Risk-Based Solvency Testing for Insurers

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Overview

- Solvency II

- Market consistent valuation

- Swiss Solvency Test (SST)

An International Structure and a Variety of Associations

Each level has its associations of supervisors, actuaries and insurers

- International: IAIS (International Association of Insurance Supervisors), IAA (International Actuarial Association) with sections ASTIN (Actuarial Studies In Non-life insurance) and AFIR (Actuarial Approach for FInancial Risks)

- Europe: The EC (European Commission = executive body of the EU) recently established the EIOPC (European Insurance and Occupational Pension Committee), supporting the IC (Insurance Committee), and CEIOPS (Committee of European Insurance and Occupational Pensions Supervisors). The Groupe Consultatif represents the actuarial profession in discussion with EU legislation. CEA stands for Comité Européen des assurances.

Risk-based Supervision

- Worldwide project: new system for assessing the overall solvency of insurance undertakings.
- Who is looking for it? → Supervisors and risk officers
- New system should be risk-based. → Encouragement for risk measurement and management.
- Traditional systems and rating agencies failed in early warning: Equitable Life Assurance Society UK (2000), HIH Insurance Group Australia (2001), ...
  - IAIS → IAA Insurer Solvency Assessment Working Party
  - EC (Solvency II) → CEIOPS, Groupe Consultatif, IAA, London Working Group (Sharma Report)
  - BPV (Swiss Solvency Test) → working parties in cooperation with insurers

Solvency in Europe

Documentation on http://europa.eu.int/comm/internal_market/insurance/solvency_en.htm

1970s: first EU non-life and life directives on solvency margins
(= extra capital as a buffer against unforeseen events such as higher than expected claims levels or unfavourable investment results)

1997: Müller Report "Solvency of insurance undertakings": review of solvency rules
→ Solvency I project initiated, completed 2002, in force 2004

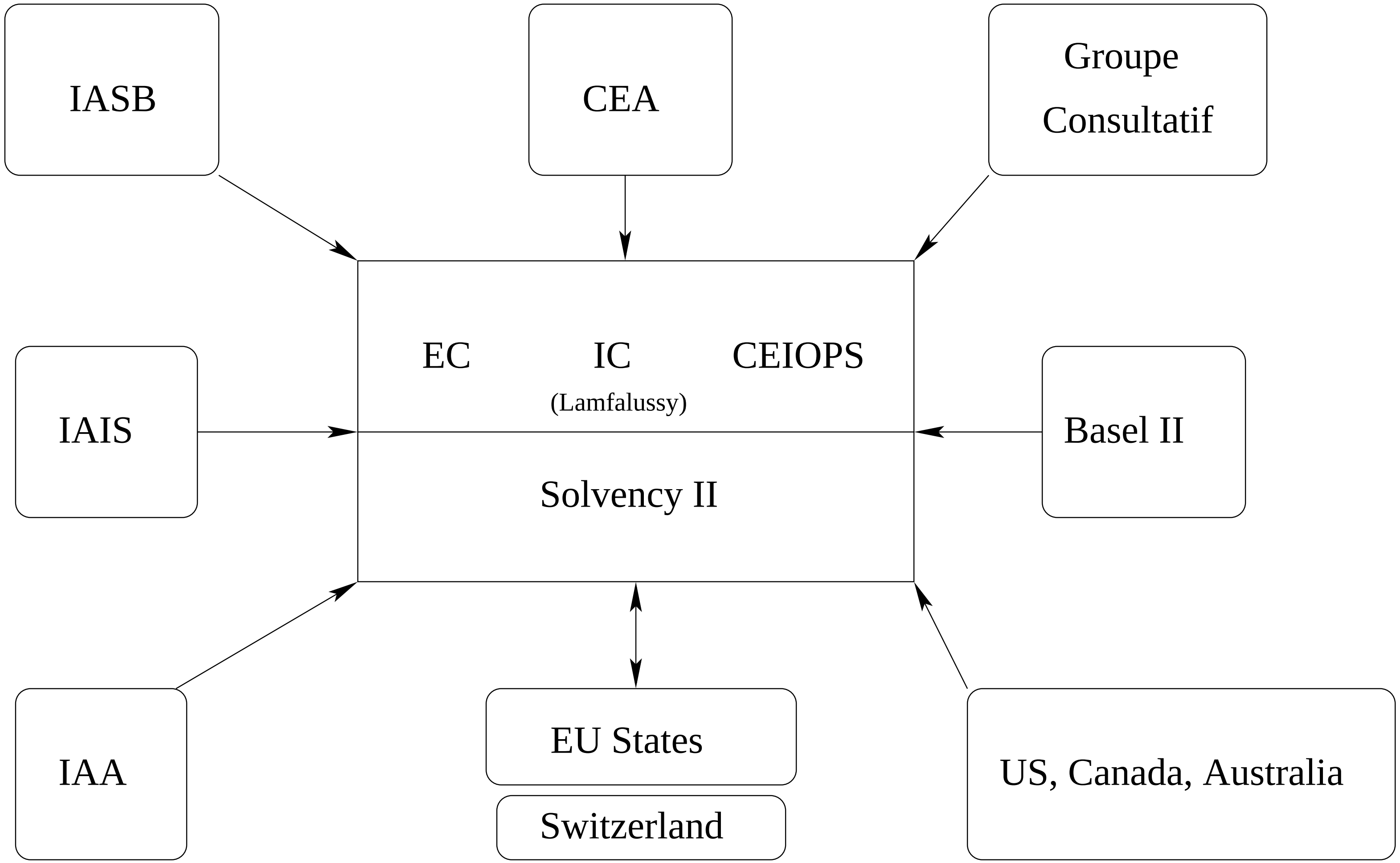
2001: Solvency II project initiated (Sharma Report, IAA WP), 2003 end of phase 1 (design of the system)

