From Theoretical Results to Real-World Applications in Bonds, FX, Commodities and Cryptocurrencies: An Overview on Market Making Models

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What is market making?

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Remark: I mainly focused on market making in OTC markets. Not market making in limit order books (no tick size, no queue, no priority).

The early literature

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Models regarding inventory cost / management

- Stoll (1978)
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An economic literature about the determinants of bid-ask spreads in the 1980s and 1990s: Hasbrouck, Huang and Stoll, MRR, etc.

From economists to mathematicians

The financial mathematics community only got interested in market making from 2008 following the paper by Avellaneda and Stoikov.

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High-frequency trading in a limit order book

MARCO AVELLANEDA and SASHA STOIKOV*

Mathematics, New York University, 251 Mercer Street, New York, NY 10012, USA

Overview of my journey

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- OTC trading (neglected area of academic research): Contacted by bond dealers and FX+commodity dealers in London and NYC, for adapting models to match real-world trading environments.

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Multiple interactions with the industry

- OTC trading (neglected area of academic research): Contacted by bond dealers and FX+commodity dealers in London and NYC, for adapting models to match real-world trading environments.
- DeFi: More recently contacted by decentralized finance players to build new Automated Market Makers.

Setup of the model (I)

One asset: reference price process ("mid"-price) $(S_t)_t$

Brownian dynamics

$$dS_t = \sigma dW_t.$$

$\rightarrow$ Can be the CBBT / CP+ for corporate bonds or a homemade reference price.
$\rightarrow$ Can be EBS / Refinitiv mid price or a homemade composite.

Setup of the model (II)

Bid and ask prices proposed by the MM

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for two point processes N^b and N^a.

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Competition and demand are modeled indirecty through the probability / intensity of jumps.

Setup of the model (III)

Intensities $(\lambda_t^b)_t$ and $(\lambda_t^a)_t$ of N^b and N^a

$$\lambda_t^b = \Lambda^b(\delta_t^b)1_{q_{t-}<Q} \text{ and } \lambda_t^a = \Lambda^a(\delta_t^a)1_{q_{t-}>-Q}.$$

They depend on the distance to the reference price: Λ^b, Λ^a decreasing (of course!)

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Cash process $(X_t)_t$

$$dX_t = \Delta S_t^a dN_t^a - \Delta S_t^b dN_t^b = -S_t dq_t + \delta_t^a \Delta dN_t^a + \delta_t^b \Delta dN_t^b.$$

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Three state variables: X (cash), q (inventory), and S (price).

PnL and objective function

PnL at time T of a market maker

$$PnL_T = X_T + q_T S_T = X_0 + q_0 S_0$$

$$+ \int_0^T \underbrace{\delta_t^a \Delta dN_t^a + \delta_t^b \Delta dN_t^b}_{\text{spread capture}} + \underbrace{\sigma q_t dW_t}_{\text{inventory+price risk}}$$

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The original Avellaneda-Stoikov's model considers a CARA utility (Model A):

CARA objective function

$$\sup_{(\delta_t^a)_t, (\delta_t^b)_t \in \mathcal{A}} \mathbb{E}\left[-\exp\left(-\gamma(X_T + q_T S_T)\right)\right],$$

where γ is the absolute risk aversion parameter, and $\mathcal{A}$ the set of predictable processes bounded from below.

HJB equation

HJB equation

In what follows, u is a candidate for the value function.

Hamilton-Jacobi-Bellman

$$
\begin{aligned}
\text{(HJB)} \qquad 0 = {} & \partial_t u(t,x,q,S) + \frac{1}{2}\sigma^2 \partial^2_{SS} u(t,x,q,S) \\
& + 1_{q<Q} \sup_{\delta^b} \Lambda^b(\delta^b) \left[u(t, x - \Delta S + \Delta\delta^b, q + \Delta, S) - u(t,x,q,S) \right] \\
& + 1_{q>-Q} \sup_{\delta^a} \Lambda^a(\delta^a) \left[u(t, x + \Delta S + \Delta\delta^a, q - \Delta, S) - u(t,x,q,S) \right]
\end{aligned}
$$

with final condition:

$$
u(T,x,q,S) = -\exp\left(-\gamma(x+qS)\right)
$$

Change of variables

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Ansatz

$$u(t,x,q,S) = -\exp(-\gamma(x + qS + \theta(t,q)))$$

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$$u(t,x,q,S) = -\exp(-\gamma(x+qS+\theta(t,q)))$$

New equation

$$0 = \partial_t\theta(t,q) - \frac{1}{2}\gamma\sigma^2q^2$$

$$+1_{q<Q}\sup_{\delta^b}\frac{\Lambda^b(\delta^b)}{\gamma}\left(1-\exp\left(-\gamma\left(\Delta\delta^b+\theta(t,q+\Delta)-\theta(t,q)\right)\right)\right)$$

$$+1_{q>-Q}\sup_{\delta^a}\frac{\Lambda^a(\delta^a)}{\gamma}\left(1-\exp\left(-\gamma\left(\Delta\delta^a+\theta(t,q-\Delta)-\theta(t,q)\right)\right)\right)$$

with final condition $\theta(T,q) = 0$.

