Model-free and data-driven methods in mathematical finance

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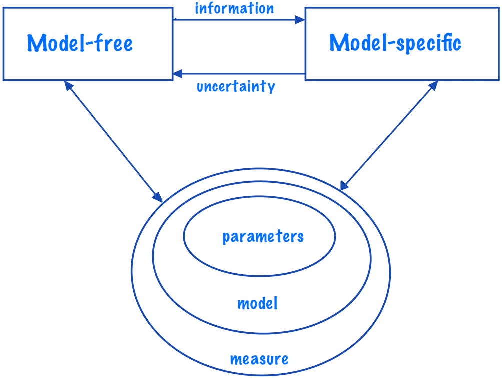
‘New’ paradigm:

You are not given the model and your task is to say something about option prices, value-at-risk, . . . $\rightsquigarrow$ compute bounds

A paradigm shift in mathematical finance, II



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Coin tossing / Dice rolling

We are rolling two dices D_1, D_2 and are interested in the distribution of the sum.

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  - $D_1, D_2 = D_1$ (comonotonicity)
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  - $D_1, D_2 = D_1 + 1$ ("permutation")
  - ...

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▶ Dependence uncertainty: the marginal distributions are known, the dependence structure is not known

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Motivation

Random variables

- $(X_1, \ldots, X_d)$: random variables with marginal distributions $(F_1, \ldots, F_d)$
- Dependence structure: determined by joint distribution F or copula C
- Sklar's Theorem: given $F, F_1, \ldots, F_d$, there exists C s.t.

$$F(x_1, \ldots, x_d) = C(F_1(x_1), \ldots, F_d(x_d)) \quad \text{for all } x \in \mathbb{R}^d$$

- Dependence uncertainty: the marginal distributions are known, the dependence structure is not known

▶ Main question: f 'nice' function, compute

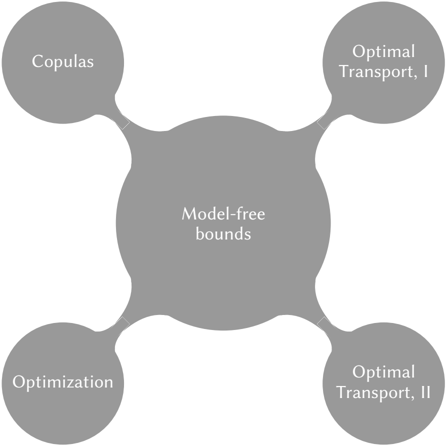
$$\inf\{\mathbb{E}_C[f] : C \text{ copula}\} \quad \text{and} \quad \sup\{\mathbb{E}_C[f] : C \text{ copula}\}$$

- Recently, the problem was reformulated under additional constraints by Tankov

$$\inf / \sup \{\mathbb{E}_C[f] : C \text{ copula } + \text{partial information on } C\}$$

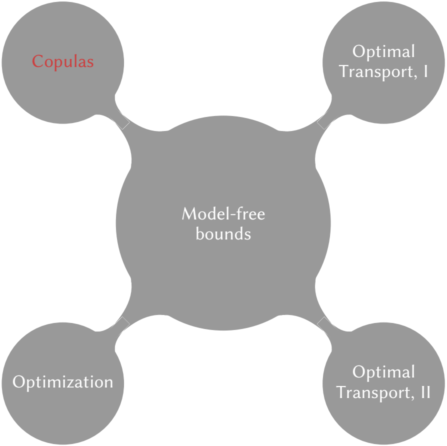
- Math Finance: d'Aspremont, Bertsimas, Deelstra, Denuit, Hobson, Laurence, Vyncke, Wang, …
- QRM / Insurance Math: Bernard, Embrechts, Puccetti, Rüschendorf, Vanduffel, Wang, …

Outline


Copulas
Optimal Transport, I
Model-free bounds
Optimization
Optimal Transport, II