2004: Calls of advice from CEIOPS (3 waves)

2006: Draft guidelines

2008: Inforcement by European jurisdictions

Solvency II: organisation/compatibility



Purpose of Capital

- Economic capital: minimum amount of equity to ensure on-going operations of the firm

- Solvency capital requirement (SCR) = “target capital”: appropriate amount of capital to protect policyholders from (the consequences of) insolvency

- Minimum capital requirement (MCR): final threshold requiring maximum supervisory measures (“insolvency”)

IAA WP: key principles

- 3 pillar approach (target capital requirements necessary for solvency assessment but not sufficient by itself)
- all types of risks relevant for insurer to be included (insurance, credit, market and operational risk(. . .))
- principles- vs. rules-based approach
- total balance sheet approach (one risk measure on "assets minus liabilities". Avoids different levels of conservatism inherent in accouting systems.)
- appropriate risk measure (expected shortfall)
- dependencies/diversification taken into account
- standardized approaches proposed (internal models based on a set of uniform methods)
- advanced approaches (internal models based on company-specific measures of risk. Need approval)

Formal Setup

- $(\Omega, \mathcal{F}, (\mathcal{F}_t), \mathbb{P})$, $\mathbb{P} =$ real-world measure, final time horizon $T \in \mathbb{N}$
- Financial instruments $S(t) = (S_1(t), \ldots, S_m(t))$, yielding dividends $D(t) = (D_1(t), \ldots, D_m(t))$ (e.g. coupons)
- Insurance policies $P(t) = (P_1(t), \ldots, P_n(t)) =$ market-consistent values, receiving cashflows $Z(t) = (Z_1(t), \ldots, Z_n(t))$

  (no new bussines after $t = 1$)
- $G(t) := S(t) + \sum_{0<s\leq t} D(s)$ and $V(t) := P(t) + \sum_{0<s\leq t} Z(s)$ $=$ gain processes
- All future cash values are discounted with numeraire $\widetilde{S}_0(t)$: e.g. nominal value $\widetilde{S}_i(t) = \widetilde{S}_0(t) \times S_i(t)$, $\widetilde{Z}_j(t) = \widetilde{S}_0(t) \times Z_j(t)$, etc.

A simple arbitrage result

Assumptions:

- can reinsure any fraction $1 - b_j(t) \in [0, 1]$ of risk P_j at $t - 1$
- predictable trading strategy $a_i(t)$ in instrument S_i
- market value $W(t) = a(t+1) \cdot S(t) - b(t+1) \cdot P(t)$
- (a, b) is self-financing

Final result for insurer

$$W(T) - W(0) = \sum_{0<t\leq T} (a(t) \cdot \Delta G(t) - b(t) \cdot \Delta V(t))$$

Definition: (a,b) is **(negative) arbitrage** if

$$W(T) - W(0) \overset{(\leq)}{\geq} 0 \quad \text{and} \quad \mathbb{P}\left[W(T) - W(0) \overset{(<)}{>} 0\right] > 0.$$

Theorem: There is no (negative) arbitrage iff $\exists\ \mathbb{Q}_u \sim \mathbb{P}$ ($\mathbb{Q}_l \sim \mathbb{P}$) such that

1. G is a $\mathbb{Q}_u$-martingale ($\mathbb{Q}_l$-martingale)
2. V is a $\mathbb{Q}_u$-submartingale ($\mathbb{Q}_l$-supermartingale)

Remark: $P(T) = 0$

$$\Longrightarrow \mathbb{E}_{\mathbb{Q}_l}\left[\sum_{t<s\leq T} Z(s) \mid \mathcal{F}_t\right] \leq P(t) \leq \mathbb{E}_{\mathbb{Q}_u}\left[\sum_{t<s\leq T} Z(s) \mid \mathcal{F}_t\right]$$

