Modelling Energy Forward Curves

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Energy markets

- Pre-1980s: regulated energy markets
- 1980s: deregulation of oil and natural gas industries
- 1990s: deregulation of electricity industries worldwide

- Energy is the world's largest traded commodity class
- Crude oil is the world's largest commodity
- Energy markets are extremely volatile (annual volatilities: Oil 40+%, NG 60+%, Electricity 100+% $\longrightarrow$ comp. with 15+% for equity indices)

$\Longrightarrow$ need for efficient risk management

- Energy prices are negatively correlated to the stock prices and indices

$\Longrightarrow$ perfect diversification tools

What is traded?

Physical crude oil, oil products, NG (spot markets); forward contracts (OTC);

Futures contracts on ICE, NYMEX: volumes 9-10 times higher than those in spot markets!

Futures contracts and forward curves

- *Futures*: standardized contracts for delivery of a commodity (e.g. crude oil) at different time points (expiries) in the future.

- Prices for futures with different expiries (e.g., for oil, up to 72 months ahead) are recorded daily.

- The collection of these futures prices on any particular day is called the *forward curve*.

- The set $\{F(t,T), T > t\}$ is the forward curve prevailing at date t for a given commodity in a given location; T indicates the expiry, or maturity date (month).

- The forward curve is the fundamental tool when trading commodities, as spot prices may be unobservable and options illiquid.

Benefits of forward curves

- Forward curves reflect market fundamentals and anticipated price trends

- Benchmark for Valuation: Deal Pricing, P & L

- Internal Consistency in the desk or the firm with other derivatives

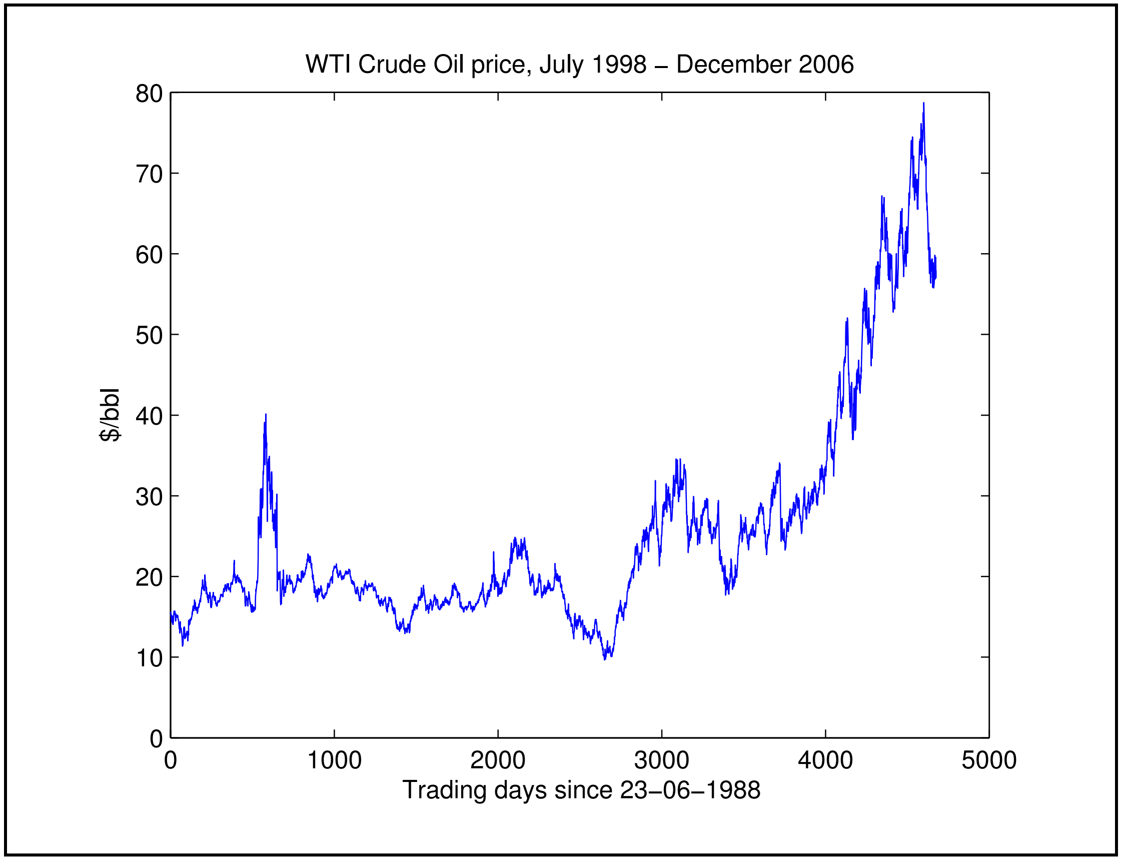
- Mark To Market, Stop Loss, VaR

- The forward curves provide the calibration of the model parameters under the pricing measure

- Commodity portfolios contain futures with different expiries
$\longrightarrow$ risk exposure to movements of the entire forward curve

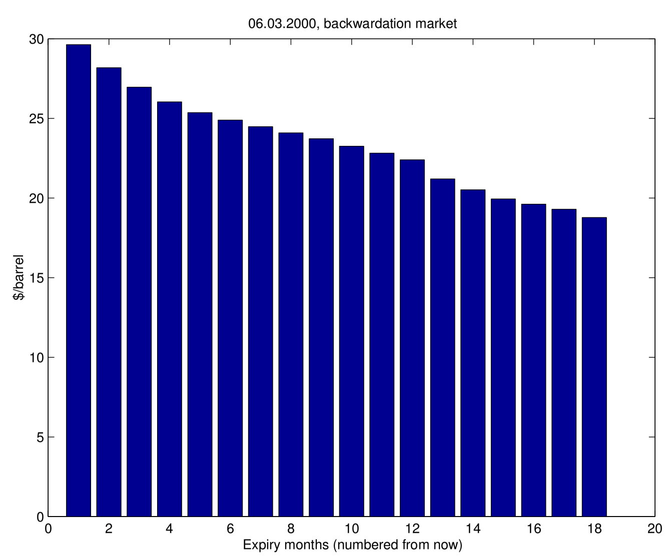
- Pricing of derivatives on futures and forwards requires forward curve models

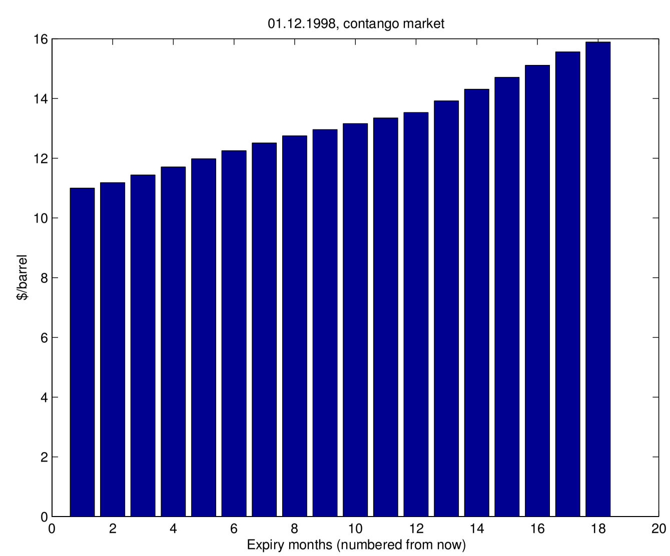
The oil price in the last two decades



Oil forward curves: two fundamental market states Backwardation and Contango

Anticipated value of the future spot price is lower (B) or higher (C) than the current one. Influenced by: current price and inventory levels, transportation and storage costs, supply/demand, strategic and political reasons, ...