Outline

T. Lux



Theorem

Let $S \subseteq \mathbb{I}^d$ be a compact set and Q^* be a d-quasi-copula. Consider the set

$$\mathcal{Q}^{S,Q^*} := \left\{ Q \in \mathcal{Q}^d : Q(\mathbf{x}) = Q^*(\mathbf{x}) \text{ for all } \mathbf{x} \in S \right\}.$$

Then it holds that

$$\begin{aligned} &\underline{Q}^{S,Q^*}(\mathbf{u}) \leq Q(\mathbf{u}) \leq \overline{Q}^{S,Q^*}(\mathbf{u}) \quad \text{for all } \mathbf{u} \in \mathbb{I}^d \\ \text{and} \quad &\underline{Q}^{S,Q^*}(\mathbf{u}) = Q(\mathbf{u}) = \overline{Q}^{S,Q^*}(\mathbf{u}) \quad \text{for all } \mathbf{u} \in S \end{aligned} \tag{1}$$

for all $Q \in \mathcal{Q}^{S,Q^*}$, where the bounds $\underline{Q}^{S,Q^*}$ and $\overline{Q}^{S,Q^*}$ are provided by

$$\begin{aligned} \underline{Q}^{S,Q^*}(\mathbf{u}) &= \max\left(0, \sum_{i=1}^d u_i - d + 1, \max_{\mathbf{x} \in S} \left\{ Q^*(\mathbf{x}) - \sum_{i=1}^d (x_i - u_i)^+ \right\}\right) \\ \overline{Q}^{S,Q^*}(\mathbf{u}) &= \min\left(u_1, \ldots, u_d, \min_{\mathbf{x} \in S} \left\{ Q^*(\mathbf{x}) + \sum_{i=1}^d (u_i - x_i)^+ \right\}\right). \end{aligned} \tag{2}$$

Furthermore, the bounds $\underline{Q}^{S,Q^*}$, $\overline{Q}^{S,Q^*}$ are d-quasi-copulas.

Improved Fréchet–Hoeffding bounds, II

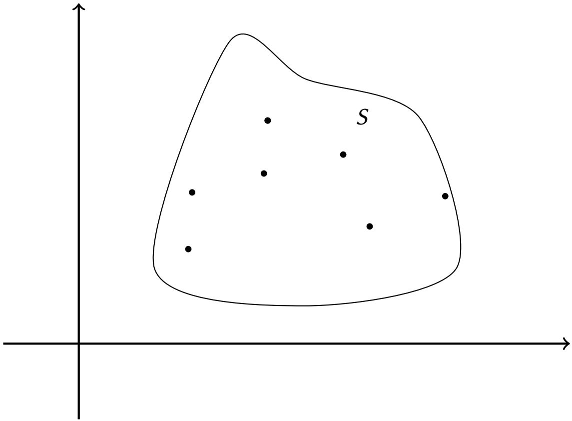


Figure: Illustration of the set S.

Measures of association / option prices

Let $\rho\colon \mathcal{Q}^d \to \mathbb{R}$ non-decreasing and continuous, and consider

$$\mathcal{Q}^\theta := \{Q \in \mathcal{Q}^d \colon \rho(Q) = \theta\}.$$

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Lower-dimensional copulas

$$\mathcal{Q}^I = \left\{Q \in \mathcal{Q}^d \colon \underline{Q}_j \preceq Q_{I_j} \preceq \overline{Q}_j,\ j = 1, \ldots, k\right\}$$