Problem: $P(t)$ not known, no linear pricing rule

Towards market-consistent valuation

- $(\mathcal{G}_t)$ and $(\mathcal{H}_t)$ $\mathbb{P}$-independent filtrations with $\mathcal{F}_t = \mathcal{G}_t \vee \mathcal{H}_t$.
- S, D are $(\mathcal{G}_t)$-adapted ("financial risk")
- $\mathcal{H}_t$= "insurance risk", but $Z(t) = S_0(t) \times \widetilde{Z}(t)$ is $\mathcal{G}_t \vee \mathcal{H}_t$-measurable in general
- pricing measure $\mathbb{Q} \sim \mathbb{P}$ with

$$\frac{d\mathbb{Q}}{d\mathbb{P}}|_{\mathcal{F}_t} = \frac{d\mathbb{Q}}{d\mathbb{P}}|_{\mathcal{G}_t} \times \frac{d\mathbb{Q}}{d\mathbb{P}}|_{\mathcal{H}_t}$$

  such that
  - G is a $\mathbb{Q}$-martingale
  - $\mathbb{Q}|_{\mathcal{H}_t} = \mathbb{P}|_{\mathcal{H}_t}$: no risk premium in $\mathbb{Q}$ for insurance risk (LLN)

$\Rightarrow$ $(\mathcal{G}_t)$ and $(\mathcal{H}_t)$ also $\mathbb{Q}$-independent

- Market-consistent = risk-neutral valuation of cashflows:

$$S(t) = \mathbb{E}_{\mathbb{Q}}\left[S(T) + \sum_{t<s\leq T} D(s) \mid \mathcal{F}_t\right]$$

- **Define** expected liability $L_j(t)$ (=“best estimate”) by

$$\begin{aligned} L_j(t) &:= \mathbb{E}_{\mathbb{Q}}\left[\sum_{t<u\leq T} Z_j(u) \mid \mathcal{F}_t\right] \\ &= \sum_{t<u\leq T} P(t,u)\mathbb{E}[\widetilde{Z}_j(u) \mid \mathcal{H}_t] \quad \text{if } \widetilde{Z}_j \text{ and } (\mathcal{G}_t) \text{ are independent} \end{aligned}$$

with $P(t,u) := \mathbb{E}_{\mathbb{Q}}\left[\frac{1}{S_0(u)} \mid \mathcal{F}_t\right]$ = discounted zero-coupon bond price

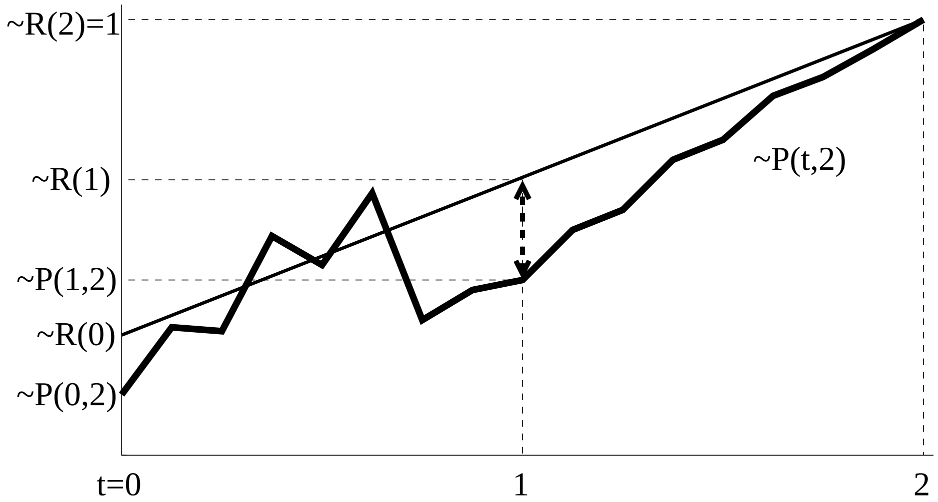
- $L_j(t)$ is not a prudential provision (LLN, no safety loading yet)

- Idea: $P_j(t) = L_j(t) + M_j(t)$, $M_j(t)$ = risk/market value margin

- **Convention:** $Z(t) \equiv \sum_j Z_j(t)$, $L(t) \equiv \sum_j L_j(t)$

Embedded options are taken into account: A simple example

- endowment insurance with term $T = 2$
- $\widetilde{R}(t)$ = statutory reserve = surrender value, $\widetilde{R}(2) = 1$
- surrender option at $t = 1$
- τ = life time of insured ($\mathcal{H}_t$-stopping time)