A new transform

$$H^b_\xi(p) = \sup_\delta \frac{\Lambda^b(\delta)}{\xi\Delta}\left(1 - \exp\left(-\xi\Delta\left(\delta - p\right)\right)\right)$$

$$H^a_\xi(p) = \sup_\delta \frac{\Lambda^a(\delta)}{\xi\Delta}\left(1 - \exp\left(-\xi\Delta\left(\delta - p\right)\right)\right)$$

Equation for θ

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New equation

$$0 = \partial_t\theta(t,q) - \frac{1}{2}\gamma\sigma^2q^2 + 1_{q<Q}\Delta H^b_\gamma\left(\frac{\theta(t,q) - \theta(t,q+\Delta)}{\Delta}\right)$$

$$+1_{q>-Q}\Delta H^a_\gamma\left(\frac{\theta(t,q) - \theta(t,q-\Delta)}{\Delta}\right)$$

with final condition $\theta(T,q) = 0$.

Another objective function

Variant (Cartea, Jaimungal *et al.*) with a running penalty:

Risk-neutral with running penalty (Model B)

$$\sup_{(\delta^a_t)_t,(\delta^b_t)_t\in\mathcal{A}} \mathbb{E}\left[X_T + q_T S_T - \frac{\gamma}{2}\sigma^2 \int_0^T q_t^2 dt\right]$$

i.e.

$$\sup_{(\delta^a_t)_t,(\delta^b_t)_t\in\mathcal{A}} \mathbb{E}\left[\int_0^T \left(\Delta\delta^a_t \Lambda^a(\delta^a_t) 1_{q_{t-}>-Q} + \Delta\delta^b_t \Lambda^b(\delta^b_t) 1_{q_{t-}<Q} - \frac{\gamma}{2}\sigma^2 q_t^2\right) dt\right]$$

where γ is a kind of absolute risk aversion parameter.

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$\rightarrow$ Optimal control on a very simple finite graph (truncated $\Delta\mathbb{Z}$)

Value function θ (Model B)

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Hamilton-Jacobi equation (Model B)

$$0 = \partial_t \theta(t,q) - \frac{1}{2}\gamma\sigma^2 q^2 + 1_{q<Q}\Delta H_0^b\left(\frac{\theta(t,q)-\theta(t,q+\Delta)}{\Delta}\right)$$
$$+1_{q>-Q}\Delta H_0^a\left(\frac{\theta(t,q)-\theta(t,q-\Delta)}{\Delta}\right)$$

with final condition $\theta(T,q) = 0$.

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Same kind of transform

$$H_0^b(p) = \sup_\delta \Lambda^b(\delta)(\delta - p)$$

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Analysis

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Light assumptions of the intensity functions

1. $\Lambda^{b/a}$ is C^2.
2. $\Lambda^{b/a\prime} < 0$.
3. $\lim_{\delta \to +\infty} \Lambda^{b/a}(\delta) = 0$.
4. The intensity functions $\Lambda^{b/a}$ satisfy:

$$\sup_{\delta} \frac{\Lambda^{b/a}(\delta)\Lambda^{b/a\prime\prime}(\delta)}{\left(\Lambda^{b/a\prime}(\delta)\right)^2} < 2.$$

The functions H_ξ^b and H_ξ^a

Proposition

- $\forall \xi \geq 0$, $H_\xi^{b/a}$ is a decreasing function of class C^2.
- In the definition of $H_\xi^{b/a}(p)$, the supremum is attained at a unique $\tilde{\delta}_\xi^{b/a*}(p)$ characterized by

$$\tilde{\delta}_\xi^{b/a*}(p) = {\Lambda^{b/a}}^{-1}\left(\xi \Delta H_\xi^{b/a}(p) - {H_\xi^{b/a}}'(p)\right).$$

- The function $p \mapsto \tilde{\delta}_\xi^{b/a*}(p)$ is increasing.

Existence and uniqueness

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Results for θ

There exists a unique C^1 (in time) solution $t \mapsto (\theta(t,q))_{|q|\leq Q}$ to

$$0 = \partial_t \theta(t,q) - \frac{1}{2}\gamma\sigma^2 q^2 + 1_{q<Q}\Delta H^b_\xi\left(\frac{\theta(t,q)-\theta(t,q+\Delta)}{\Delta}\right)$$

$$+1_{q>-Q}\Delta H^a_\xi\left(\frac{\theta(t,q)-\theta(t,q-\Delta)}{\Delta}\right)$$

with final condition $\theta(T,q) = 0$.

Solution of the initial problem (verification argument)

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By using a verification argument, the function u is the value function associated with the problem.

