthum and Ludger Rüschendorf. Comparison results for path-dependent options. *Statist. Decisions*, 26(1):53–72, 2008.
- [EKJPS98] Nicole El Karoui, Monique Jeanblanc-Picqué, and Steven E. Shreve. Robustness of the Black and Scholes formula. *Math. Finance*, 8(2):93–126, 1998.
- [Haj85] Bruce Hajek. Mean stochastic comparison of diffusions. *Z. Wahrsch. Verw. Gebiete*, 68(3):315–329, 1985.
- [Hob98] David Hobson. Robust hedging of the lookback option. *Finance Stoch.*, 2(4):329–347, 1998.
- [Hob10] David Hobson. Comparison results for stochastic volatility models via coupling. *Finance Stoch.*, 14(1):129–152, 2010.
- [JP22a] Benjamin Jourdain and Gilles Pagès. Convex ordering for the stochastic volterra equation and its Euler schemes, 2022.

- [JP22b] Benjamin Jourdain and Gilles Pagès. Convex ordering of solutions to one-dimensional sdes, 2022.
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- [Kel72] Hans G. Kellerer. Markov-Komposition und eine Anwendung auf Martingale. *Math. Ann.*, 198:99–122, 1972.
- [LP20] Yating Liu and Gilles Pagès. Functional convex order for the scaled McKean-Vlasov processes. *arXiv e-prints (to appear in Annals of Applied Probability)*, page arXiv:2005.03154, May 2020.
- [LP22] Yating Liu and Gilles Pagès. Monotone convex order for the McKean-Vlasov processes. *Stochastic Process. Appl.*, 152:312–338, 2022.
- [Pag16] Gilles Pagès. Convex order for path-dependent derivatives: a dynamic programming approach. In *Séminaire de Probabilités XLVIII*, volume 2168 of *Lecture Notes in Math.*, pages 33–96. Springer, Cham, 2016.
- [Str65] Volker Strassen. The existence of probability measures with given marginals. *Ann. Math. Statist.*, 36:423–439, 1965.

SPECIAL INVITED LECTURES

José Manuel Corcuera (Universitat de Barcelona)

Path-dependent Kyle equilibrium model

We consider an auction type equilibrium model with an insider in line with the one originally introduced by Kyle in 1985 and then extended to the continuous time setting by Back in 1992. The novelty introduced in this talk is that we deal with a general price functional depending on the whole past of the aggregate demand, i.e. we work with path-dependency. By using the functional Itô calculus, we provide necessary and sufficient conditions for the existence of an equilibrium. Furthermore, we consider both the cases of a risk-neutral and a risk-averse insider.

Christoph Reisinger (University of Oxford)

title

abstract

Luitgard Veraart (London School of Economics and Political Science)

Systemic Risk in Markets with Multiple Central Counterparties

We provide a framework for modelling risk and quantifying payment shortfalls in cleared markets with multiple central counterparties (CCPs). Building on the stylised fact that clearing membership is shared among CCPs, we show that stress in this shared membership can transmit across markets through multiple CCPs. We provide stylised examples to lay out how such stress transmission can take place, as well as empirical evidence to illustrate that the mechanisms we study could be relevant in practice. Furthermore, we show how stress mitigation mechanisms such as variation margin gains haircutting by one CCP can have spillover effects on other CCPs. The framework can be used to enhance CCP stress-testing, which currently relies on the “Cover 2” standard requiring CCPs to be able to withstand the default of their two largest clearing members. We show that

who these two clearing members are can be significantly affected by higher-order effects arising from interconnectedness through shared clearing membership. This is joint work with Iñaki Aldasoro (BIS).

SHORT CONTRIBUTIONS

Mike Derksen (Deep Blue Capital)

Stochastic price formation in call auctions

In modern financial markets, most stock exchanges facilitate intraday continuous trading, where buy and sell orders are immediately matched if possible. However, to start and stop trading and determine opening and closing prices, a call auction is usually conducted. In a call auction, orders are aggregated for a while without immediately giving rise to transactions, after which all possible transactions are executed against a single clearing price. While modelling continuous trading is extensively studied in the (quantitative) finance literature, the call auction has received very little attention, despite the fact that the portion of the daily volume that is transacted in the closing auction increased strongly in recent years. In this talk I will discuss a stochastic model of the call auction. The model considers random buy and sell orders, placed following order placement distributions, leading to analytical expressions for the distribution of the clearing price. Order placement distributions and distributions of bid and ask volumes are left as free parameters, permitting possibly heavy-tailed or very skewed order flow conditions. Results and open questions on the relationship between order flow and clearing price will be discussed. This talk is based on joint work with Bas Kleijn and Robin de Vilder [1, 2].