Cashflow (discounted):

$$Z(1) = P(1)1_{\{\tau \le 1\}} + P(1)1_{\{P(1)>P(1,2)\}}1_{\{\tau>1\}}$$
$$Z(2) = P(2)1_{\{P(1)\le P(1,2)\}}1_{\{\tau>1\}}$$

Expected liability at $t = 0$:

$$\begin{aligned}
L(0) &= \mathbb{E}_{\mathbb{Q}}[Z(1) + Z(2)] = \mathbb{E}_{\mathbb{Q}}[Z(1) + \mathbb{E}_{\mathbb{Q}}[Z(2) \mid \mathcal{F}_1]] \\
&= \mathbb{E}_{\mathbb{Q}}[P(1)]\mathbb{Q}[\tau \le 1] + \mathbb{E}_{\mathbb{Q}}[P(1)1_{\{P(1)>P(1,2)\}}]\mathbb{Q}[\tau > 1] \\
&\quad + \mathbb{E}_{\mathbb{Q}}[P(1,2)(1 - 1_{\{P(1)>P(1,2)\}})]\mathbb{Q}[\tau > 1] \\
&= P(0,1)\widetilde{P}(1)\mathbb{P}[\tau \le 1] && \text{death by } t = 1 \\
&\quad + \mathbb{E}_{\mathbb{Q}}\left[(P(1) - P(1,2))^+\right]\mathbb{P}[\tau > 1] && \text{caplet} \\
&\quad + P(0,2)\mathbb{P}[\tau > 1] && \text{terminal cashflow}
\end{aligned}$$

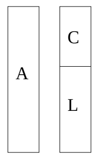
Capital structure

- Value of financial assets

$$\text{at } t-1: \quad A(t-1) = a(t) \cdot S(t-1)$$

$$\begin{aligned} \text{at } t: \quad A(t) &= a(t) \cdot (\Delta S(t) + D(t)) - Z(t) \\ &= a(t+1) \cdot S(t) \qquad \text{self-financing} \end{aligned}$$

$$\Rightarrow \Delta A(t) = \underbrace{a(t) \cdot \Delta S(t)}_{\text{price change}} + \underbrace{a(t) \cdot D(t) - Z(t)}_{\text{cashflow}}$$



- Risk-bearing capital $C(t) := A(t) - L(t)$

- Annual result

$$\begin{aligned} \Delta C(t) &= \Delta A(t) - \Delta L(t) \\ &= \underbrace{a(t) \cdot (\Delta S(t) + D(t))}_{\text{financial result}} + \underbrace{L(t-1) - Z(t) - L(t)}_{\text{insurance result}} \end{aligned}$$

$\Rightarrow$ **Theorem:** C is a $\mathbb{Q}$-martingale (Hattendorf et al).

- L is a $\mathbb{Q}$-supermartingale $\Leftrightarrow A = C + L$ is a $\mathbb{Q}$-supermartingale

- $L(T) = 0 \Rightarrow A(T) = C(T)$

$\Rightarrow C(t) = \mathbb{E}_{\mathbb{Q}}[A(T) \mid \mathcal{F}_t]$ functional of A

$\Rightarrow L(t) = \mathbb{E}_{\mathbb{Q}}[A(t) - A(T) \mid \mathcal{F}_t]$ functional of A

How to define solvency?

- Cashflow solvency: $A(t) \geq 0$ for all t.

- Balance sheet solvency: $C(t) \geq 0$ for all t

$\Rightarrow$ in both cases: target capital

$$TC = C(0) + \rho(A(0), A(1), \ldots, A(T))$$

ρ: multi-period risk measure

Coherent multiperiod risk measures (Artzner et al.)

- $\mathcal{G} = \{X \text{ bounded adapted processes}\} = L^\infty(\Omega', \mathcal{F}', \mathbb{P}')$ with
  - $\Omega' = \Omega \times \{0, 1, 2 \ldots, T\}$
  - $\mathcal{F}' = \sigma\{B_t \times \{t\} \mid B_t \in \mathcal{F}_t\}$
  - $\mathbb{P}'\left[\bigcup_{0 \le t \le T} B_t \times \{t\}\right] = \sum_{0 \le t \le T} \mu_t \mathbb{P}[B_t], \quad \mu_t \ge 0, \quad \sum_{0 \le t \le T} \mu_t = 1$

- Representation result: ρ coherent and satisfies Fatou property

$$\Rightarrow \quad \rho(X) = \sup_{f \in \mathcal{P}} \sum_{0 \le t \le T} \mu_t \mathbb{E}[-f(t)X(t)]$$

  for $\mathcal{P} \subset L^1_+(\mathbb{P}')$ closed convex with $\sum_{0 \le t \le T} \mu_t \mathbb{E}[f(t)] = 1$ for all $f \in \mathcal{P}$