Optimal quotes

The optimal quotes in models A ($\xi = \gamma$) and B ($\xi = 0$) are:

$$\delta_t^{b*} = \tilde{\delta}_\xi^{b*}\left(\frac{\theta(t, q_{t-}) - \theta(t, q_{t-} + \Delta)}{\Delta}\right)$$

$$\delta_t^{a*} = \tilde{\delta}_\xi^{a*}\left(\frac{\theta(t, q_{t-}) - \theta(t, q_{t-} - \Delta)}{\Delta}\right)$$

where

$$\tilde{\delta}_\xi^{b/a*}(p) = {\Lambda^{b/a}}^{-1}\left(\xi \Delta H_\xi^{b/a}(p) - {H_\xi^{b/a}}'(p)\right).$$

The case $\Lambda^b(\delta) = \Lambda^a(\delta) = Ae^{-k\delta}$ (I)

The functions $H_\xi^{b/a}$ and $\tilde{\delta}_\xi^{b/a*}$

If $\Lambda^b(\delta) = \Lambda^a(\delta) = Ae^{-k\delta}$, then $H_\xi^{b/a}(p) = \frac{A}{k} C_\xi \exp(-kp)$, with

$$C_\xi = \begin{cases} \left(1 + \frac{\xi\Delta}{k}\right)^{-\frac{k}{\xi\Delta}-1} & \text{if } \xi > 0 \\ e^{-1} & \text{if } \xi = 0. \end{cases}$$

and

$$\tilde{\delta}_\xi^{b/a*}(p) = \begin{cases} p + \frac{1}{\xi\Delta} \log\left(1 + \frac{\xi\Delta}{k}\right) & \text{if } \xi > 0 \\ p + \frac{1}{k} & \text{if } \xi = 0, \end{cases}$$

The case $\Lambda^b(\delta) = \Lambda^a(\delta) = Ae^{-k\delta}$ (II)

The system of ODEs

$$0 = \partial_t \theta(t,q) - \frac{1}{2}\gamma\sigma^2 q^2 +$$

$$+\frac{A\Delta}{k} C_\xi \left(1_{q<Q} e^{k\frac{\theta(t,q+\Delta)-\theta(t,q)}{\Delta}} + 1_{q>-Q} e^{k\frac{\theta(t,q-\Delta)-\theta(t,q)}{\Delta}} \right),$$

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with final condition $\theta(T,q) = 0$.

Change of variables: $v_q(t) = \exp\left(\frac{k\theta(t,q)}{\Delta}\right)$

The case $\Lambda^b(\delta) = \Lambda^a(\delta) = Ae^{-k\delta}$ (III)

A linear system of ODEs

$$v_q'(t) = \alpha q^2 v_q(t) - \eta_\xi \left(1_{q<Q} v_{q+\Delta}(t) + 1_{q>-Q} v_{q-\Delta}(t)\right),$$

with

$$\alpha = \frac{k}{2\Delta}\gamma\sigma^2, \qquad \eta_\xi = AC_\xi$$

and the terminal condition $v(T, q) = 1$.

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and the terminal condition $v(T, q) = 1$.

This simplifies a lot the equations of Avellaneda and Stoikov. See the paper Guéant-Lehalle-Fernandez-Tapia (2013) (when $\Delta = 1$ and $\xi = \gamma$).

The case $\Lambda^b(\delta) = \Lambda^a(\delta) = Ae^{-k\delta}$ (IV)

Optimal quotes

The optimal quotes in models A ($\xi = \gamma$) and B ($\xi = 0$) are:

$$\delta_t^{b*} = \delta^{b*}(t, q_{t-}) := D_\xi + \frac{1}{k}\ln\left(\frac{v_{q_{t-}}(t)}{v_{q_{t-}+\Delta}(t)}\right)$$

$$\delta_t^{a*} = \delta^{a*}(t, q_{t-}) := D_\xi + \frac{1}{k}\ln\left(\frac{v_{q_{t-}}(t)}{v_{q_{t-}-\Delta}(t)}\right)$$

$$D_\xi = \begin{cases} \frac{1}{\xi\Delta}\log\left(1 + \frac{\xi\Delta}{k}\right) & \text{if } \xi > 0 \\ \frac{1}{k} & \text{if } \xi = 0, \end{cases}$$

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The optimal quotes are made of two components:

- D_ξ corresponds to the static trade-off.
- $\frac{1}{k} \ln \left(\frac{v_q(t)}{v_{q+\Delta}(t)} \right)$ or $\frac{1}{k} \ln \left(\frac{v_q(t)}{v_{q-\Delta}(t)} \right)$: dynamic aspects.