Changing face of oil market: arrival of hedge funds

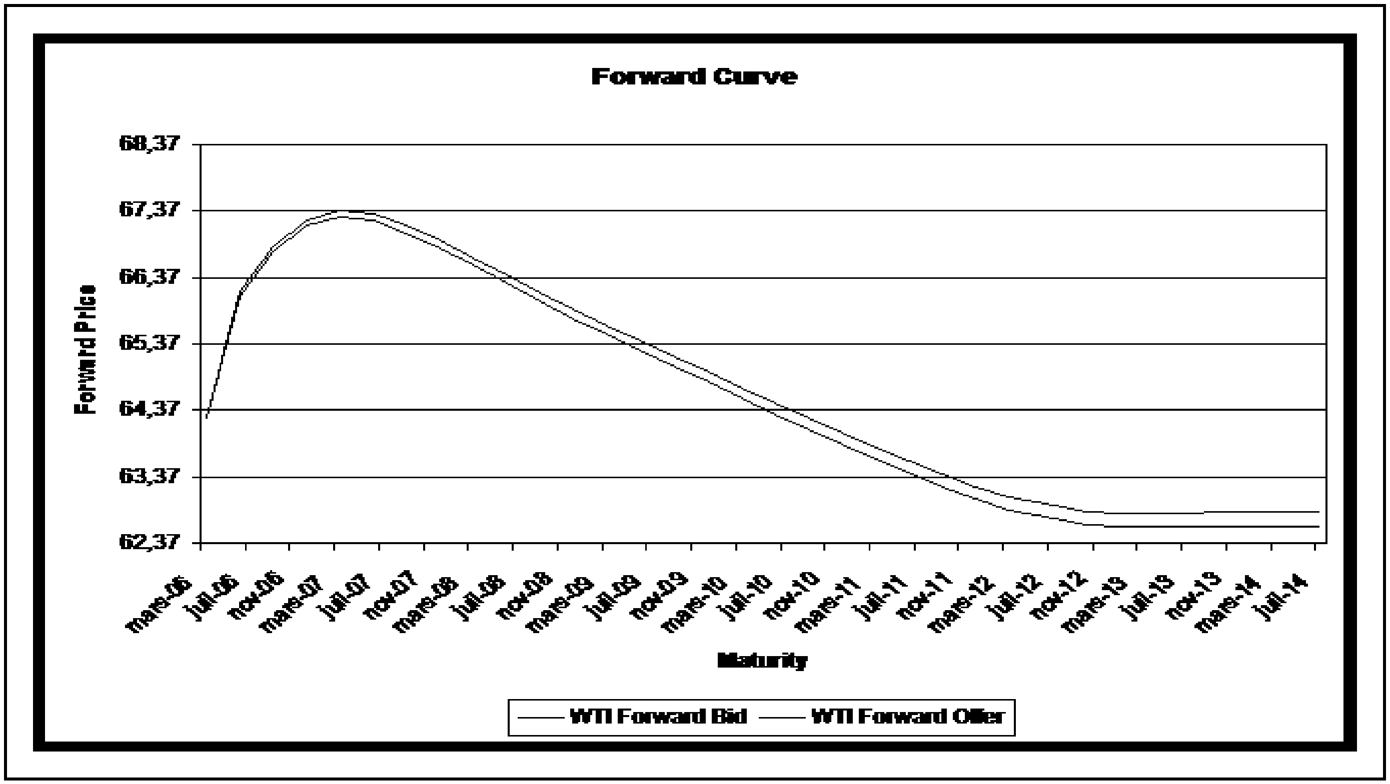
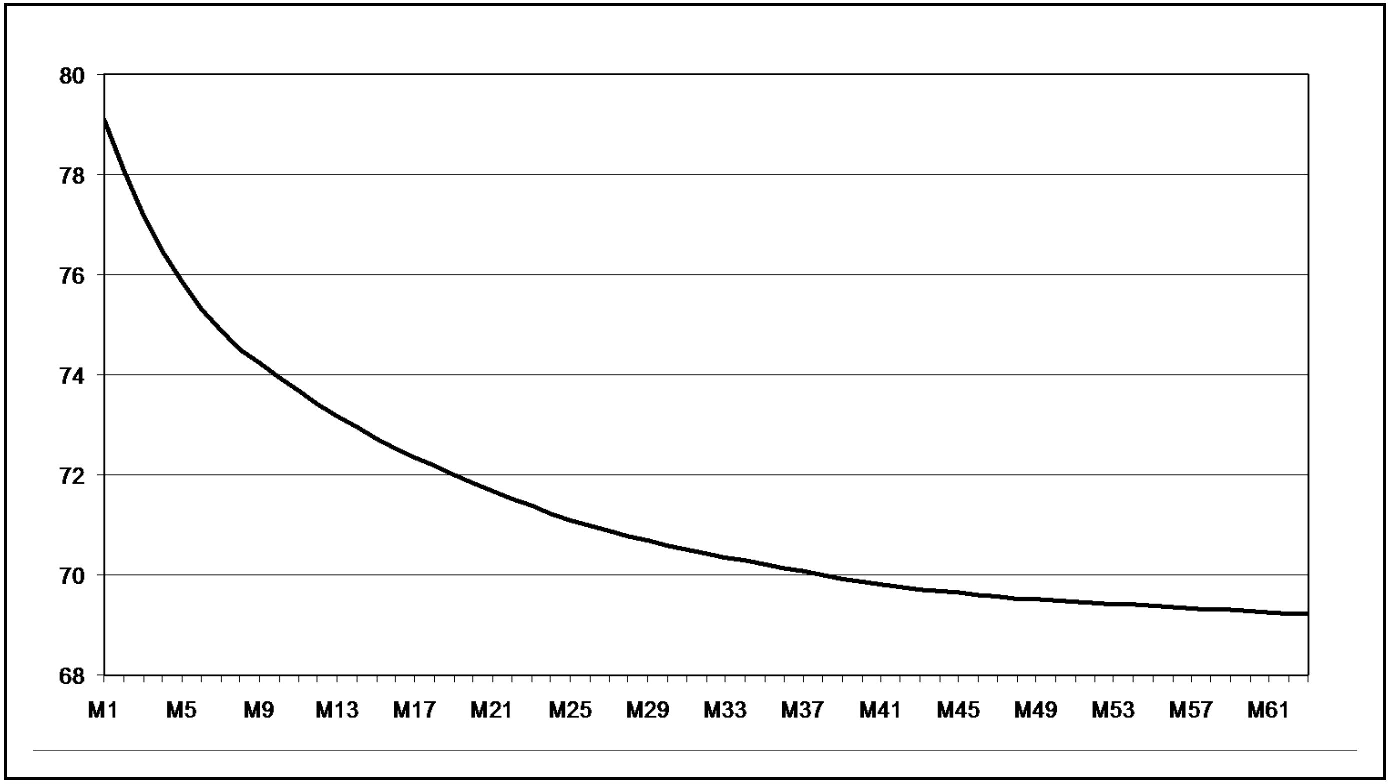


Figure 1: WTI Oil forward curve, March 2006

September 2007



Seasonality in commodity prices

- For oil, seasonality is not significant, since tankers are rerouted to satisfy a surge of demand in a given region

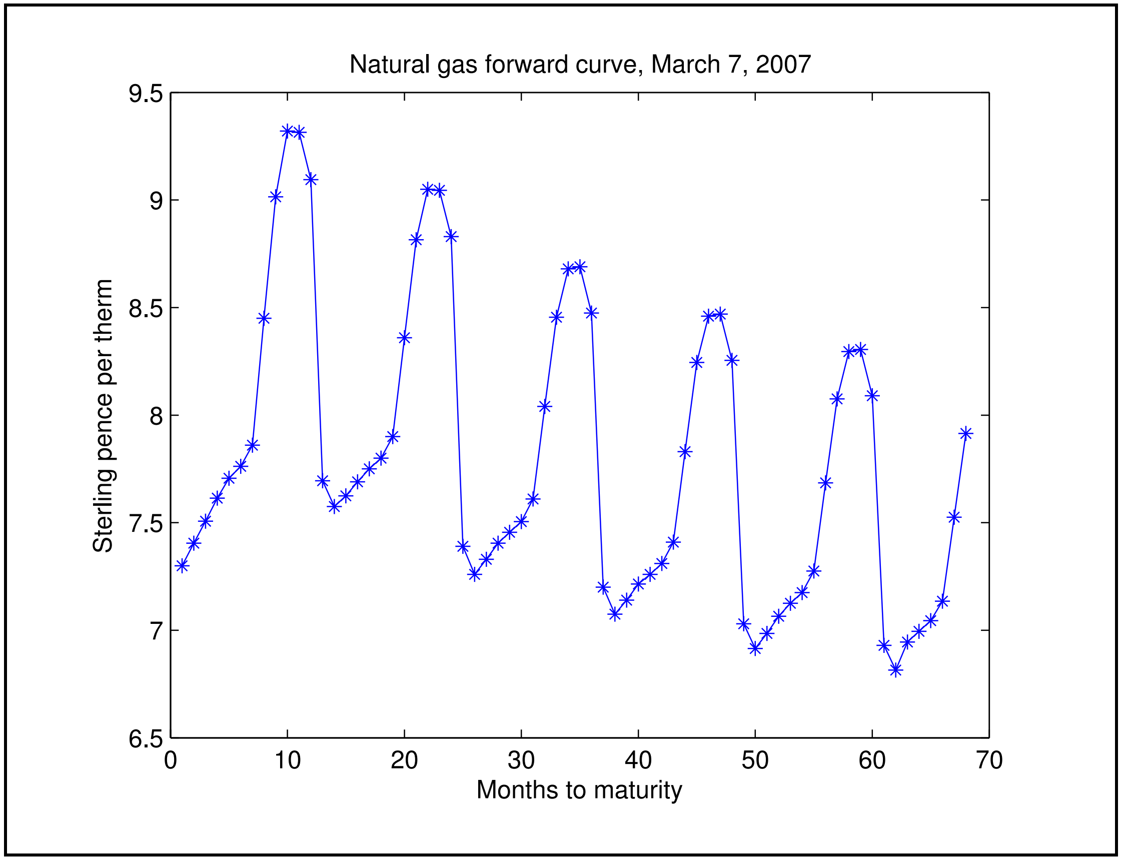
- Energy (electricity, natural gas, spark spread) - governed by seasonal demand

- Agricultural commodities (wheat, soybean, soymeal, crush spread, coffee, cocoa) - governed by seasonal supply

- Seasonality in energy or agricultural spot prices:
well-understood and easily modelled (e.g. mean-reversion with seasonally varying mean, seasonal component + autoregression)

- Seasonality in forward curves:
much less studied; no "explicit" models

Example of Natural Gas forward curve:



Futures vs. spot prices: Theory of Storage

Cost-of-carry relationship (no-arbitrage arguments):

$$F(t,T) = S(t)e^{[r(t)+w(t)](T-t)} \qquad (*)$$

$r(t)$: spot interest rate, $w(t)$: marginal storage costs per $ of spot, per time unit.