- Example: multiperiod expected shortfall at level α

$$ES_\alpha[X] = \frac{1}{\alpha} \sum_{0 \le t \le T} \mu_t \mathbb{E}\left[(q_\alpha(X) - X(t))^+\right] - q_\alpha(X)$$

  where $q_\alpha(X)$ is an α-quantile of X

$$\mathbb{P}'[X < q_\alpha(X)] \le \alpha \le \mathbb{P}'[X \le q_\alpha(X)]$$

Swiss Solvency Test: minimal amount

$M(t) = \sum_j M_j(t)$
:= minimal amount that allows a healthy insurer to take over the portfolio at no additional cost
= segregated fund covering cost of future target capital (spread $sp > 0$) given financial distress:

$$M(t) = sp \sum_{t<s\leq T} ES[C(s) \mid C(s-1) = 0] \approx sp \sum_{t<s\leq T} ES[\Delta C(s)]$$

Target capital

$$TC = C(0) + ES[C(1)] + M(1) = C(0) + \rho(C)$$

where

$$\rho(C) := ES[C(1)] + sp \sum_{1<s\leq T} ES[\Delta C(s)]$$

Swiss Solvency Test: risk measure

$\rho = ES[C(1)] + sp \sum_{1<s\leq T} ES[\Delta C(s)]$ satisfies

- Translation invariance: $\rho(C + a) = \rho(C) - a$, $a \in \mathbb{R}$
- Positive homogeneity: $\rho(\lambda C) = \lambda\rho(C)$, $\lambda \geq 0$
- Sub-additivity: $\rho(C_1 + C_2) \leq \rho(C_1) + \rho(C_2)$

But ρ is not monotone: $\exists C \geq 0$ with $\rho(C) > 0$

In fact, ρ is too conservative:

$$\rho(C) \geq \underbrace{(1 - sp) \cdot ES[C(1)] + sp \cdot ES[C(T)]}_{\text{coherent risk measure}}$$

Swiss Solvency Test: computation

Computational approximation:

$$ES[\Delta C(t)] \approx ES[C(t) \mid C(t-1) = 0] \approx c(t-1)\mathbb{E}_{\mathbb{Q}}[L(t-1)]$$

where

$$c(0) = \frac{ES[\Delta C(1)]}{L(0)}$$

and

$$c(t) = w(t)\frac{ES[\Delta C(1)]}{L(0)} + (1 - w(t))\frac{ES[\Delta L(1) - Z(1)]}{L(0)}$$

linear interpolation ($w(t) \downarrow 0$) between total risk $ES[\Delta C(1)]$ and stand alone insurance risk $ES[\Delta L(1) - Z(1)]$ (optimal portfolio)

$\rightarrow$ Computational formula:

$$TC \approx ES[\Delta C(1)] + sp \sum_{1<t\leq T} c(t-1) \underbrace{\mathbb{E}_{\mathbb{Q}}[L(t-1)]}_{\text{run-off pattern}}$$

Swiss Solvency Test: scenarios

Modelling of $\Delta C(1)$: under two hypotheses ($\Theta = \theta_i,\ i = 0,1$)

- normal year ($\Theta = \theta_0$): empirical distribution
- extremal year ($\Theta = \theta_1$): scenarios $\mathcal{S}_1 \dot\cup \ldots \dot\cup \mathcal{S}_d = \{\Theta = \theta_1\}$

$$\Rightarrow ES[\Delta C(1)] = \frac{1}{\alpha}\Big((1-p)\mathbb{E}_{\theta_0}[-\Delta C(1)1_A] + p\mathbb{E}_{\theta_1}[-\Delta C(1)1_A]\Big)$$

where

$$A := \{\Delta C(1) < q_\alpha\}, \quad \mathbb{P}[\Delta C(1) = q_\alpha] = 0, \quad p := \mathbb{P}[\Theta = \theta_1]$$

Hence

$$\mathbb{E}_{\theta_1}[-\Delta C(1)1_A] = \sum_{i=1}^{d} \mathbb{E}[-\Delta C(1)1_A \mid \mathcal{S}_i]\frac{\mathbb{P}[\mathcal{S}_i]}{p}$$

Industrial

Explosion with emission of toxic gases

Example: fertiliser plant in Toulouse, 2001

Scenario:

injureds, disableds, deads, property damage, environment damage, production interrupt

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Pandemic ("Spanish Flu" 1918 in 2004)

	Kinder	Gesunde Erwachs. (15-49)	Gesunde Erwachs. (50-65)	Ältere	Erwachs. mit hohem Risiko (15-65)	Erwachs. mit hohem Risiko (>66)	Personen im Gesundheitswesen	Total
Bevölkerung	1'249'000	3'155'000	1'080'000	700'000	383'000	328'000	269'000	**7'164'000**
Anzahl Kranke	1'001'136	2'242'890	485'603	228'701	226'314	107'163	173'252	**4'465'059**
Arztvisiten	508'549	966'972	210'059	123'902	128'886	66'497	78'093	**2'082'958**
Hospitalisierung	2'928	13'287	1'884	2'824	8'317	2'570	1'411	**33'221**
Betttage	20'555	25'592	6'404	25'641	76'694	58'961	8'857	**222'704**
Tote	4'831	10'295	3'521	3'072	4'995	14'190	1'096	**42'000**
verlorene Arbeitstage	0	8'519'486	1'836'142	0	921'977	0	849'512	**12'127'117**

Insurers determine impact of pandemic on their portfolio.