The case $\Lambda^b(\delta) = \Lambda^a(\delta) = Ae^{-k\delta}$ (V)

The optimal quote functions far from T only depend on q:

Asymptotics

$$\delta_\infty^{b*}(q) = \lim_{T\to\infty} \delta^{b*}(0,q) = D_\xi + \frac{1}{k}\ln\left(\frac{f_q^0}{f_{q+\Delta}^0}\right)$$

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$f^0 \in \mathbb{R}^{2Q+1}$ is characterized by:

$$\underset{\|f\|_2=1}{\operatorname{argmin}} \sum_{|q|\leq Q} \alpha q^2 f_q^2 + \eta_\xi \left(\sum_{q=-Q}^{Q-\Delta} (f_{q+\Delta} - f_q)^2 + (f_Q)^2 + (f_{-Q})^2\right).$$

The case $\Lambda^b(\delta) = \Lambda^a(\delta) = Ae^{-k\delta}$ (VI)

Continuous counterpart

$\tilde{f}^0 \in L^2(\mathbb{R})$ characterized by:

$$\underset{\|\tilde{f}\|_{L^2(\mathbb{R})}=1}{\operatorname{argmin}} \int_{-\infty}^{\infty} \left(\alpha x^2 \tilde{f}(x)^2 + \eta_\xi \Delta^2 \tilde{f}'(x)^2\right) dx.$$

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$$\tilde{f}^0(x) \propto \exp\left(-\frac{1}{2\Delta}\sqrt{\frac{\alpha}{\eta_\xi}} x^2\right)$$

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$$\tilde{f}^0(x) \propto \exp\left(-\frac{1}{2\Delta}\sqrt{\frac{\alpha}{\eta_\xi}} x^2\right)$$

Hence, we get an approximation of the form:

$$f_q^0 \propto \exp\left(-\frac{1}{2\Delta}\sqrt{\frac{\alpha}{\eta_\xi}} q^2\right)$$

The case $\Lambda^b(\delta) = \Lambda^a(\delta) = Ae^{-k\delta}$ (VII)

Using the continuous counterpart, we get:

Closed-form approximations: optimal quotes (Model A: $\xi = \gamma$)

$$\delta^{b*}_{\infty}(q) \simeq \frac{1}{\Delta\xi}\ln\left(1+\frac{\Delta\xi}{k}\right) + \frac{2q+\Delta}{2}\sqrt{\frac{\gamma\sigma^2}{2kA\Delta}\left(1+\frac{\Delta\xi}{k}\right)^{1+\frac{k}{\Delta\xi}}}$$

$$\delta^{a*}_{\infty}(q) \simeq \frac{1}{\Delta\xi}\ln\left(1+\frac{\Delta\xi}{k}\right) - \frac{2q-\Delta}{2}\sqrt{\frac{\gamma\sigma^2}{2kA\Delta}\left(1+\frac{\Delta\xi}{k}\right)^{1+\frac{k}{\Delta\xi}}}$$

Remark: these formulas are used by many practitioners in Europe and Asia on quote-driven markets.

The case $\Lambda^b(\delta) = \Lambda^a(\delta) = Ae^{-k\delta}$ (VIII)

Using the continuous counterpart, we get:

Closed-form approximations: optimal quotes (Model B: $\xi = 0$)

$$\begin{aligned}
\delta_\infty^{b*}(q) &\simeq \frac{1}{k} + \frac{2q+\Delta}{2}\sqrt{\frac{\gamma\sigma^2 e}{2kA\Delta}} \\
\delta_\infty^{a*}(q) &\simeq \frac{1}{k} - \frac{2q-\Delta}{2}\sqrt{\frac{\gamma\sigma^2 e}{2kA\Delta}}
\end{aligned}$$

The case $\Lambda^b(\delta) = \Lambda^a(\delta) = Ae^{-k\delta}$ (IX)

A good way to analyze the result is to consider the spread $\psi = \delta^b + \delta^a$ and the skew $\zeta = \delta^b - \delta^a$.

Closed-form approx.: spread and skew (Model A, $\xi = \gamma$)

$$\psi^*_\infty(q) \simeq \frac{2}{\Delta\xi}\ln\left(1+\frac{\Delta\xi}{k}\right) + \Delta\sqrt{\frac{\gamma\sigma^2}{2kA\Delta}\left(1+\frac{\Delta\xi}{k}\right)^{1+\frac{k}{\Delta\xi}}}$$

$$\zeta^*_\infty(q) \simeq 2q\sqrt{\frac{\gamma\sigma^2}{2kA\Delta}\left(1+\frac{\Delta\xi}{k}\right)^{1+\frac{k}{\Delta\xi}}}$$

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Extensions

Basic ideas

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- Other objective functions.

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- Including a drift and / or price jumps (easy).
- Including stoch. vol. models (easy but lead to a system of parabolic PDEs in dimension depending of the number of factors).
- Modeling price by microstructural models / point processes: $\rightarrow$ lead to a system of PDEs with nonlocal terms.

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- Price signalling: need to model the impact of streamed prices.
- D2C vs. D2D (internalization vs. externalization) + market impact on the D2D segment.