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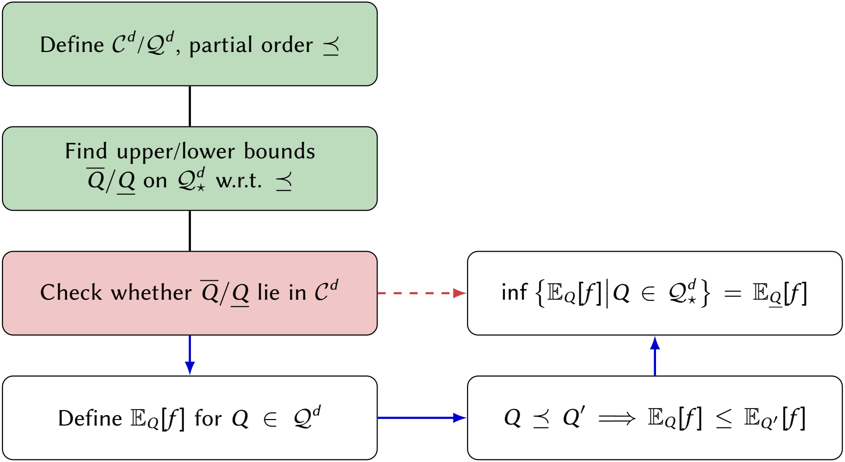
Distance to a reference copula

Let $\mathcal{D}\colon \mathcal{Q}^d \times \mathcal{Q}^d \to \mathbb{R}_+$ be a statistical distance, C^* be a reference copula, and consider

$$\mathcal{Q}^{\mathcal{D},\delta} := \{Q \in \mathcal{Q}^d \colon \mathcal{D}(Q, C^*) \leq \delta\}.$$

Roadmap


Define $\mathcal{C}^d/\mathcal{Q}^d$, partial order $\preceq$
Find upper/lower bounds $\overline{Q}/\underline{Q}$ on $\mathcal{Q}^d_\star$ w.r.t. $\preceq$
Check whether $\overline{Q}/\underline{Q}$ lie in $\mathcal{C}^d$
$\inf \left\{ \mathbb{E}_Q[f] \middle| Q \in \mathcal{Q}^d_\star \right\} = \mathbb{E}_{\underline{Q}}[f]$
Define $\mathbb{E}_Q[f]$ for $Q \in \mathcal{Q}^d$
$Q \preceq Q' \implies \mathbb{E}_Q[f] \leq \mathbb{E}_{Q'}[f]$

- Let (S^1, S^2, S^3) be asset prices that follow the Black–Scholes model, with $S_0^i = 10$, $r = 0$ and $\sigma_i = 1$.
- Observe market prices of single asset options $\rightsquigarrow$ known marginals
- Observe market prices of bivariate options $H(S^i, S^j) = 1_{\{\max\{S^i, S^j\} < K\}}$ for $K = 2, 4, \ldots, 16$ and $i, j = 1, 2, 3$
- Observed market prices are expectations under a risk-neutral measure $\mathbb{Q}$:

$$\mathbb{E}_{\mathbb{Q}}[H(S^j, S^j)] = \mathbb{Q}(S^i < K, S^j < K) \quad \Longrightarrow \quad \text{Prescription on compact set}$$

- Reference model: multivariate log-normal (Gaussian copula) with

$$\rho^{i,j} = \mathrm{Corr}(S^i, S^j)$$

▶ Arbitrage bounds for $f(S^1, S^2, S^3) = 1_{\{\max\{S^1, S^2, S^3\} < K\}}$

Numerical illustration

Example: $\rho^{i,j} = -0.3$ (left) and $\rho^{1,2} = -0.5$, $\rho^{1,3} = 0.5$, $\rho^{2,3} = 0$ (right)

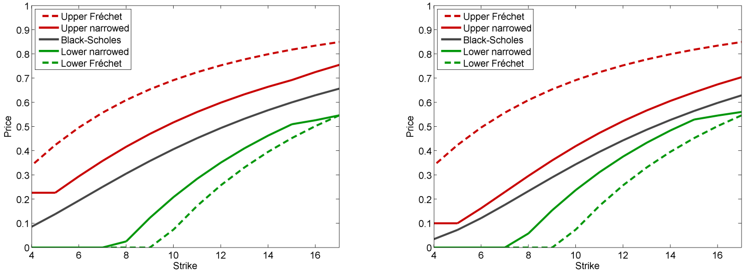


Figure: Arbitrage bounds as functions of K

- Application I: bounds for VaR (Lux & P., IME, 2019)
- Application II: detection of arbitrages (P. & Yanez, DEMO, 2021)

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Questions — open problems

1. The 'nice' functions are Δ-tonic — basket options are excluded …

2. The improved Fréchet–Hoeffding bounds are not sharp for $d > 2$, although …
   - Tankov showed that they are copulas for $d = 2$,
   - Bernard et al. strengthened this result ($d = 2$).