In practice $(*)$ almost never holds (e.g. backwardation or hump-shaped forward curves): strategic importance (oil); limited or non-storability (agricultural, electricity)

Convenience of having physical commodity as opposite to futures contract
$\longrightarrow$ concept of *convenience yield* $y(t)$:

$$F(t,T) = S(t)e^{[r(t)-y(t)](T-t)}$$

- *premium (as perceived on the day t) earned by an owner of physical commodity as opposite to an owner of the futures contract with maturity T.*

Convenience yield

- Often considered net marginal storage costs: $y(t) = \tilde{y}(t) - w(t)$.
- Convenience yield proportional to the spot price $y(S)$: Brennan & Schwartz (1985).

- Stochastic convenience yield $y(t) = y(t, \omega)$: Gibson & Schwartz (1990), Schwartz (1997).

- Dependence on t: "premium" to owner of physical commodity changes with inventories (stocks) and hence, with agents' preference for physical rather than paper.

- At a fixed date t, a single value of the process $(y(t))_t$ *for all maturities* is not compatible with the hump-shaped forward curve observed in 2006 in the oil market (and other commodity markets), or with seasonal features of the forward curve.

Theory of Storage revisited

- One possible modelling answer is to introduce a term structure $y(t,T)$ of convenience yields at date t, deterministic in the maturity argument T and stochastic in t (Borovkova & Geman, 2006, 2007)

- This approach is certainly beneficial in the case of seasonal commodities such as natural gas where, assuming today $=$ January 2008, $y(t,T)$ should be different for $T =$ September 2008 or $T =$ December 2008.

- Dependence of convenience yield on maturity T $(y(t,\underline{T}))$: to emphasize seasonality of $F(t,T)$ in

$$F(t,T) = S(t)e^{[r(t)-y(t,T)](T-t)}$$

e.g. futures expiring at "desirable" season (e.g. NG futures expiring in December)

Emphasizes the *time-spread option* feature of convenience.

Forward curve models

One, two and three factor models: spot price, convenience yield and interest rate (Black (1976), Gibson & Schwartz (1990), Schwartz (1997))
Futures prices are derived by no-arbitrage arguments: $F(t,T) = E_Q[S(T)|\mathcal{F}_t]$.

Seasonal commodities (Sorensen (2002) and Lucia & Schwartz (2002)):
Two-factor models with seasonal spot price and a long-term equilibrium price.
Seasonality enters the futures price, but not in an explicit and consistent way.

One step forward: Amin, Ng & Pirrong (1994): seasonal (but deterministic) convenience yield, one fundamental factor: spot price, cost-of-carry relationship.

Main drawbacks of all above models:
Spot price is not a good indicator of overall state of the market.
Forward curve's seasonal features are not taken into account explicitly $\Longrightarrow$
Models do not match observed forward curves.

Seasonal cost-of-carry model: First fundamental factor

The average level of the forward curve, or the average forward price prevailing at date t:

$$\bar{F}(t) = \sqrt[N]{\prod_{T=1}^{N} F(t,T)}, \quad \text{or} \quad \ln \bar{F}(t) = \frac{1}{N} \sum_{T=1}^{N} \ln F(t,T),$$

where N: maximum liquid maturity.

- Assume: $(N \text{ mod } 12) = 0$, i.e. consider maturities up to a (number of) year(s)

$\longrightarrow$ that way $\bar{F}(t)$ is not seasonal.

- Other ways of constructing a non-seasonal $\bar{F}(t)$, so the assumption can be relaxed
- Not limited to regularly spaced maturities but can include all traded liquid maturities
- Can include all (not liquid) maturities, by considering traded-volume–weighted average

Seasonal cost-of-carry model: Seasonal premium

The *seasonal premium* $(s(M))_{M=1,\ldots,12}$ is the collection of *long-term–average premia* (expressed in %) *on futures expiring in the calendar month* M $(M = 1, \ldots, 12)$ *with respect to the average forward price* $\bar{F}(t)$.

- Assume $(s(1), \ldots, s(12))$ is the deterministic collection of 12 parameters;
- Require that $\sum_{M=1}^{12} s(M) = 0$;
- Seasonal premium is an absolute quantity and not a rate: premium on futures expiring in July is the same whether today is June or December. Premium on July 2008 futures is the same as on July 2009 futures.
- Can be defined as a continuous-time periodic function (e.g. trigonometric); however less appropriate for monthly expiries.

Seasonal cost-of-carry model: The model

For any maturity T, we write

$$F(t,T) = \bar{F}(t)e^{[s(T)-\gamma(t,T)(T-t)]}, \qquad (*)$$

where $\gamma(t,T)$, *defined by the relationship* $(*)$, is called the *stochastic convenience yield net of seasonal premium*, for maturity T, as perceived on the day t.

Seasonal (monthly) premium (or discount): in $s(T)$
Stochastic factors influencing forward prices: in $\gamma(t,T)$

The relationship $(*)$ involves *only* forward prices, hence *no* interest rates.

Features of the model

Relationship to classic convenience yield models:

$$\gamma(t,T) - \frac{s(T)}{T-t} = y(t,T) - \frac{1}{N}\sum_{K=1}^{N} y(t,K)$$

$\longrightarrow$ $\gamma(t,T)$ can be interpreted as *the relative convenience yield net of the (scaled) seasonal premium.*

• Convenience yield γ can be used for non-storable commodities (e.g. electricity), since spot price plays no role
• If $\gamma(t,T) \equiv 0$ $\longrightarrow$ one-factor model driven by $\bar{F}(t)$ and deterministic $s(T)$
• If $s(T) = 0 \quad \forall T$, then no deterministic seasonality (e.g. oil) and $\gamma(t,T)$ is the "relative convenience yield" $\longrightarrow$ two-factor model similar to Gibson & Schwartz (1990) but with $\bar{F}(t)$ instead of the spot price.

Dynamics of fundamental factors and futures prices

$\bar{F}(t)$ is not seasonal by construction
$\longrightarrow$ can be modelled as a mean-reversion with constant mean, or GBM.
$\gamma(t,T)$ is essentially zero (on average), since all systematic deviations are in $s(T)$
$\longrightarrow$ can be modelled as a mean-reversion with mean zero.

All stochastic convenience yields $(\gamma^T(t))_{T=1,\dots,N}$ are driven by the same Brownian motion, independent of the BM driving the average forward price.

Seasonal cost-of-carry + dynamics of $(\bar{F}(t), \gamma^T(t))$ = dynamics of $(F(t,T))_T$.
Resulting futures prices $F(t,T)$ are log-normal with instantaneous proportional variance

$$\xi^2(t,T) = \sigma^2 + (\eta^T(T-t))^2 - 2\sigma\rho\eta^T(T-t)$$

Model estimation

Historical data of daily forward curves $(F(t,1), \ldots, F(t,12))_{t=1,\ldots,n}$.

Estimate

- the daily average forward price by $\ln \bar{F}(t) = \frac{1}{12}\sum_{M=1}^{12} \ln F(t,M)$;
- the seasonal premia $(s(M))_M$, according to the definition, by

$$\hat{s}(M) = \frac{1}{n}\sum_{t=1}^{n} [\ln F(t,M) - \ln \bar{F}(t)], \quad M = 1, \ldots, 12,$$