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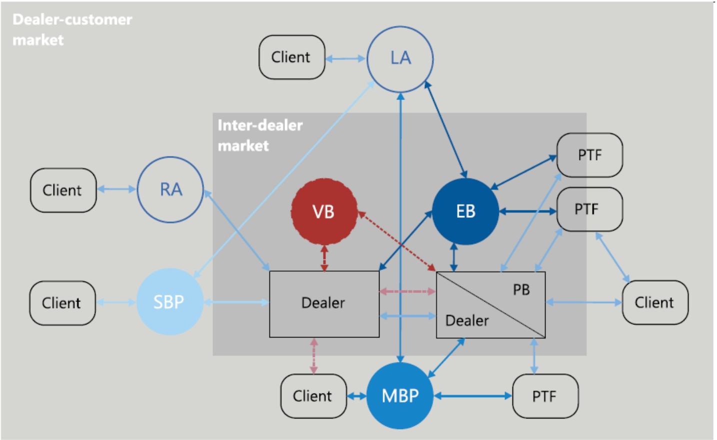
  $\rightarrow$ Less than a year later, we released a paper to build AMM based on price oracles or offchain reference prices (Bergault, Bertucci, Bouba, Guéant).
- Not really satisfying for FX (correlations + triplets) or corporate bonds (many securities for one issuer).
  $\rightarrow$ diversification and liquidity differences must be taken into account.

Organization of the FX market (Schrimpf-Sushko)



EB = electronic broker; LA = liquidity aggregator; MBP = multi-bank platform; PB = prime broker; PTF = principal trading firm; RA = retail aggregator; SBP = single-bank platform; VB = voice broker. Dashed lines indicate voice execution; solid lines indicate electronic execution.

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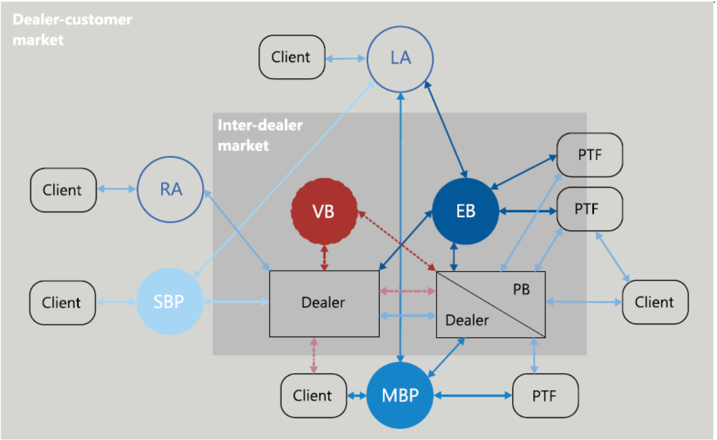

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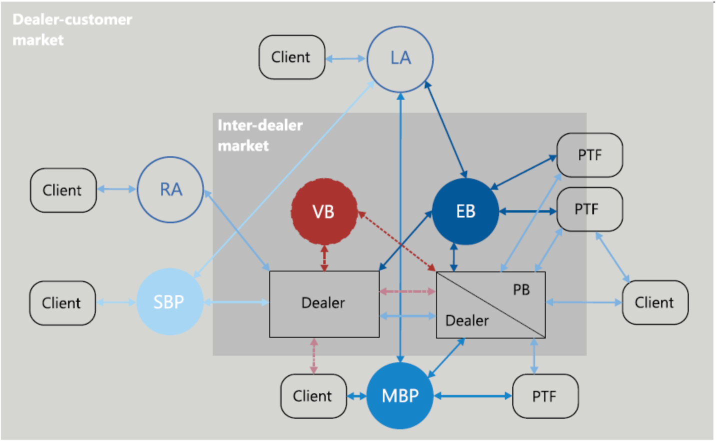

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RFSs and RFQs (D2C) and access to multiple platforms (D2D and all-to-all) $\rightarrow$ dealers can internalize or externalize the flow.

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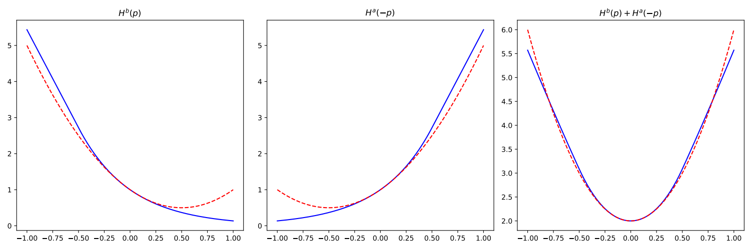
  - $\tilde{\theta}$ is plugged in the above equations to get great pricing and hedging strategies.

