UNIVERSITÄT
BIELEFELD

Faculty of Business Administration
and Economics

IMW

Knightian Uncertainty in Economics and Finance

22nd Winter School on Mathematical Finance
January 20-22, 2025
Soesterberg

Frank Riedel

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Risk versus Uncertainty Sharing

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Risk versus Uncertainty Sharing

Outline

1. Risk Sharing

2. Intertemporal Risk Sharing

3. Uncertainty Sharing

The Risk Sharing Problem

- Risk sharing involves distributing the financial consequences of risks among various parties. This concept is fundamental in insurance, finance, and business.

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- The process by which multiple parties agree to take on portions of risk to reduce the burden on any single entity.
- To mitigate the impact of adverse events by spreading potential losses across a wider base, making them more manageable.

Mechanisms of Risk Sharing

- Insurance: Policyholders pay premiums to an insurer, which in return assumes the risk of specific events (e.g., accidents, natural disasters). When claims arise, the insurer covers the losses.

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- Financial Instruments: Products like derivatives, options, and swaps can redistribute financial risk between parties. For example, a company might use a currency swap to hedge against exchange rate fluctuations.

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- Increased Risk Capacity: Entities can undertake larger projects or investments since risks are distributed.
- Encouragement of Innovation: By mitigating potential losses, risk sharing encourages investment in new ventures and technologies.

Risk Sharing: A Model

Bob and Alice

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- both know the probability distribution P as in roulette
- Alice and Bob have strictly concave (risk-averse) Bernoulli utility u_A and u_B
- find a risk sharing agreement $(\xi_A, Z - \xi_A)$ that maximizes
$$E^P u_A(\xi_A) + E^P u_B(Z - \xi_A)$$

Risk Sharing: Simple Examples

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Example

- X and Y i.i.d.
- sharing risk should mean $\frac{1}{2}Z$ for both?

Risk Sharing: Solution

FOCs and Optimum

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FOCs and Optimum

- we can maximize pointwise under P

$$\max \mathbb{E}^P[u_A(\xi_A)+u_B(Z-\xi_A)] = \mathbb{E}^P[\max_{\xi\in\mathbb{R}_+} u_1(\xi)+u_2(Z(\omega)-\xi)]$$

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- if $Z(\omega) = const$, ...
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- Constant absolute risk aversion ...
- the solution is comonotone, i.e. ξ_A and $Z - \xi_A$ are both monotone functions of Z

A General Version of Risk Sharing

- Risk sharing lead to efficient allocations in the sense of economics

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- we present a general version based on *Dana*, Econometrica 1992

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  - $\lim_{c \downarrow 0} u_i'(c) = \infty$

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- $(X_i)_i$ is feasible if $\sum X_i \leq e$,
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Remark

Convex Analysis: optima can be found by maximizing a weighted sum

$$U(e;\lambda) = \max_{(X_i)\in\left(L_+^p\right)^I : \sum X_i \leq e} \sum_i \lambda_i E^P u_i(X_i)$$

o[illegible]ficient allocations

Pareto Optima: Representative Agent

Consider the (ex-post) pointwise maximization problem

$$u(x;\lambda) := \max_{(x_i)\in\mathbb{R}^I_+ : \sum x_i = x} \sum \lambda_i u_i(x_i)$$

With our assumptions, the solution vector $c = (c_i)$ is unique and determined by the system

$$\begin{aligned} \lambda_1 u_1'(c_1) &= \mu \\ \vdots &= \vdots \\ \lambda_I u_I'(c_I) &= \mu \\ \sum c_i &= x \end{aligned}$$

for some Lagrange multiplier μ.

Pareto Optima: Representative Agent

Theorem

The solutions $c_i = c_i(x;\lambda)$ *are continuously differentiable on* $(0,\infty)$*. The function* u *is continuously differentiable on* $(0,\infty)$ *and satisfies*

$$u'(x;\lambda) = \lambda_i u_i'(c_i(x;\lambda))$$

The optimal risk sharing plans $c_i = c_i(x;\lambda)$ *are continuous, monotone functions of aggregate endowment.*

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The solutions $c_i = c_i(x; \lambda)$ are continuously differentiable on $(0, \infty)$. The function u is continuously differentiable on $(0, \infty)$ and satisfies

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The optimal risk sharing plans $c_i = c_i(x; \lambda)$ are continuous, monotone functions of aggregate endowment.

Remark

Optimal risk sharing: everybody gets a continuous, monotone increasing share of the total income. If there is no aggregate risk, everybody is fully insured.