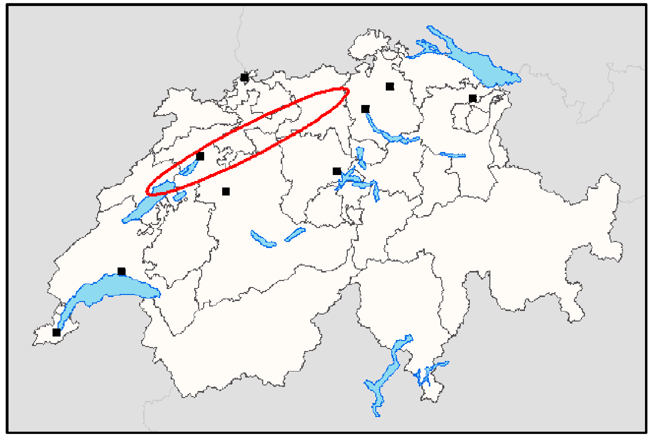
Scenario is based on a study of the Swiss Federal Office of Public Health.

The Economics of Pandemic Influenza in Switzerland, Prepared by MAPI VALUES for The Swiss Federal Office of Public Health, Division of Epidemiology and Infectious Diseases, Section of Viral Diseases and Sentinel Systems, James Piercy / Adrian Miles, March 2003

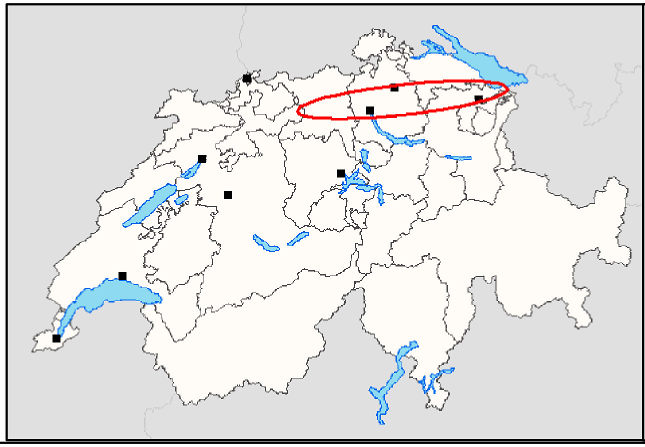
Hail

Hail scenario: 4 hail tracks with given loss severities for ZIP codes (footprints)

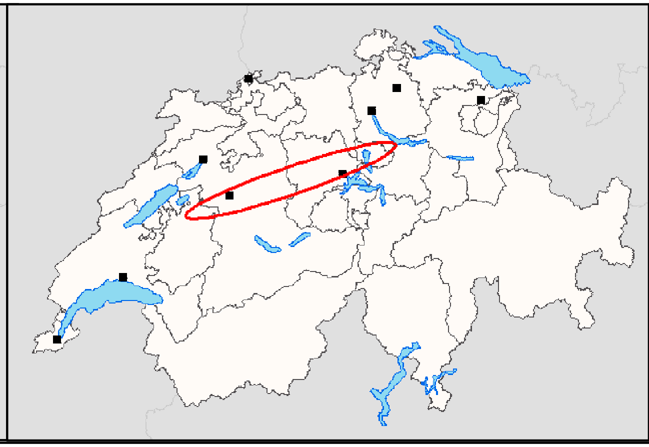
Neuchâtel - Biel - Grenchen - Solothurn - Olten - Aarau



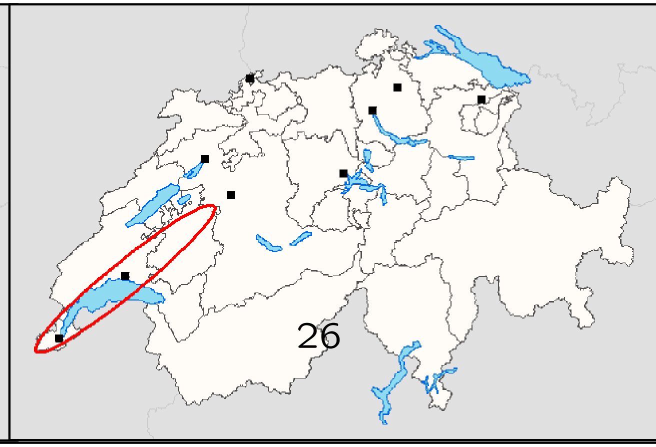
Aarau - Zürich - Winterthur - St. Gallen