Approximation of the Hamiltonians

$$H^b\left(\frac{\theta(t,q)-\theta(t,q+\Delta)}{\Delta}\right)+H^a\left(\frac{\theta(t,q)-\theta(t,q-\Delta)}{\Delta}\right)$$

$$\rightsquigarrow \quad H^b(p)+H^a(-p)$$



Example (video)

Time for a short animation

The problem with precious metals

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- In many cases, market making on the spot market and hedging on both the (illiquid) spot market and the (liquid) futures market.
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- Spot: $dS_t = \sigma_S dW_t^S, \qquad \sigma_S > 0.$
- Futures: $F_t = S_t + E_t$.
  We want prices with linear dynamics to stay in the quadratic value case:

$$\begin{aligned} dE_t &= -k_E \left(E_t - D_t\right) dt + \sigma_E dW_t^E, \qquad k_E, \sigma_E > 0, \\ dD_t &= -k_D \left(D_t - \bar{D}\right) dt + \sigma_D dW_t^D, \qquad k_D, \sigma_D \geq 0, \quad \bar{D} \in \mathbb{R}, \end{aligned}$$

where $(W_t^S, W_t^E, W_t^D)_t$ is a three-dimensional Brownian motion with correlation matrix R (covariance matrix: Σ).

Other state variables

Inventories

- Spot – jumps (trade with clients) and execution:

$$dq_t^S = \int_{z=0}^{\infty} zJ^b(dt, dz) - \int_{z=0}^{\infty} zJ^a(dt, dz) + v_t^S dt.$$

- Futures – execution: $dq_t^F = v_t^F dt$.

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where the intensities are

$$\Lambda^b(z,\delta) = \Lambda^a(z,\delta) = \Lambda(z,\delta) = \lambda(z) f(\delta) \quad \text{with} \quad f(\delta) = \frac{1}{1 + e^{\alpha + \beta\delta}}.$$

Cash

The resulting cash process $(X_t)_t$ follows:

$$\begin{aligned} dX_t &= \int_{z=0}^{\infty} S^a(t,z) z J^a(dt,dz) - \int_{z=0}^{\infty} S^b(t,z) z J^b(dt,dz) \\ &\quad - v_t^S S_t dt - L^S(v_t^S)dt - v_t^F F_t dt - L^F(v_t^F)dt \end{aligned}$$

where $L^S(v_t^S)$ and $L^F(v_t^F)$ account for execution costs upon externalizing.

Stochastic optimal control

Objective function

The goal is now to maximize

$$\mathbb{E}\left[-\exp\left(-\gamma\left(X_T + q_T^S S_T + q_T^F F_T - K\left((q_T^S)^2 + (q_T^F)^2\right)\right)\right)\right]$$

by selecting δ^b, δ^a, v^S and v^F optimally.

Hamilton-Jacobi-Bellman equation

$$\begin{aligned} 0 &= \partial_t u - k_E\,(E - D)\,\partial_E u - k_D\left(D - \bar{D}\right)\partial_D u \\ &\quad + \frac{1}{2}\mathrm{Tr}(\Sigma\nabla^2_{SED} u) + \mathcal{L}^b u + \mathcal{L}^a u \\ &\quad + \sup_{v^S}\left(v^S\partial_{q^S} u - \left(L^S(v^S) + v^S S\right)\partial_x u\right) \\ &\quad + \sup_{v^F}\left(v^F\partial_{q^F} u - \left(L^F(v^F) + v^F(S+E)\right)\partial_x u\right), \end{aligned}$$

with terminal condition

$$\begin{aligned} & u(T, x, q^S, q^F, S, E, D) \\ = & -\exp\left(-\gamma\left(x + q^S S + q^F(S+E) - K\left((q^S)^2 + (q^F)^2\right)\right)\right). \end{aligned}$$

Hamilton-Jacobi-Bellman equation

Nonlocal jump operators:

$$
\begin{aligned}
&\mathcal{L}^b u(t, x, q^S, q^F, S, E, D) \\
= \quad & \int_0^\infty \sup_{\delta^b} f(\delta^b) \left(u(t, x - z(S - \delta^b), q^S + z, q^F, S, E, D) \right. \\
& \qquad\qquad \left. - u(t, x, q^S, q^F, S, E, D) \right) \lambda(z)\, dz
\end{aligned}
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$$
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&\qquad\qquad \left. - u(t, x, q^S, q^F, S, E, D) \right) \lambda(z)\, dz
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Change of variables

Ansatz

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\begin{aligned}
& u(t, x, q^S, q^F, S, E, D) \\
= \quad & -\exp\left(-\gamma\left(x + q^S S + q^F (S + E) + \theta(t, q^S, q^F, E, D)\right)\right)
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New equation for θ

The equation for θ becomes:

$$\begin{aligned} 0 \quad = \quad & \partial_t \theta - k_E\,(E - D)\left(q^F + \partial_E \theta\right) - k_D\left(D - \bar{D}\right)\partial_D \theta + \frac{1}{2}\mathrm{Tr}(\widetilde{\Sigma}\nabla^2_{ED}\theta) \\ & -\frac{\gamma}{2}\begin{pmatrix} q^S + q^F \\ q^F + \partial_E \theta \\ \partial_D \theta \end{pmatrix}^{\mathsf{T}} \Sigma \begin{pmatrix} q^S + q^F \\ q^F + \partial_E \theta \\ \partial_D \theta \end{pmatrix} + \mathcal{J}_H \theta + \mathcal{H}^S\left(\partial_{q^S}\theta\right) + \mathcal{H}^F\left(\partial_{q^F}\theta\right) \end{aligned}$$