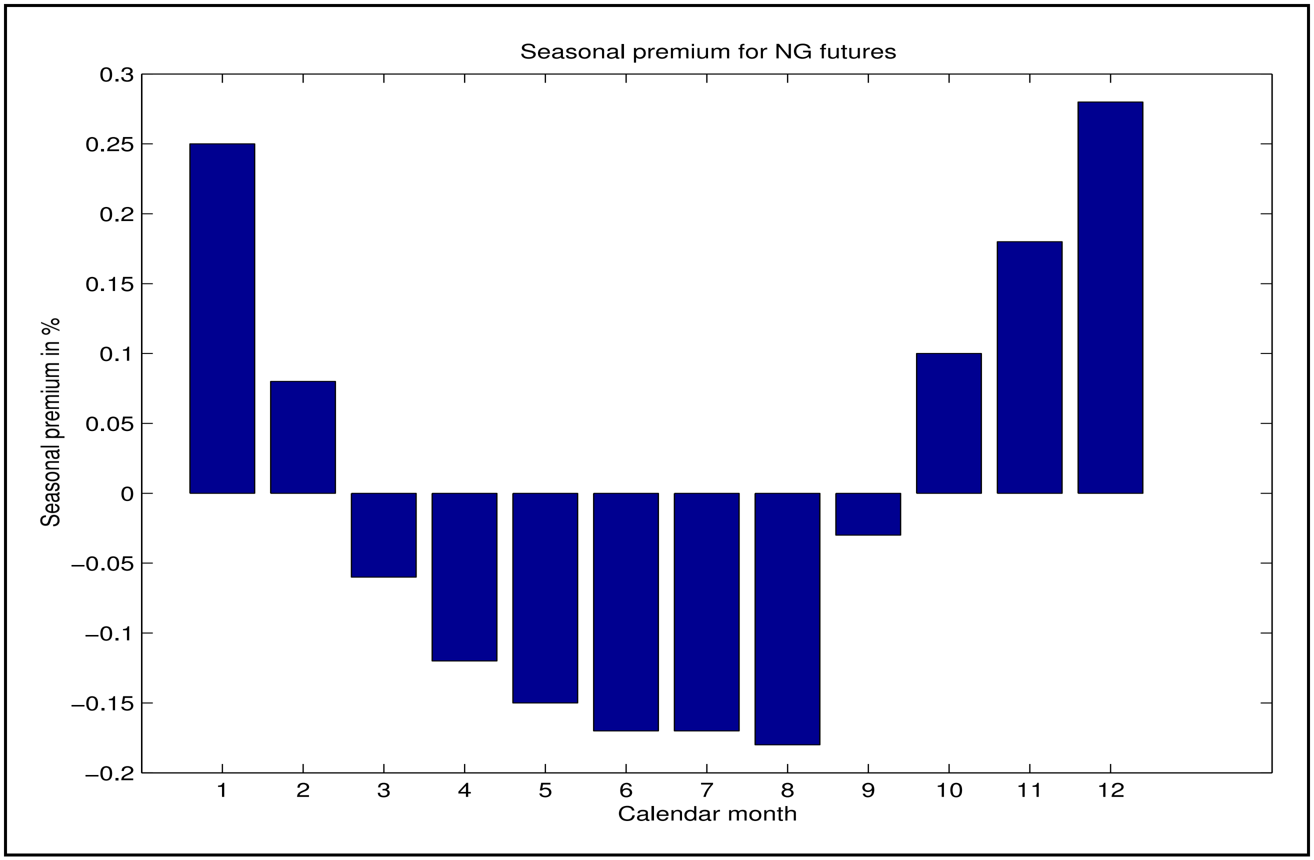
- the stochastic convenience yield by

$\hat{\gamma}(t,T) = (-\ln(F(t,T)/\bar{F}(t)) + \hat{s}(T))/((T-t))$.

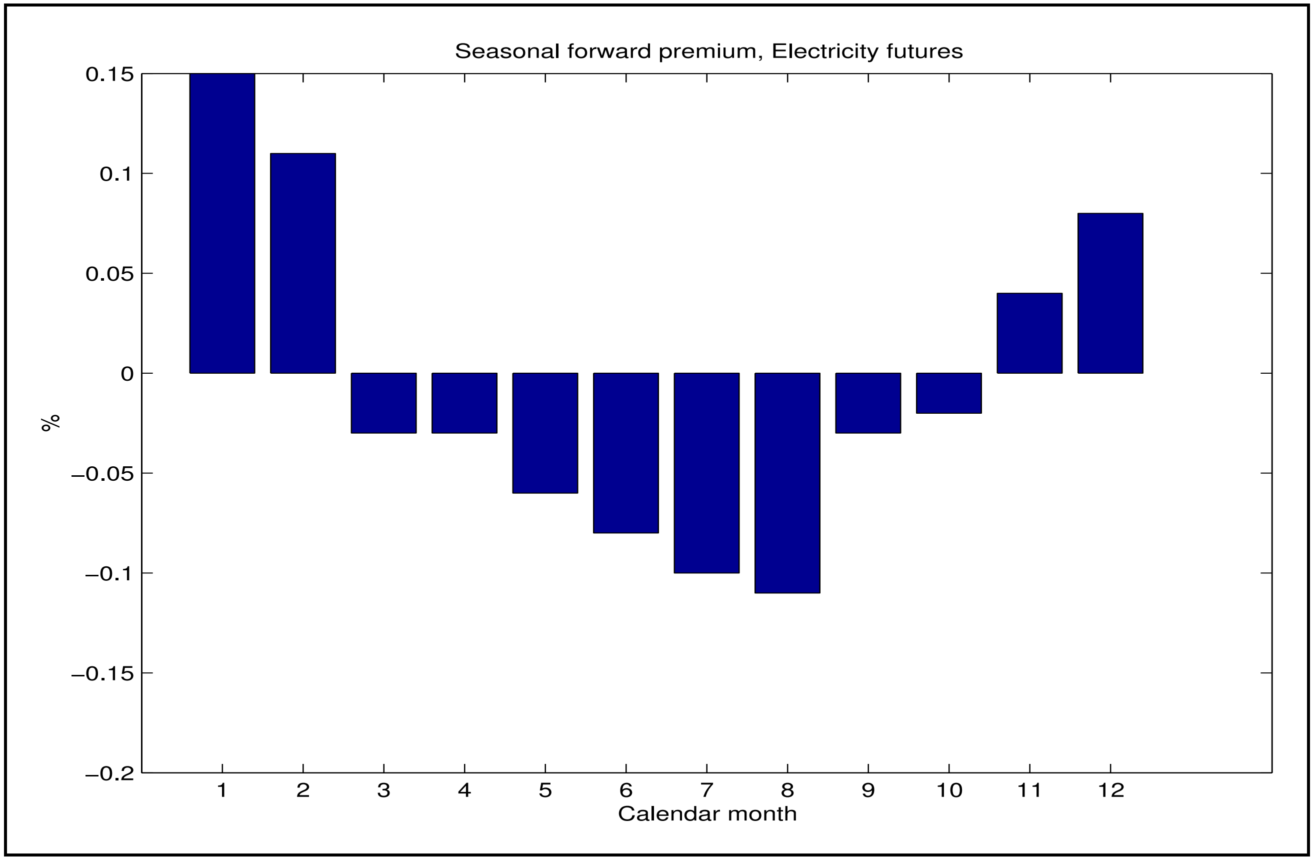
More than 12 maturities: easily incorporated, but if fewer than 12 maturities, the unbiased estimate for $\bar{F}(t)$ is not available

$\longrightarrow$ a more complicated estimation procedure.

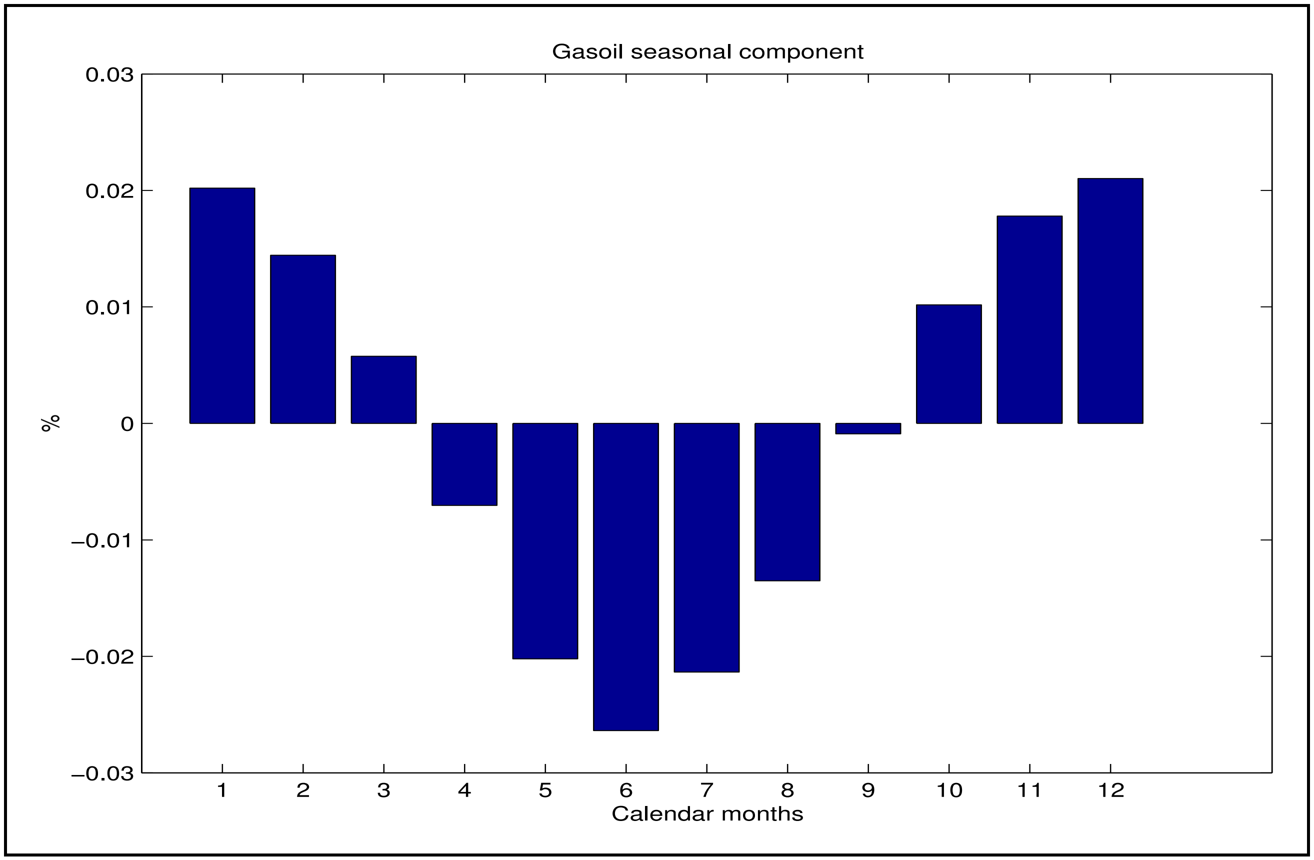
Seasonal premium for Natural Gas futures