References

- [1] Derksen, M., Kleijn, B. and De Vilder, R. (2020), Clearing price distributions in call auctions, *Quantitative Finance* 20(9): 1475-1493.
- [2] Derksen, M., Kleijn, B. and De Vilder, R. (2022), Heavy tailed distributions in closing auctions, *Physica A: Statistical Mechanics and its Applications* 593: 126959.

Jian He (ING, University of Amsterdam)

A Bayesian filter based dimension reduction approach for the pricing grid

Pricing grids are widely used among financial institutions to re-evaluate portfolios when assessing the potential risks, for instance credit risk or market risks. When applying the pricing grid for the revaluation, the biggest challenge is the “curse of dimensionality”. Therefore, a dimension reduction approach is usually required to project the higher dimensional risk factors to the lower dimensional factors. In this talk, we propose a Bayesian filter based dimension reduction approach and we will specially focus on the application in the credit risk calculations.

Matteo Michielon (ABN AMRO, University of Amsterdam)

Implied risk-neutral default probabilities via conic finance

Despite financial modelling often takes place under risk-neutral settings where trading activities are assumed to obey the law of one price, in practice quoted market prices are direction-dependent. Nonetheless, many valuation models require input parameters, implied from observed market quotes, that are consistent with the risk-neutral paradigm.

In this presentation, a methodology allowing to extract risk-neutral quantities directly from bid and ask quotes, without relying on mid-quote approximations, is presented. The approach outlined relies on some monotonicity- and liquidity-related assumptions and is based on the conic finance framework, which enables to calculate bid and ask prices of financial securities by employing Choquet expectations with respect to distorted versions of the relevant pricing measures as valuation functionals. In particular, as far as the credit default swap market is concerned, it will be shown how to compute risk-neutral default probabilities from quoted bid and ask premia under well-known dynamics, and at the same time how to calculate the implied liquidity level of the market. This talk is based on [1].

References

[1] Michielon, M., Khedher, A. and Spreij, P. (2021). From bid-ask credit default swap quotes to risk-neutral default probabilities using distorted expectations. *International Journal of Theoretical and Applied Finance* 24(3).

Stan Olijslagers (CPB, Netherlands Bureau for Economic Policy Analysis)

Discounting the Future: on Climate Change, Ambiguity Aversion and Epstein-Zin preferences

We show that deviations from standard expected time separable utility have a major impact on estimates of the willingness to pay to avoid future climate change risk. We propose a relatively standard integrated climate/economy model but add stochastic climate disasters. The model yields closed form solutions up to solving an integral, and therefore does not suffer from the curse of dimensionality of most numerical climate/economy models. We analyze the impact of substitution preferences, risk aversion (known probabilities), and specifically ambiguity aversion (unknown probabilities) on the social cost of carbon (SCC). Introducing ambiguity aversion leads to two offsetting effects on the social cost of carbon: a positive direct effect and a negative effect through discounting. Our numerical results show that for reasonable calibrations, the direct effect dominates and that ambiguity aversion gives substantially higher estimates of the SCC.

Registration procedure

To register for the winter school, please use the electronic registration form that is available at the web page of the winter school, <https://staff.fnwi.uva.nl/p.j.c.spreij/winterschool/winterschool.html>.

The registration fee includes accommodation (single room) for the nights of January 23–24 and 24–25, all meals starting with lunch on Monday up to and including lunch on Wednesday, and tea and coffee during breaks. Payment can be made by transfer to IBAN account number: NL27 INGB 0007388994 of Winter School Amsterdam, Secretariaat Korteweg–De Vries Instituut, Amsterdam and (for international money transfers) BIC: INGBNL2A.

The fee schedule is as follows:

	early registration (before December 1)	late registration (after December 1)
industry professional	€1295	€1450
full-time academic	€445	€495

Inquiries concerning fees for partial attendance may be directed to Aaaa Bbbb at the address given below. Registration will be valid after full payment has been received. Refunds can be given only for cancellations received before January 1, 2023. However, **should the situation of the spreading of the Corona virus deteriorate in such a way that governmental rules force the organization to cancel the winter school, all fully paid registrations will be completely restituted.**

Please note that PhD students and postdocs which receive an FWO grant should also register at the website, to ensure accommodation.

Accommodation at the venue is limited. Therefore, reservations will be treated on a first-come-first-served basis with priority for full arrangements. Participants who cannot be lodged at the venue will be accommodated in a hotel nearby. Transportation from the hotel to the venue and vice versa will be taken care of by the organization.

Further information

For further information regarding the scientific program, please contact one of the members of the organizing committee. For information concerning registration please contact:

ms. Aaaa Bbbb
Korteweg–De Vries Institute for Mathematics
University of Amsterdam
e-mail: secr-kdv-science@uva.nl
tel.: +31-(0)20-5255217