Notations

- $\widetilde{\Sigma}$ is the submatrix of Σ obtained by removing the first row and the first column
- $\mathcal{H}^S$ and $\mathcal{H}^F$ are the Hamiltonian functions defined by

$$\mathcal{H}^S : p \in \mathbb{R} \mapsto \sup_{v^S} \left(v^S p - L^S(v^S) \right)$$

$$\mathcal{H}^F : p \in \mathbb{R} \mapsto \sup_{v^F} \left(v^F p - L^F(v^F) \right)$$

- Nonlocal jump operators:

$$\begin{aligned} &\mathcal{J}_H \theta(t, q^S, q^F, E, D) \\ = &\int_0^{\infty} zH\left(z, \frac{\theta(t, q^S, q^F, E, D) - \theta(t, q^S + z, q^F, E, D)}{z}\right) \lambda(z) dz \\ &+ \int_0^{\infty} zH\left(z, \frac{\theta(t, q^S, q^F, E, D) - \theta(t, q^S - z, q^F, E, D)}{z}\right) \lambda(z) dz \end{aligned}$$

with $H : (z, p) \in (0, +\infty) \times \mathbb{R} \mapsto \sup_{\delta} \frac{f(\delta)}{\gamma z}(1 - e^{-\gamma z(\delta - p)})$.

Solution

If we approximate the Hamiltonian terms by polynomials of degree 2, the resulting approximation of θ will be a polynomial of degree 2, with coefficients solving simple ODEs (Riccati and linear).

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Optimal strategy

$$\begin{cases} \delta^{b*}(t,z) = \bar{\delta}\left(z, \dfrac{\theta(t, q^S_{t-}, q^F_t, E_t, D_t) - \theta(t, q^S_{t-} + z, q^F_t, E_t, D_t)}{z}\right) \\ \delta^{a*}(t,z) = \bar{\delta}\left(z, \dfrac{\theta(t, q^S_{t-}, q^F_t, E_t, D_t) - \theta(t, q^S_{t-} - z, q^F_t, E_t, D_t)}{z}\right) \\ v^{S*}_t = \mathcal{H}^{S'}\left(\partial_{q^S}\theta(t, q^S_{t-}, q^F_t, E_t, D_t)\right) \\ v^{F*}_t = \mathcal{H}^{F'}\left(\partial_{q^F}\theta(t, q^S_{t-}, q^F_t, E_t, D_t)\right) \end{cases}$$

where $\bar{\delta}(z,p) = f^{-1}\left(\gamma z H(z,p) - \partial_p H(z,p)\right)$.

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The dynamics after filtering – Same Nested OU structure

Assuming $d\langle W^S, W^E\rangle = \rho dt$ and $\langle W^S, W^D\rangle = \langle W^E, W^D\rangle = 0$, we get:

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$$\begin{aligned} dE_t &= -k_E\left(E_t - \widehat{D}_t\right)dt + \sigma_E d\widehat{W}_t^E \\ d\widehat{D}_t &= -k_D\left(\widehat{D}_t - \bar{D}\right)dt + \frac{1}{\sqrt{1-\rho^2}}\frac{k_E}{\sigma_E}\nu_t^2 d\widehat{W}_t^D \end{aligned}$$

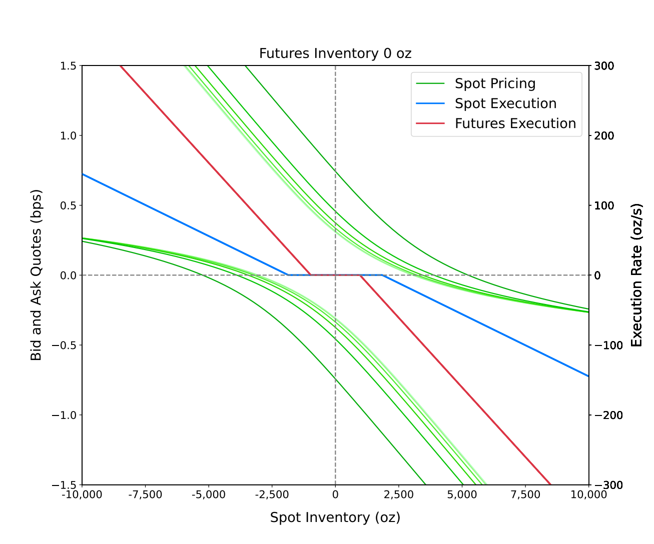
where

$$\widehat{D}_t = \mathbb{E}\left[D_t|(S_s)_{s\le t},(E_s)_{s\le t}\right], \quad \nu_t^2 = \mathbb{V}\left(D_t|(S_s)_{s\le t},(E_s)_{s\le t}\right),$$