   Are they pointwise sharp, e.g. $\overline{Q}(u) = \sup_{Q \in \mathcal{Q}_\star} Q(u)$?

3. The marginals are known. Is that realistic?

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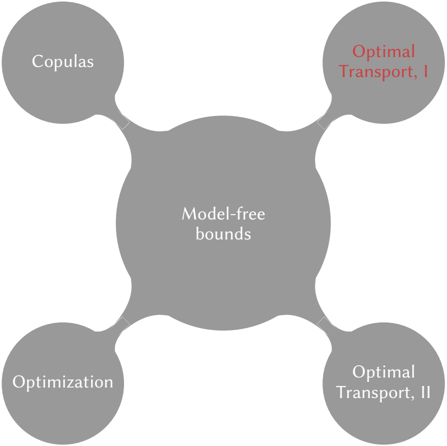
Outline

D. Bartl, M. Kupper, T. Lux, S. Eckstein



Transport and relaxed transport duality

Aim: upper bound – superhedging strategy for $f(\mathbf{X}) \rightsquigarrow \mathbb{E}[f(\mathbf{X})]$

Classical ingredients:

- $f_1, \ldots, f_d : \mathbb{R} \to \mathbb{R}$ bounded, measurable functions ('put options')
- $\nu_1, \ldots, \nu_d$ marginal distributions, μ joint distribution

Then

$$\sup_{\mu \in \ldots} \int f \mathrm{d}\mu = \inf \left\{ \int f_1 \mathrm{d}\nu_1 + \cdots + \int f_d \mathrm{d}\nu_d : f_1 + \cdots + f_d \geq f \right\}$$

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New ingredients

- π^i price of multi-asset digital 1_{A^i}, $A^i = \times_{j=1}^{d}(-\infty, A^i_j]$, $i \in I$
- a^i amount invested in 1_{A^i}

Transport and relaxed transport duality, II

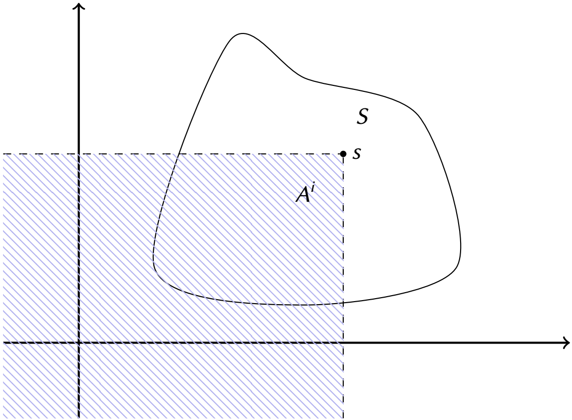

Figure: Illustration of the relation between the sets S and $(A^i)_{i \in I}$.

Define

$$\Theta(f) := \left\{(f_1,\dots,f_d,a) : f_1(x_1)+\cdots+f_d(x_d)+\sum_{i\in I} a^i 1_{A^i}(x) \geq f(x), \text{ for all } x\in\mathbb{R}^d\right\},$$

and

$$\pi(f_1,\dots,f_d,a) := \int_{\mathbb{R}} f_1\,d\nu_1 + \cdots + \int_{\mathbb{R}} f_d\,d\nu_d + \sum_{i\in I}\left(a^{i+}\overline{\pi}^i - a^{i-}\underline{\pi}^i\right).$$