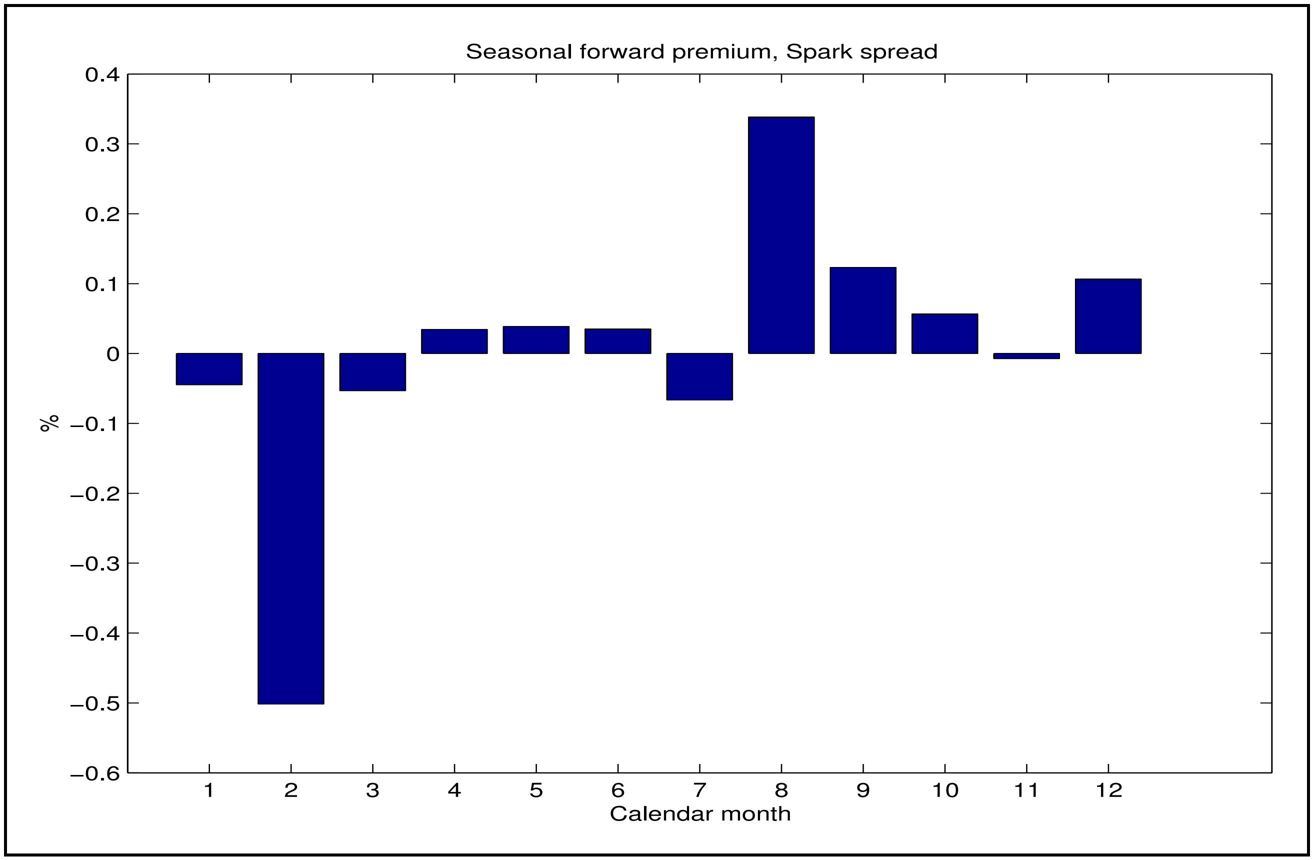
Seasonal premium for electricity futures