Pareto Optima: Representative Agent

Theorem

The representative agent is of expected utility type and has Bernoulli utility function $u(\cdot;\lambda)$, i.e.

$$U(e;\lambda) = E^P u(e;\lambda).$$

Full Insurance is Optimal under Risk - Second Proof

Suppose that Bob and Alice agree that the probability measure P is correct, i.e.

$$U_A(Z) = E^P u(Z), U_B(Z) = E^P v(Z)$$

Theorem

An allocation $(Z, 1 - Z)$ is optimal if and only if Z is constant.

Second Proof: If Z is not constant, replace Z by $z = E^P Z$. By no aggregate uncertainty, $(z, 1 - z)$ is a feasible. By (strict) concavity, $(z, 1 - z)$ is better.

Outline

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- Now let us consider two agents with intertemporal utility

$$U_1(c) = \int_0^\infty \exp(-\rho_i s) u_i(c_i(s)) ds$$

Deterministic Time--Additive Model

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- Yes, but there is an analogy: the agents share time instead of states of the world
- they have an analogous interest in smoothing consumption plans over time as they have in smoothing consumption plans over states of the world under risk

First-Order Condition

- the marginal utility at time t is

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- $$c_1(t) + c_2(t) = z(t)$$

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- same allocation rule, only aggregate endowment is random

Stochastic Differential Utility

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- $$-dV_t^i = g^i(c_t^i, V_t^i)dt - Z_t^i dW_t, V_T = 0$$
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- efficient allocations solve a system of differential equations that can be solved numerically

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- measurability (optimal allocations are functions of Z alone)
- comonotonicity (optimal allocations are monotone functions of Z)

Uncertainty Sharing: Literature

Literature

- *Châteauneuf, Dana, Tallon* , Optimal risk-sharing rules and equilibria with Choquet-expected-utility. Journal of Mathematical Economics, 34(2), 2000
- *Billot, Châteauneuf, Gilboa, Tallon*, Sharing Beliefs: Between Agreeing and Disagreeing, Econometrica, 2000
- *Rigotti, Shannon, Strzalecki,* Subjective beliefs and ex ante trade, Econometrica 2008
- *Strzalecki, Werner*, Efficient allocations under ambiguity. Journal of Economic Theory, 2011
- for identifiable models, recently full solution, *Hara, Mukerji, Riedel, Tallon* , Efficient allocations under ambiguous model uncertainty. Available at SSRN 4272548.

Full Insurance under Knightian Uncertainty

Literature

- The benchmark case of no aggregate uncertainty is archetypical to discuss economic institutions (*Mirrlees*, 1971)
- Main result: Efficient allocations are full insurance allocations if agents "do not fully disagree on possible models"

Full Insurance under Knightian Uncertainty

Billot, Châteauneuf, Gilboa, Tallon, Sharing Beliefs: Between Agreeing and Disagreeing, Econometrica, 2000

- Suppose that Knightian uncertainty is described by a set of priors $\mathcal{P}$.

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General Ambiguity-Averse Preferences and Full Insurance

Rigotti, Shannon, Strzalecki, Subjective beliefs and ex ante trade, Econometrica 2008

Let $\mathcal{P}$ be the set of priors describing uncertainty. and denote by $U_i(c)$ the utility of agent i for consumption plan c.

Definition

We call $Q \in \Delta$ a (supporting) subjective belief at consumption plan c if

$$E^Q[y] \geq E^Q[c]$$

for all consumption plans y with $U_i(y) \geq U_i(c)$.

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Remark

The supporting subjective belief is a subgradient of U_i at c, normalized to be a probability.

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- *Each U_i is translation invariant at certainty: For all $h \in \mathbb{X}$ and all constant bundles $c, c' > 0$, if $U_i(c + \lambda h) \geq U_i(c)$ for some $\lambda > 0$, then there exists $\lambda' > 0$ such that $U_i(c' + \lambda' h) \geq U_i(c')$. We denote the subjective belief of agent i at any constant bundle $c > 0$ by π_i.*

Assumptions on Preferences

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- *Preferences are consistent with the set of priors $\mathbb{P}$, i.e. we have $\pi_i \subset \mathbb{P}$, and agents share some common subjective belief at certainty: $\bigcap_{i=1}^{I} \pi_i \neq \emptyset$.*

Full Insurance under Ambiguity

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- *Hara, Mukerji, R., Tallon* provide complete solution for the smooth model, Lecture 4 in identified models