Theorem

Let $f:\mathbb{R}^d\to\mathbb{R}$ be an upper semicontinuous and bounded function, then

$$\max_{\mu\in\mathcal{Q}}\int_{\mathbb{R}^d} f\,d\mu = \inf\{\pi(f_1,\dots,f_d,a) : (f_1,\dots,f_d,a)\in\Theta(f)\},$$

where

$$\mathcal{Q} := \left\{\mu\in ca_1^+(\mathbb{R}^d) : \mu_1=\nu_1,\ \dots,\ \mu_d=\nu_d \text{ and } \underline{\pi}^i \leq \mu(A^i)\leq \overline{\pi}^i, \text{ for all } i\in I\right\}.$$

Shortselling costraints:

$$\Theta_+(f) := \{(f_1,\dots,f_d,a) \in \Theta(f) : f_1,\dots,f_d \geq 0 \text{ and } a^i \geq 0, \text{ for all } i \in I\},$$

$$\pi(f_1,\dots,f_d,a) := \int_{\mathbb{R}} f_1 \,\mathrm{d}\nu_1 + \cdots + \int_{\mathbb{R}} f_d \,\mathrm{d}\nu_d + \sum_{i\in I} a^{i+}\overline{\pi}^i.$$

Theorem

Let $f : \mathbb{R}^d \to \mathbb{R}$ *be an upper semicontinuous and bounded function, then*

$$\max_{\mu\in\mathcal{Q}_+} \int_{\mathbb{R}^d} f \,\mathrm{d}\mu = \inf\{\pi(f_1,\dots,f_d,a) : (f_1,\dots,f_d,a) \in \Theta_+(f)\}, \tag{3}$$

where

$$\mathcal{Q}_+ := \Big\{\mu \in ca^+_{\leq 1}(\mathbb{R}^d) : \mu_1 \leq \nu_1, \dots, \mu_d \leq \nu_d \text{ and } \mu(A^i) \leq \overline{\pi}^i, \text{ for all } i \in I\Big\}.$$

⇝ Uncertainty in the dependence structure and the marginals!

Results

Copula bounds vs Optimal Transport bounds (á la Eckstein–Kupper) [M. Ntaoutis, MSc Thesis, NTUA]

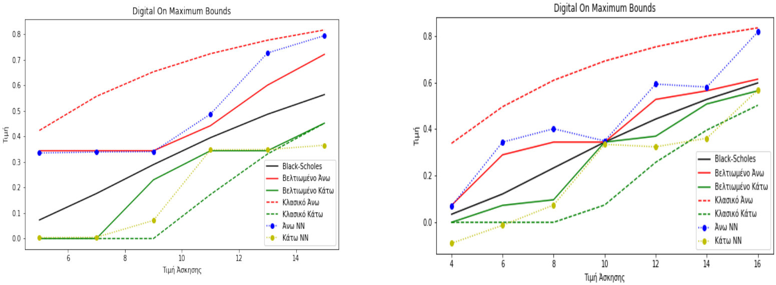

Figure: Bounds on option prices

- Pointwise sharpness of the improved Fréchet–Hoeffding bounds for relaxed Fréchet classes
- Counterexample, even for $d = 2$, when the conditions of Tankov / Bernard et al. are violated

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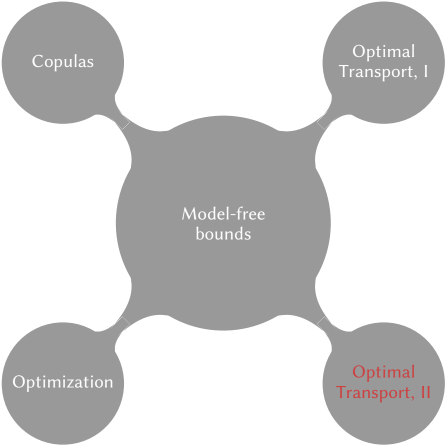
Questions — open problems

1. The additional information is not stemming from traded assets, *i.e.* multi-asset digital options are not (liquidly) traded …
2. Can we replace the additional information with traded asset prices, *e.g.* basket options?