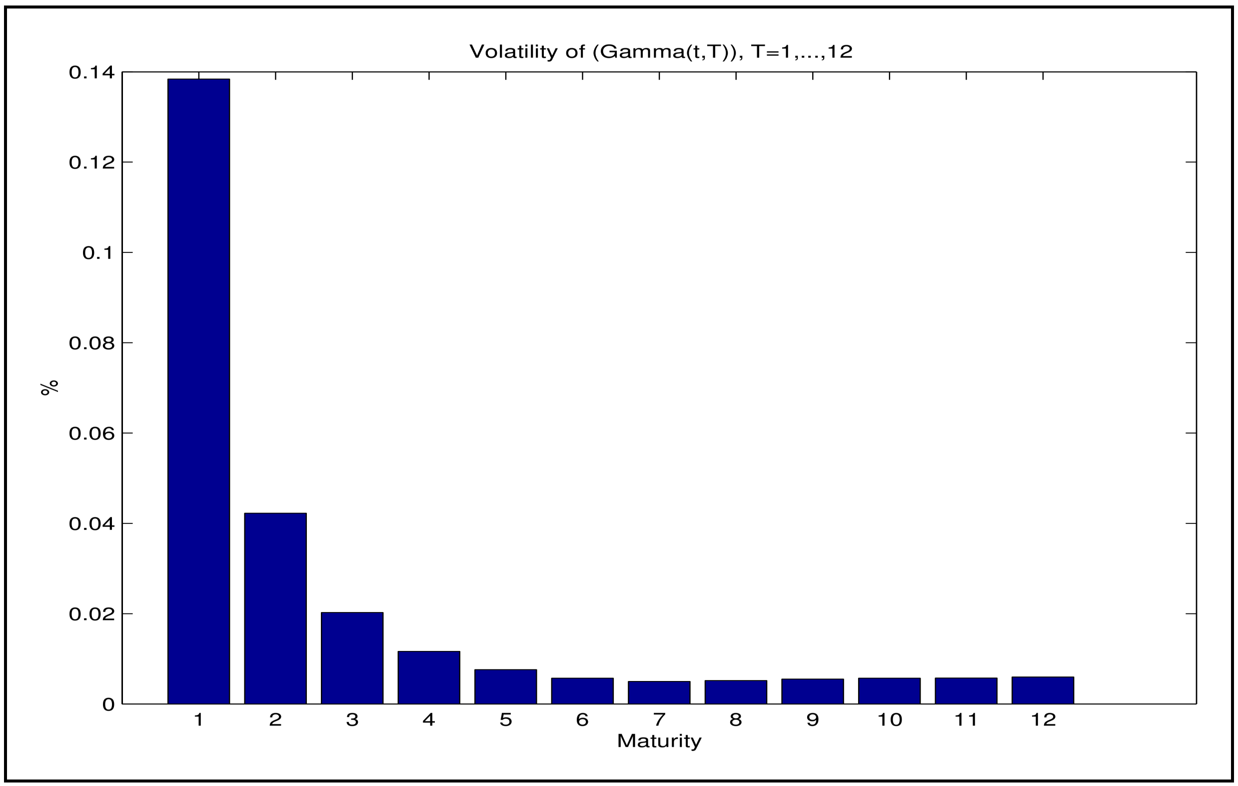
Seasonal premium for Gasoil futures



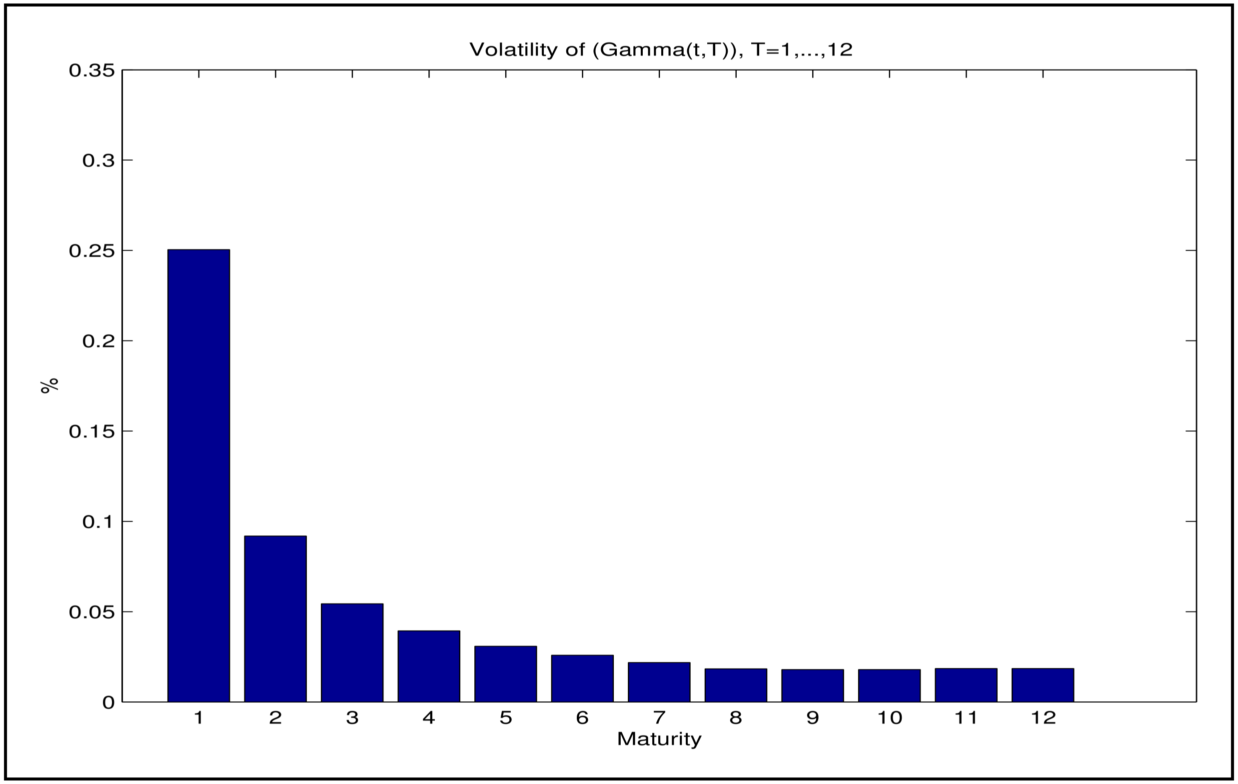
Seasonal premium for spark spread



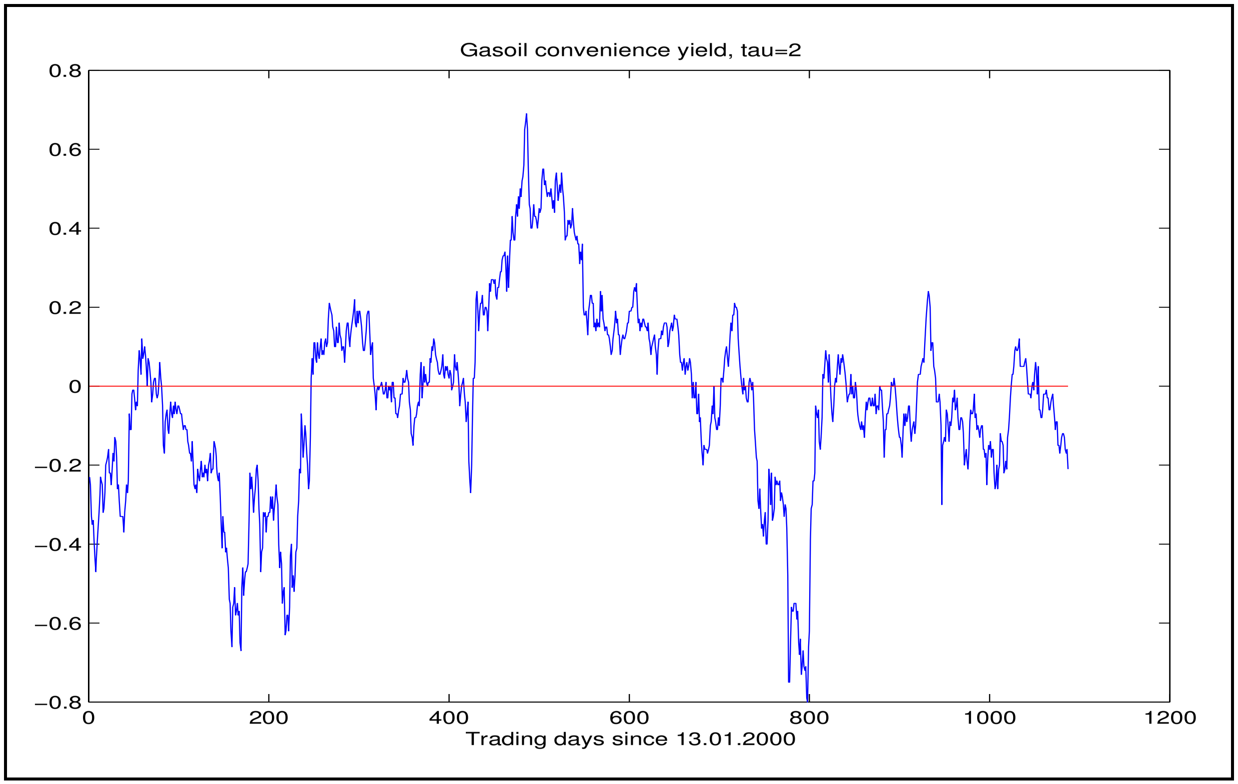
Term structure of stochastic forward premium volatilities, Gasoil futures



Term structure of stochastic forward premium volatilities, Natural Gas futures



The second state variable (stochastic forward premium), for two months to maturity, Gasoil futures, Jan. 2000 - Dec. 2004



Properties of the convenience yield

• All observed series $(\gamma(t,T))_t$ can be modelled by low-order autoregression (order 2 - 5) $\longrightarrow$ autoregressive structure can be exploited for
- forecasting the stochastic convenience yield
- forecasting market conditions
- devising market indicators
- generating profitable trading strategies

• Convenience yield can be regressed on economic fundamentals and exogenous market variables, e.g. supply/demand, volatility, ...

Mean-reversion parameters $(T = 2)$:

	electricity	gas	gasoil	oil
a^T:	0.07	0.09	0.02	0.01
η^T:	0.16	0.10	0.05	0.04

Relationship of $\gamma(t, T)$ to market indicators and economic fundamentals

Theory of storage + empirical considerations
$\longrightarrow$ conjectures about the convenience yield:

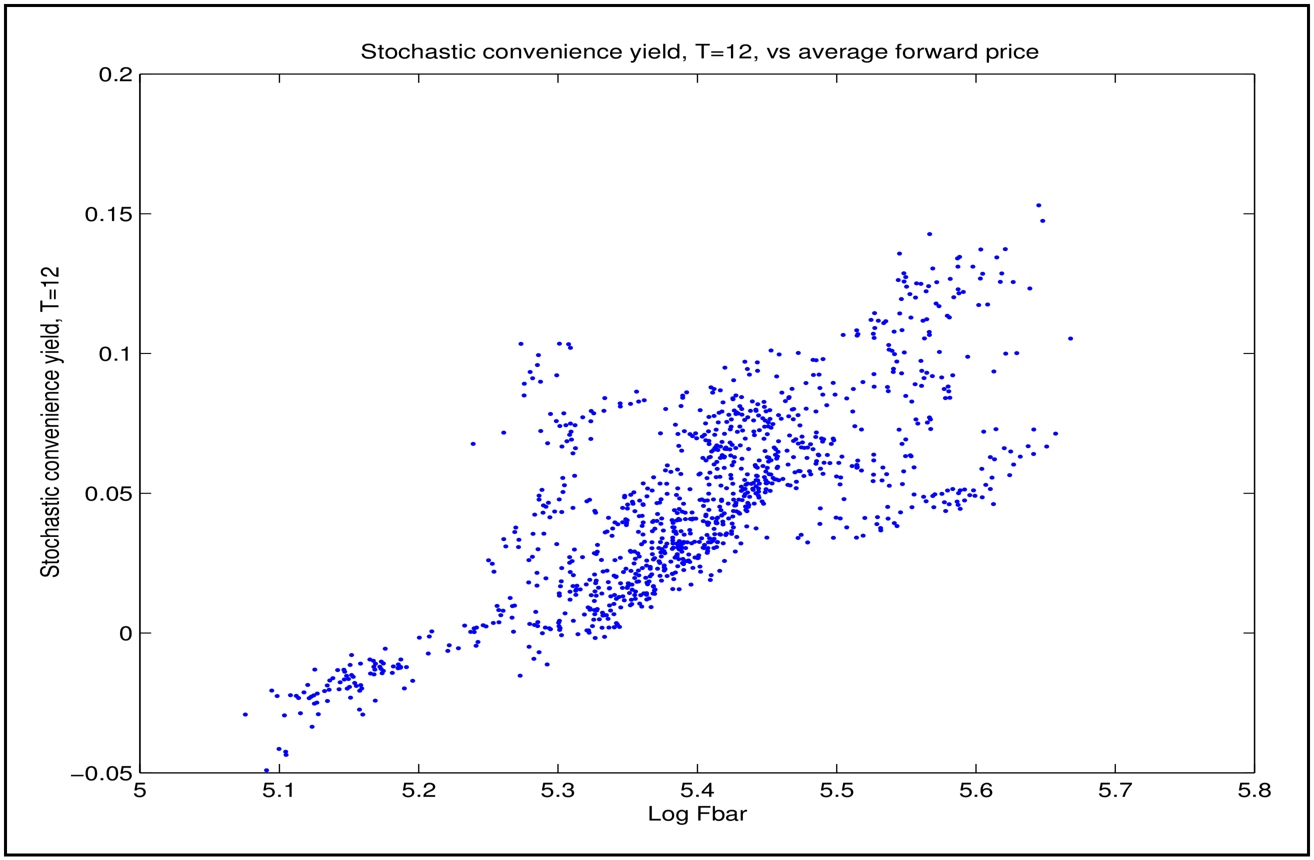
I. It is positively correlated to the overall price level (given by either spot price or average forward price)