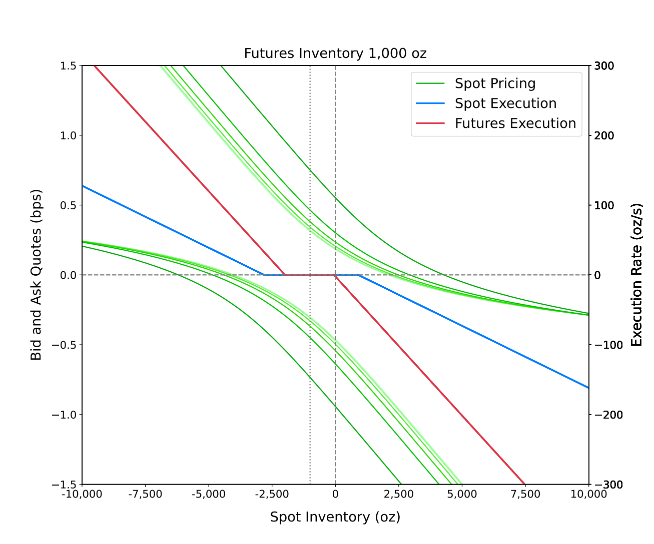
and

$$\widehat{W}_t^E = W_t^E + \frac{k_E}{\sigma_E}\int_0^t (D_s - \widehat{D}_s)ds \quad \text{and} \quad \widehat{W}_t^D = \frac{\widehat{W}_t^E - \rho W_t^S}{\sqrt{1-\rho^2}}$$

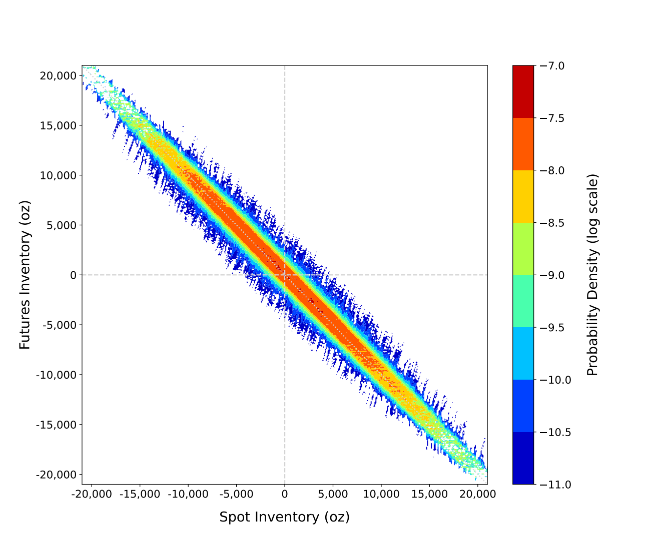
Optimal strategies



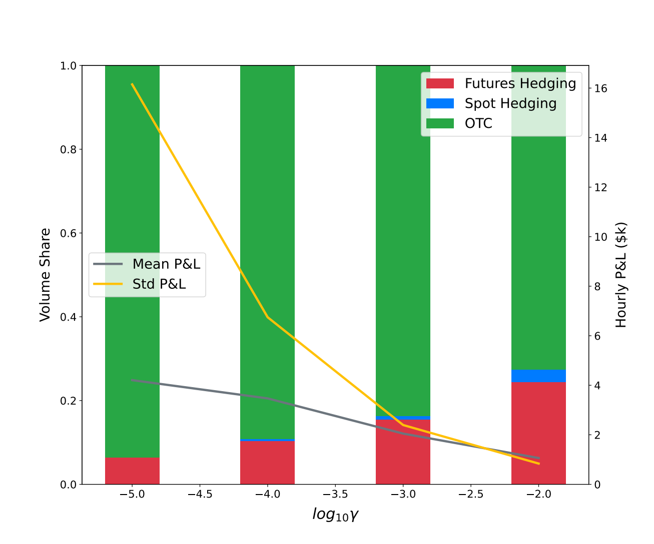
Optimal strategy



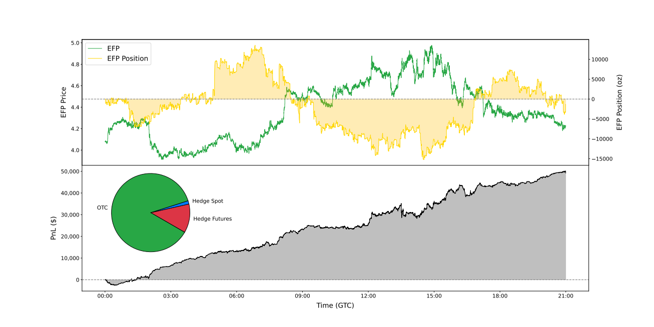
Inventory probability distribution



Volume shares



Performance



Questions

Thanks for your attention.
Questions?