Outline

E. Dragazi, S. Liu



Transport duality under option-implied information

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- p^i price of multi-asset option with payoff ϕ^i
- b^i amount invested in ϕ^i

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Transport duality under option-implied information, II

Define

$$\Theta(f) := \left\{ (f_1, \ldots, f_d, b) : f_1(x_1) + \cdots + f_d(x_d) + \sum_{i \in I} b^i \phi^i \geq f(x), \text{ for all } x \in \mathbb{R}^d \right\},$$

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Let $f : \mathbb{R}^d \to \mathbb{R}$ be an upper semicontinuous and bounded function, then

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where

$$\mathcal{Q} := \left\{ \mu \in \mathcal{P}(\mathbb{R}^d) : \mu_1 = \nu_1, \ \ldots, \ \mu_d = \nu_d \text{ and } \int \phi^i \mathrm{d}\mu = \pi^i, \text{ for all } i \in I \right\}.$$

Numerical scheme – penalization and neural networks

Eckstein & Kupper (AMO, 2019)

We would like to approximate the function

$$\Phi(f) = \inf\left\{ \sum_j \int f_j \mathrm{d}\nu_j + \sum_i b^i p^i \,\middle|\, \sum f_j + \sum b^i \phi^i \geq f \right\}$$

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Step 1: Penalization

$$\Phi_{\beta,\theta}(f) = \inf\left\{ \sum_j \int f_j \mathrm{d}\nu_j + \sum_i b^i p^i - \int \beta\Big(f - \sum f_j + \sum b^i \phi^i\Big) \mathrm{d}\theta \right\}$$

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Step 2: Neural network approximation

$$\Phi^m_{\beta,\theta}(f) = \inf_{f_j \in \mathcal{C}^m}\left\{ \sum_j \int f_j \mathrm{d}\nu_j + \sum_i b^i p^i - \int \beta\Big(f - \sum f_j + \sum b^i \phi^i\Big)\mathrm{d}\theta \right\}$$

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Results

3 assets; work in progress

- 3 assets; Black–Scholes dynamics with Gaussian copula
- Additional information (ϕ) and payoff (f): call-on-max, *i.e.*

$$\left(\max(S^1, S^2, S^3) - K, 0\right)^+$$

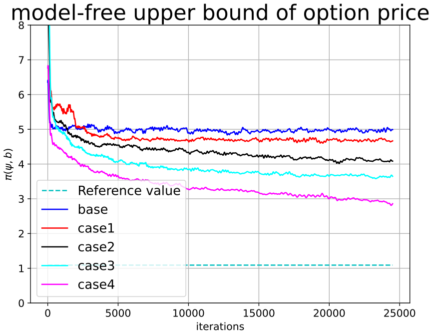


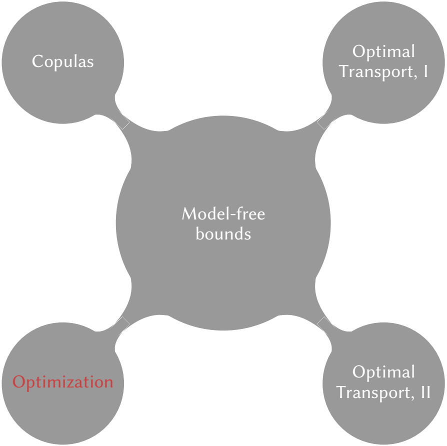
Figure: Bounds on option prices

Questions — open problems

1. We are still assuming that marginals are fully known. Is that realistic?
2. Can we have a fully data-driven approach?

Outline

A. Neufeld, Q. Xiang



Data-driven approach, I

Traded prices

- Stocks, single- and multi-asset derivatives with known bid and ask prices
- Notation: $g : \mathbb{R}^d \to \mathbb{R}$ stands for x_i, $(x_i - K)^+$, $(\sum_i x_i - K)^+, \ldots$

Modeling $\mathcal{Q}$ – option-implied measures μ

- Information on the marginals μ_i:

$$\underline{\pi}_j \leq \int_{\mathbb{R}_+} g_j \mathrm{d}\mu_i \leq \overline{\pi}_j, \quad j \in \mathcal{J}_i,\ i = 1, \ldots, d.$$