II. It is negatively correlated to inventories

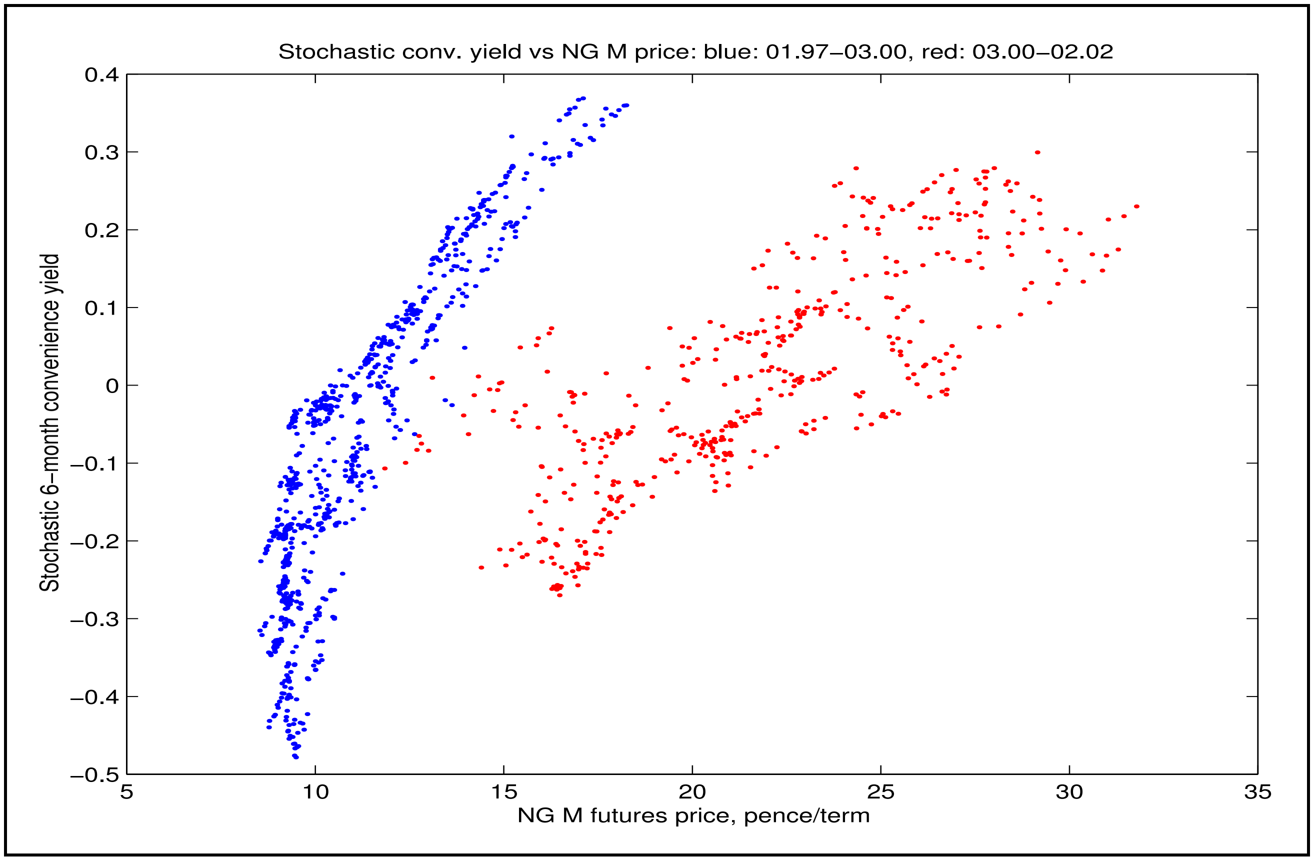
III. It is positively correlated to spot price's volatility

IV. It is negatively correlated to the correlation between spot and futures prices.

Conjecture I: true, especially for higher maturities: Gasoil

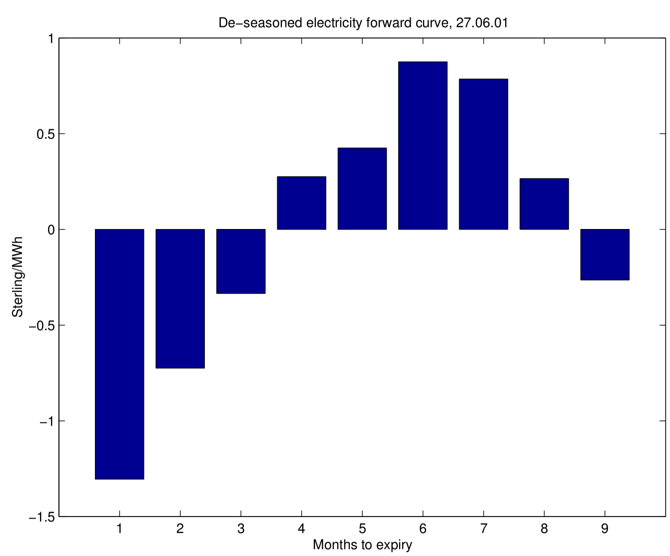


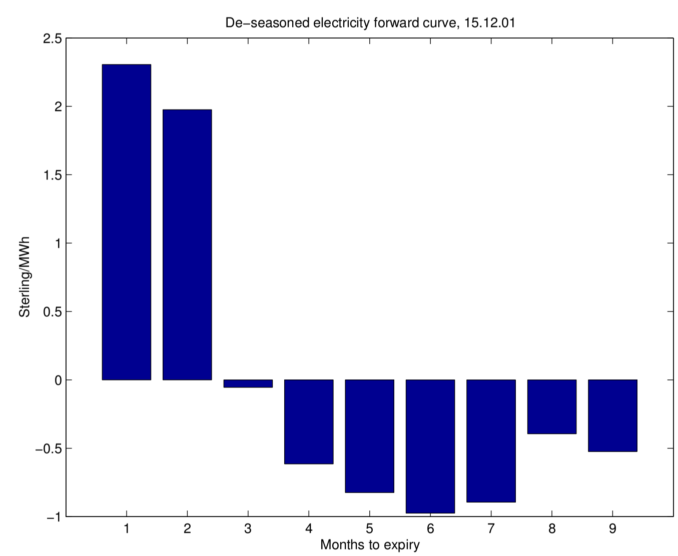
Conjecture I: true, especially for higher maturities: NG



Extracting the seasonal component

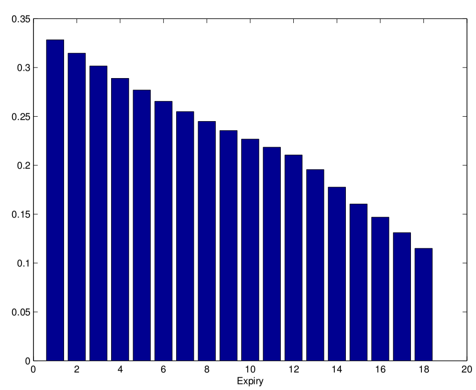
Seasonal component is "known" monthly premium $\longrightarrow$ extract it from a forward curve. If seasonality was the only determining factor, then what is left should always be flat, but it is not!
$\Longrightarrow$ situations similar to backwardation/contango arise:

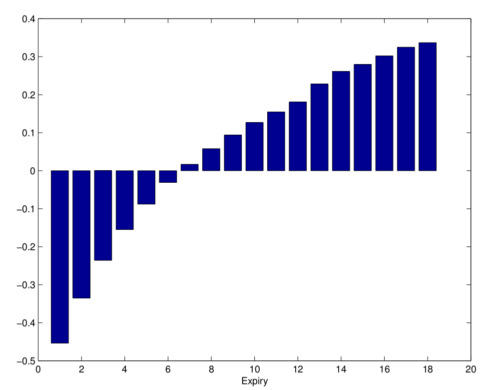


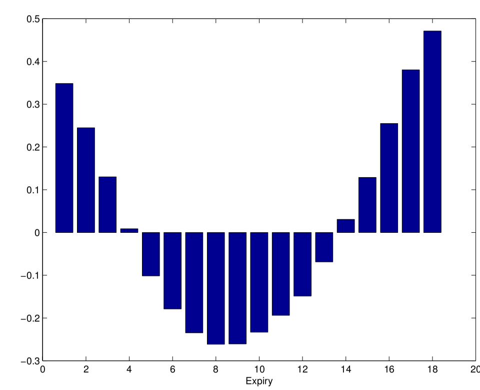


Principal Component Analysis of the forward curve
Case I: interest rates and non-seasonal commodities (oil)

A forward curve of almost any shape can be constructed by combining three simple shapes:
Level, Slope, Curvature
$\longrightarrow$ Principal Components of $(\mathbf{F}(t))_{t\in\mathbf{N}} = (F_1(t), F_2(t), \ldots, F_N(t))_{t\in\mathbf{N}}$

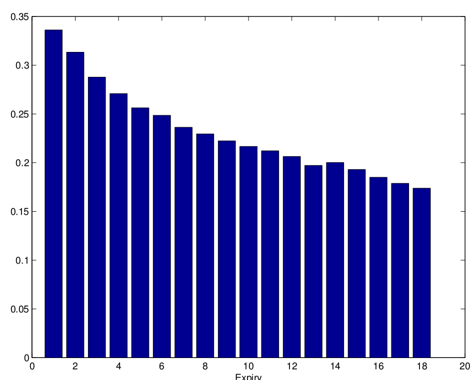


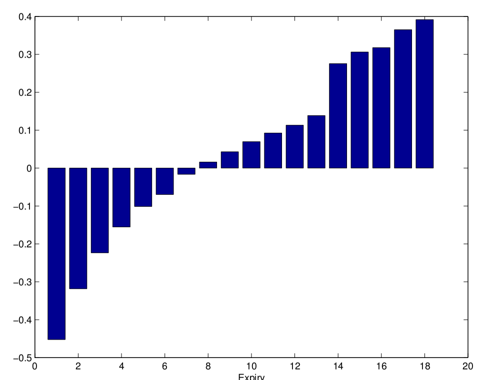


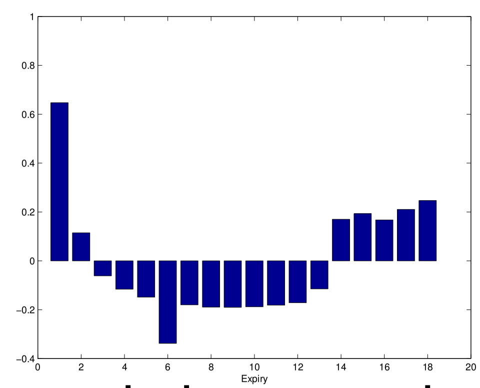


First three principal components explain approx. 99% (!) of the forward curve's variability.
(Litterman & Scheinkmann '91 for US government bonds, Cortazar & Schwartz '94 for copper)

Principal Components of daily returns







These first three principal components have clear economic interpretation, explain 95% of the total variability, can be treated as the main risk factors governing the futures prices' evolution.