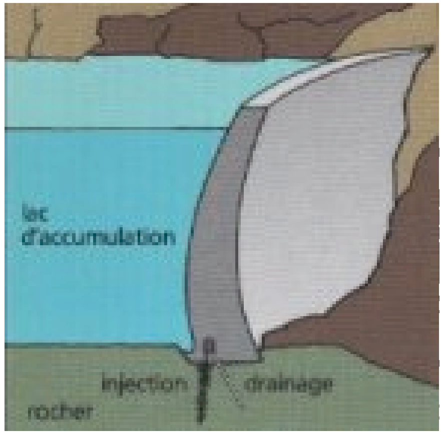
Bern - Luzern - Zug

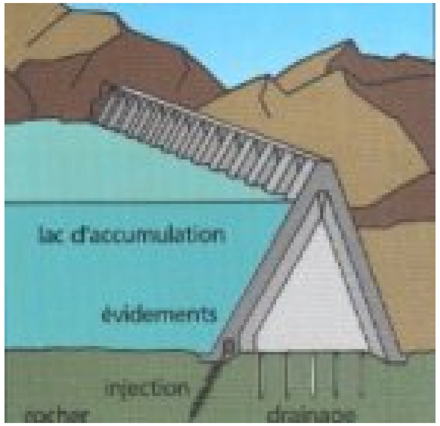


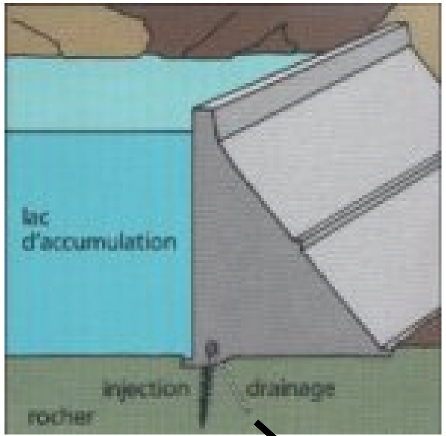
Genève - Lausanne - Fribourg

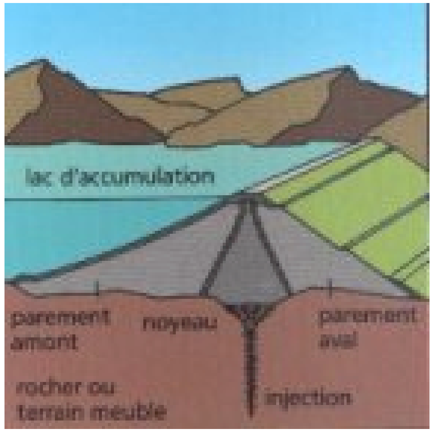


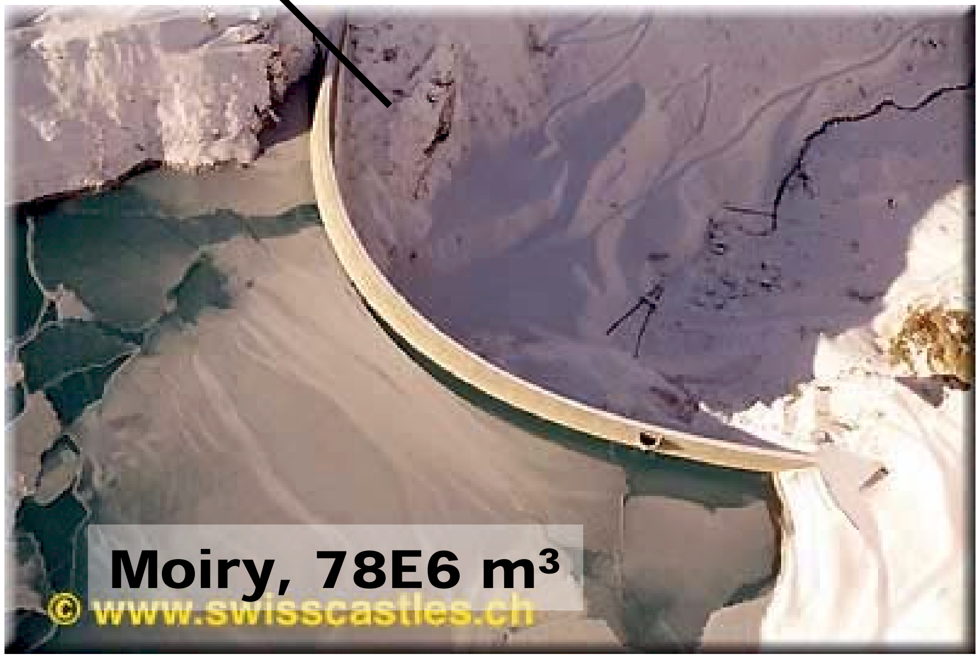
Burst of a Dam









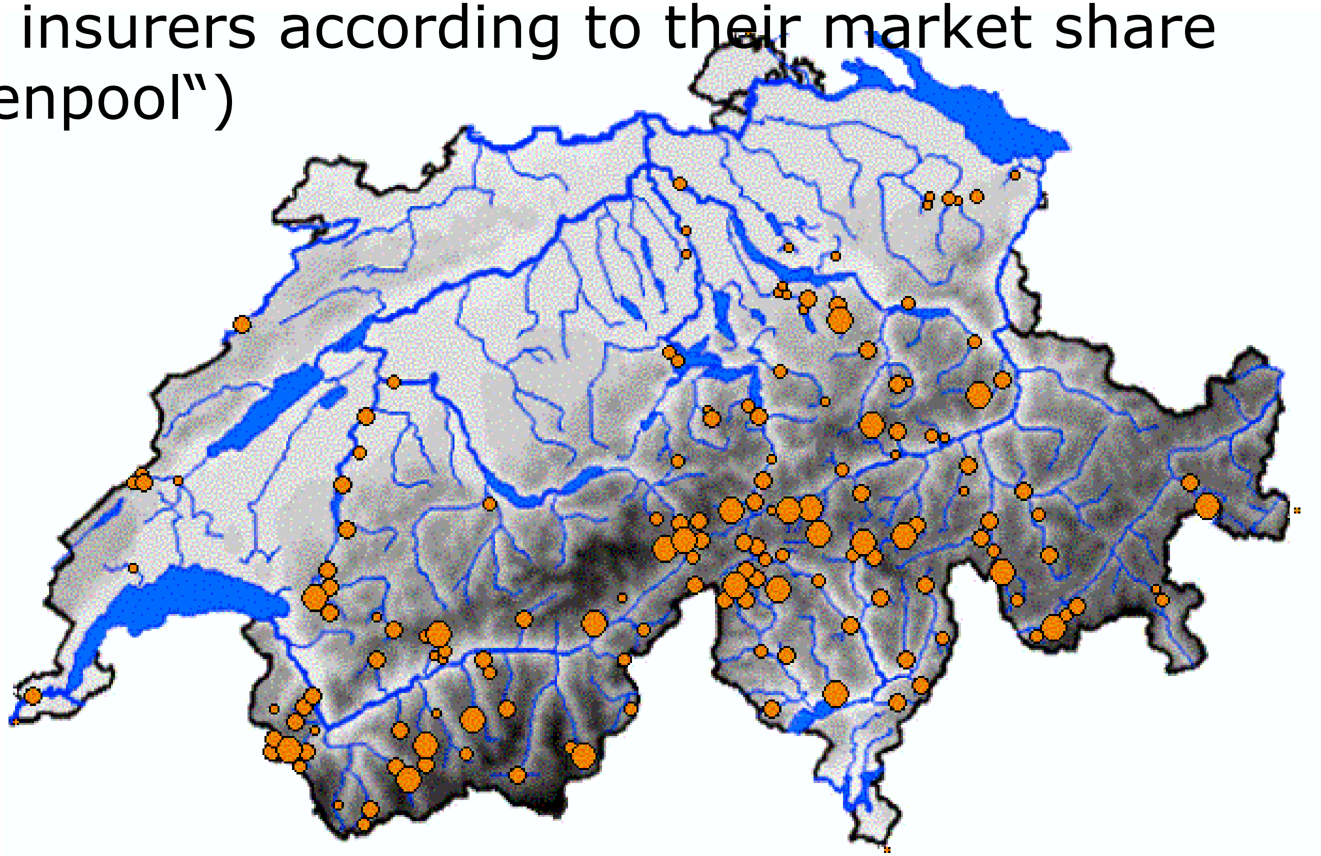


Burst of a Dam

Sum insured: 200 M CHF for each large dam,

Probability of occurance per year 0.3%

Carried by insurers according to their market share („Talsperrenpool“)



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Map of large dams in Switzerland

Swiss Solvency Test: scenario evaluation

Evaluation of scenarios $\mathbb{E}[-\Delta C(1) 1_A \mid \mathcal{S}_i]$

Simple ansatz:

$$\Delta C(1) \mid \mathcal{S}_i \sim -c_i + d_i \cdot Y$$

where

- c_i = expected additional loss amount
- d_i = scaling factor (change of volatility)
- $Y \sim \Delta C(1) \mid \{\Theta = \theta_0\}$: normal year distribution

Example hailstorm: footprints for loss severities, c_i according to market share