- Information on the joint law μ:

$$\underline{\pi}_j \leq \int_{\mathbb{R}_+^d} g_j \mathrm{d}\mu \leq \overline{\pi}_j, \quad j \in \mathcal{J}.$$

- Aggregate:

$$\mathcal{Q} := \left\{ \mu \in \mathcal{P}(\mathbb{R}_+^d) : \underline{\pi}_j \leq \int_{\mathbb{R}_+^d} g_j \mathrm{d}\mu \leq \overline{\pi}_j, \text{ for } j \in \left(\cup_i \mathcal{J}_i \right) \cup \mathcal{J} \right\}.$$

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Data-driven approach, II

Portfolios of traded assets

The value of a portfolio of traded assets with weights $y \in \mathbb{R}^m$ equals

$$\pi(y) := \sum_{j=1}^{m} y_j^+ \overline{\pi}_j - y_j^- \underline{\pi}_j,$$

and we define the functional $\phi(f)$ as follows:

$$\phi(f) := \inf \left\{ c + \pi(y) : c \in \mathbb{R}, y \in \mathbb{R}^m, c + \langle y, g \rangle \geq f \right\}.$$

Theorem: Superhedging duality

Under a no-arbitrage assumption, the following superhedging duality holds

$$\phi(f) = \sup_{\mu \in \mathcal{Q}} \int_{\mathbb{R}_+^d} f \, \mathrm{d}\mu.$$

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LSIP formulation

The minimization problem $\phi(f)$ is equivalent to the linear semi-infinite problem (LSIP)

$$\begin{aligned} \phi(f) = \quad & \text{minimize} \quad && c + \langle y^+, \overline{\pi} \rangle - \langle y^-, \underline{\pi} \rangle \\ & \text{subject to} \quad && c + \langle y^+ - y^-, g(x) \rangle \geq f(x), \forall x \in \mathbb{R}^d_+, \\ & && c \in \mathbb{R}, y^+ \geq 0, y^- \geq 0. \end{aligned} \tag{4}$$

Aim

Develop numerical methods for the computation of upper and lower bounds that are ε-optimal, *i.e.*

$$\phi(f)^{LB} \leq \phi(f) \leq \phi(f)^{UB} \text{ and } \phi(f)^{UB} - \phi(f)^{LB} \leq \varepsilon, \quad \varepsilon > 0.$$

A crucial ingredient

Continuous piece-wise affine (CPWA) functions

We call a function $h : \mathbb{R}^d \mapsto \mathbb{R}$ a CPWA function if it can be represented as

$$h(x) = \sum_{k=1}^{K} \xi_k \max \{ \langle a_{k,i}, x \rangle + b_{k,i} : 1 \leq i \leq I_k \}, \tag{5}$$

where $K \in \mathbb{N}$, $I_k \in \mathbb{N}$ for $k = 1, \ldots, K$, and $a_{k,i} \in \mathbb{R}^d$, $b_{k,i} \in \mathbb{R}$, $\xi_k \in \{-1, 1\}$ for $i = 1, \ldots, I_k$, $k = 1, \ldots, K$.

Examples

Many popular payoff functions in finance belong to the class of CPWA functions.

- Call option
$$h(x) = \max\{x_i - \kappa, 0\} = (x_i - \kappa)^+.$$
- Basket option
$$h(x) = \max \left\{ \sum_{i=1}^{d} w_i x_i - \kappa, 0 \right\} = \left(\sum_{i=1}^{d} w_i x_i - \kappa \right)^+.$$
- Spread option, call-on-max, call-on-min option, best-of-call option, ...

The exterior cutting plane (ECP) algorithm

Assumptions

- $\Omega = \{x \in \mathbb{R}^d : 0 \leq x \leq \overline{x}\}$ for $\overline{x} := (\overline{x}_1, \ldots, \overline{x}_d)^\mathsf{T} > 0$.
- f and $(g_j)_{j=1:m}$ are CPWA functions on Ω.