Applications of PCA

I. Forecasting market transitions (between backwardation and contango):

• The second principal component reflects the slope of the forward curve

• Values close to 0 indicate a flat forward curve (and hence, possible transition)

• Due to smooth time-series-like structure, it can be used to construct an indicator which anticipates possible transitions

Borovkova, EPRM magazine (June 2003).

II. Portfolio risk management and VaR estimation

• First few principal components (of returns) reflect main risk factors

$\Longrightarrow$ the number or risk factors is greatly reduced

• The distribution of portfolio returns can be approximated via the distribution of the main risk factors

• In a portfolio context, these risk factors can be hedged

Principal Component Indicator

"Raw" version: projection of the daily forward curve on the second PC

$$I(t) = \sum_{k=1}^{N} PCL_k^{(2)} F_k(t),$$

where $PCL_k^{(2)}$ $(k = 1, ..., N)$ are second principal component loadings of the futures prices series.

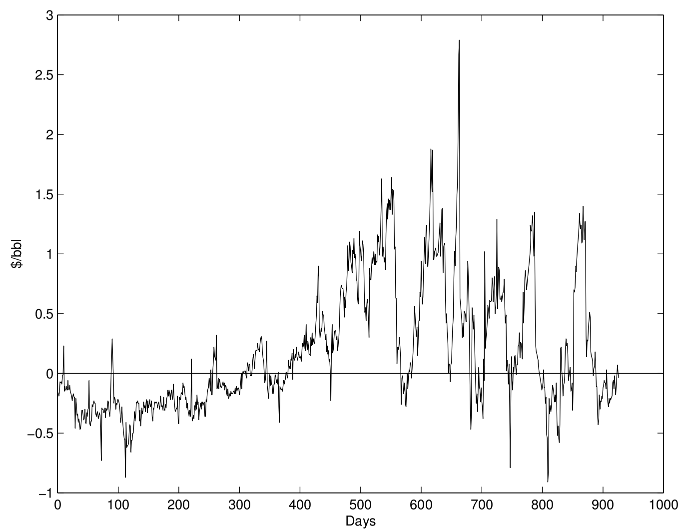
MA-smoothed version:

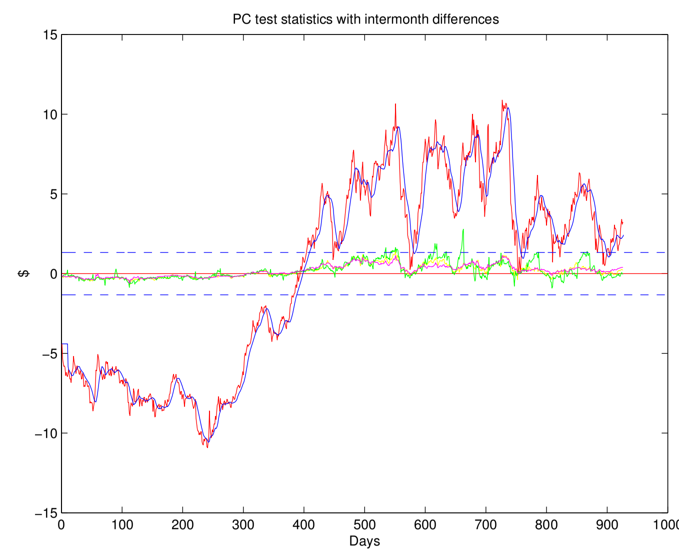
$$I^{MA}(t) = \frac{1}{M} \sum_{i=0}^{M-1} I(t - i).$$

Choice of M: take a fast and slow moving average.

Application of the PC indicator to Brent oil futures

Generate a "signal of change" when the indicator enters some ε-neighborhood of zero:

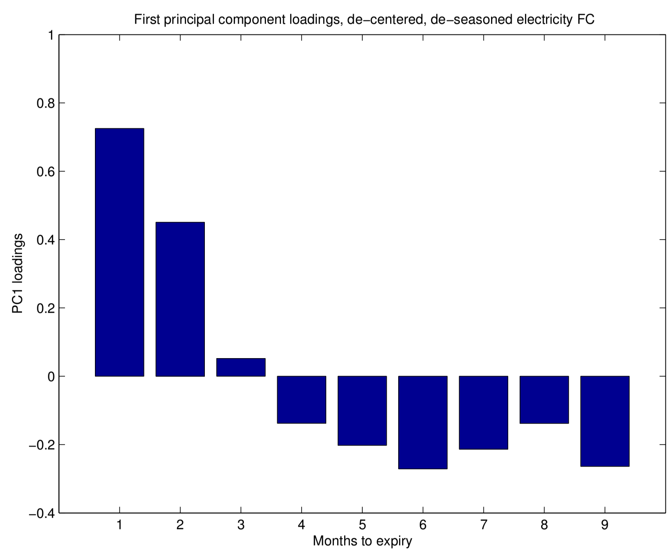


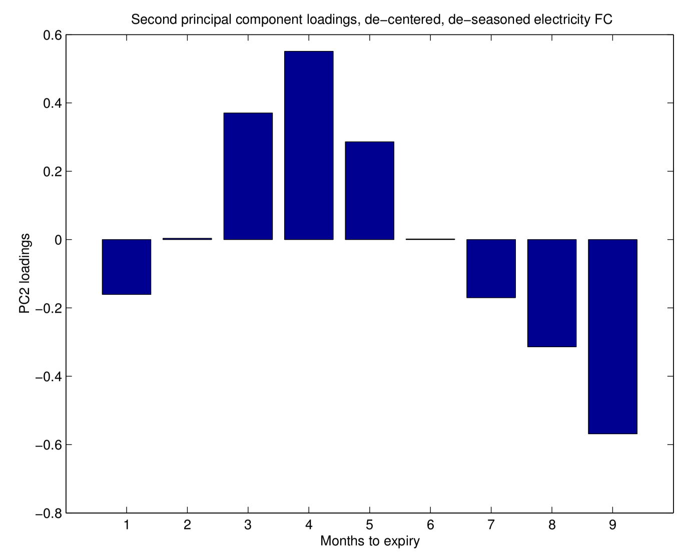


ε-neighborhood determined via the distribution of the indicator (under the null-hypothesis of no change), approximated by either Monte-Carlo or bootstrap distribution.

PCA for seasonal commodities (electricity, NG)

Apply PCA to deseasonalized forward curves $F(t,T)\exp(-s(T)))$
Second and third PCs for de-seasoned FC (first PC = level) still reflect the slope and curvature:





Applications of PCA: trading

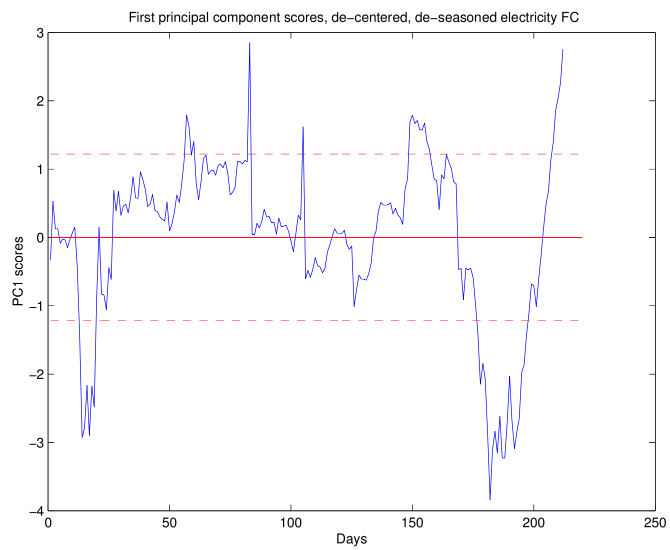
- For deseasonalized forward curves, situations analogous to backwardation/contango markets arise, in terms of deviations from the "typical" seasonal forward curve.

- High absolute values of the second PC indicates whether futures with shorter (longer) expiries are overpriced w.r.t. "typical" seasonal premium

- Again, use PC indicator: the projection of the daily deseasonalized forward curve on the second PC.

Principal Component Indicator for electricity FC

A value of the indicator far from zero signals significant deviation from the "expected" seasonal forward curve pattern:



Can construct profitable trading strategies based on the indicator. (Borovkova & Geman (SNDE 2006)).

Conclusions

- Extracting deterministic seasonality from forward curves allows to study features obscured by dominant seasonal effects (PCA, cost-of-carry, traditional term structure models)

- Average forward price is a robust identifier of the overall price level, more so than the spot price, although now need to take into account sloping forward curves

- Stochastic convenience yield is a quantity indicative of market state and economic indicators; it can be exploited to construct market indicators and generate profitable trading strategies

Perspective research directions

- Backwardation/contango-like profile in seasonal forward curves
- Applications of the model to the derivatives pricing
- Relating the stochastic convenience yield to economic and other exogenous variables such as stocks (supply), extreme weather conditions (demand), ...
- Modelling the entire term structure of convenience yields, with a number of sources of uncertainty and volatility functions
- Seasonal term structure of futures prices' volatilities
- Applications of the model to agricultural commodities