- The ECP algorithm is based on a discretization of the domain by a growing finite subset, thus relaxing the original optimization problem.
- The inner problem:

$$x^\star = \operatorname{argmin}_{x \in \mathbb{R}^d_+} c + \langle y^+ - y^-, g(x) \rangle - f(x)$$

  is solved via mixed-integer linear programming.

Theorem: Properties of ECP algorithm

- Under a no-arbitrage assumption, the ECP algorithm terminates after finitely many iterations with an ε-optimal solution $(c^\star, y^\star)$ of the LSIP problem.
- The ECP algorithm produces an ε-optimizer $\mu^\star$ of the primal problem $\sup_{\mu \in \mathcal{Q}} \int_\Omega f \, \mathrm{d}\mu$.

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Numerical experiments

Setting

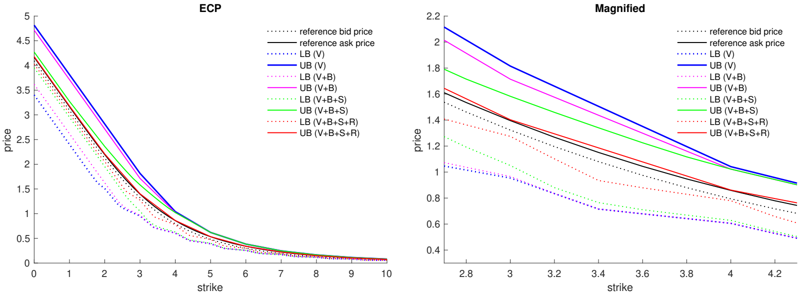
- Assets: $d = 5$ and $d = 60$
- Derivatives: $m = 439$ and $m = 400$ (include assets, vanila calls, baskets, spreads and calls-on-min)
- Target payoffs: $f(x) = \left(x_2 \vee x_3 \vee x_4 - \kappa\right)^+$ and $f(x) = \left(x_1 \wedge \cdots \wedge x_{50} - \kappa\right)^+$
- Traded prices: bid and ask prices of the traded options are simulated from a pre-specified model (log-normal + t-copula).

Notation

- V: only vanilla options;
- V+B: vanilla and basket options;
- V+B+S: vanilla, basket, and spread options;
- V+B+S+R: vanilla, basket, spread and call-on-max (rainbow) options .

Numerical experiments I

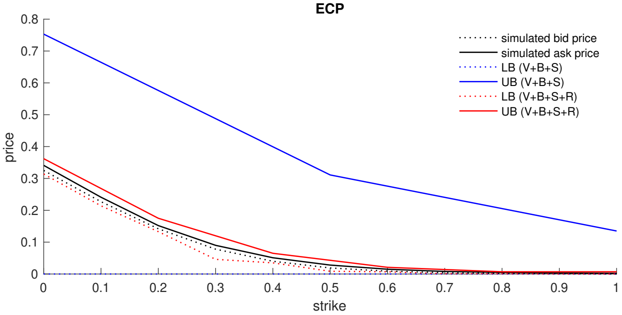
$d = 5$, "fixed" model



- Additional information (known prices) ⇝ reduction of model risk / NA gap
- Structure of additional information is important

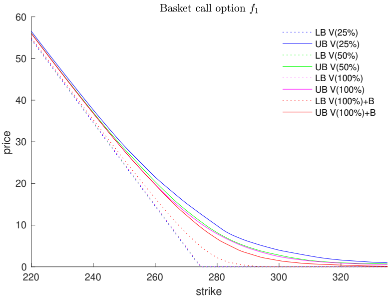
Numerical experiment II

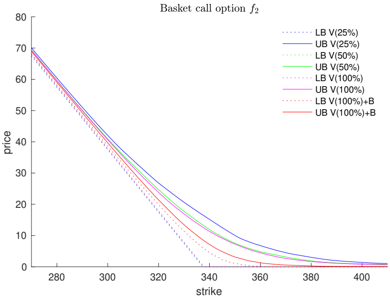
$d = 60$



Numerical experiments III

30 assets, DJIA, DIA ETF





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THANK YOU