Longevity Risk Pricing

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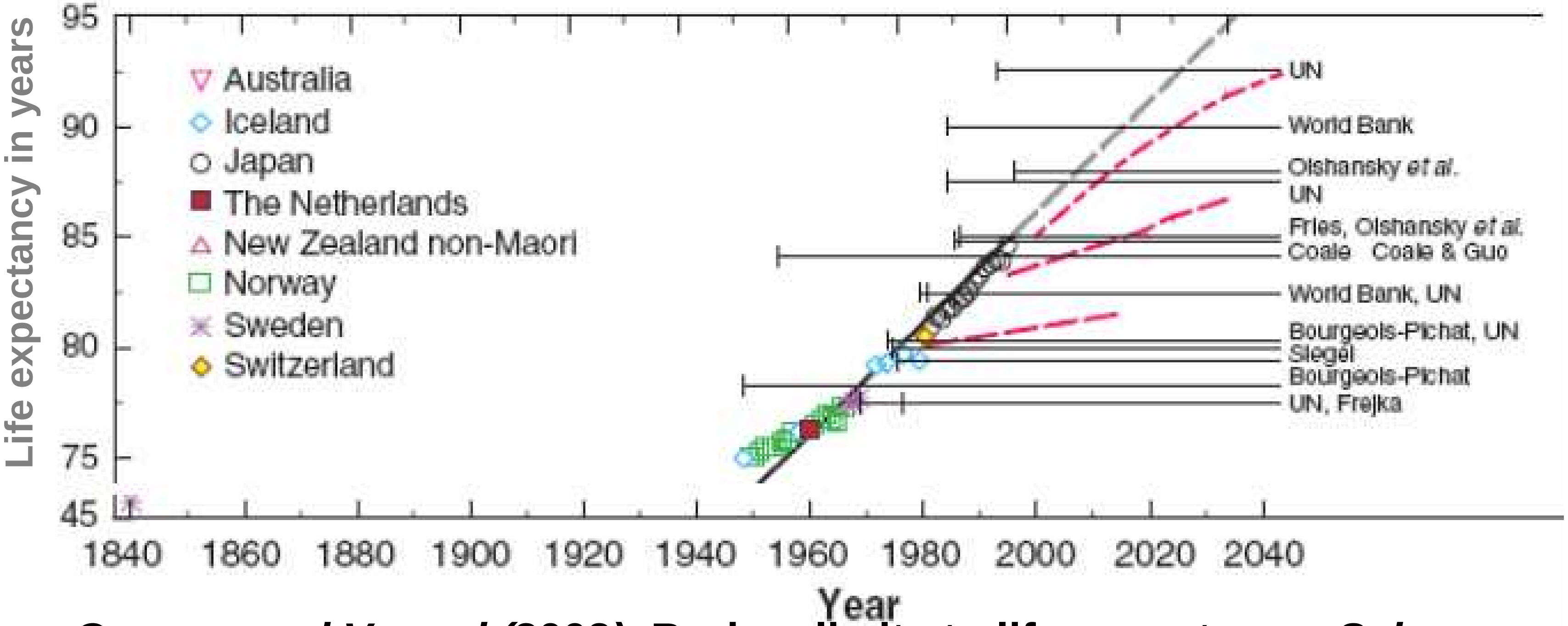
Longevity trend



Oeppon and Vaupel (*2002*),* Broken limits to life expectancy, *<u>Science</u>

- Longevity trend is difficult to predict.
- The asserted ceilings were surpassed repeatedly.

Longevity risk

Definition: Unexpected improvements in life expectancies

Severity: Even small longevity risk may lead to severe solvency issues for life insurers & pension funds

Challenges for annuity providers:

- Longevity risk is a systematic risk (macro risk)
  - Non-diversifiable (by pooling)
  - The capacity of reinsurance is limited (OECD(2005))
- Longevity linked claims cannot be replicated.

  (i.e. replicating portfolio does not exist (yet))

EIB/BNP survivor bond

- EIB/BNP longevity bond
  - *First announced in Nov 2004;*
  - *A ‘coupon-based’ bond;*
  - *Longevity risk premium of 20 basis points*
  - *Withdrawn for redesign in late 2005;*
  - Obstacles: Pricing, design, institutional issues (Blake, Cairns and Dowd (2006))
- No clear view on the ‘right’ price
  - Incomplete market, unhedgeable risk

➢ Goal: Quantify longevity risk premium

Potential market for Longevity linked securities

- Benefits to the buy side
  - Ideal protection from longevity risk
  - Avoid solvency problem at low cost
- Benefits to the sell side
  - Diversified portfolio (uncorrelated financial risks and insurance risks)
  - Earn longevity risk premium
- Pricing is difficult in the incomplete markets
  - Longevity risk is unhedgeable risk (claims can not be replicated)
  - Non-arbitrage pricing is not applicable

Methods proposed in the literatures

- CAPM (Friedberg and Webb (2006))
  - Longevity risk premium (LRP)
    = beta * market risk premium
  - 75 bp (= 0.15 * 5%), Confidence Interval. [-75, 230] bp
  - Drawback: large error of estimated risk premium
- CCAPM (Friedberg and Webb (2006))
  - LRP is proportional to the covariance of its return with per capita consumption
  - 2 bp
  - Drawback: inconsistent with market risk primia
- Sharpe Ratio (Milevsky, Promislow and Young (2006))
  - Only a proposal

$$SR^{Ins} = \frac{N(1+L) - E(W_N)}{\sigma(W_N)}$$

Desirable pricing methodology

Should be:

- Applicable for pricing unhedgeable risks under incomplete market
- Market-based method (consistent with market risk premia)
- Capable to handle real-world complications:
  e.g. natural hedging and basis risk

Therefore, utility-based pricing method

Overview

- Introduction & Motivations
- Three building blocks
  - Longevity linked securities
  - Stochastic mortality modeling
  - Equivalent Utility Pricing principle
- Pricing longevity bonds & derivatives
  - Bonds, swaps, floors, …
- Impacts of natural hedging and basis risk
- Conclusion

Longevity linked securities

- Longevity-linked zero coupon bonds
- Coupon based longevity bonds
- Deferred starting longevity bonds
- Longevity Swaps
- Floors (caps)

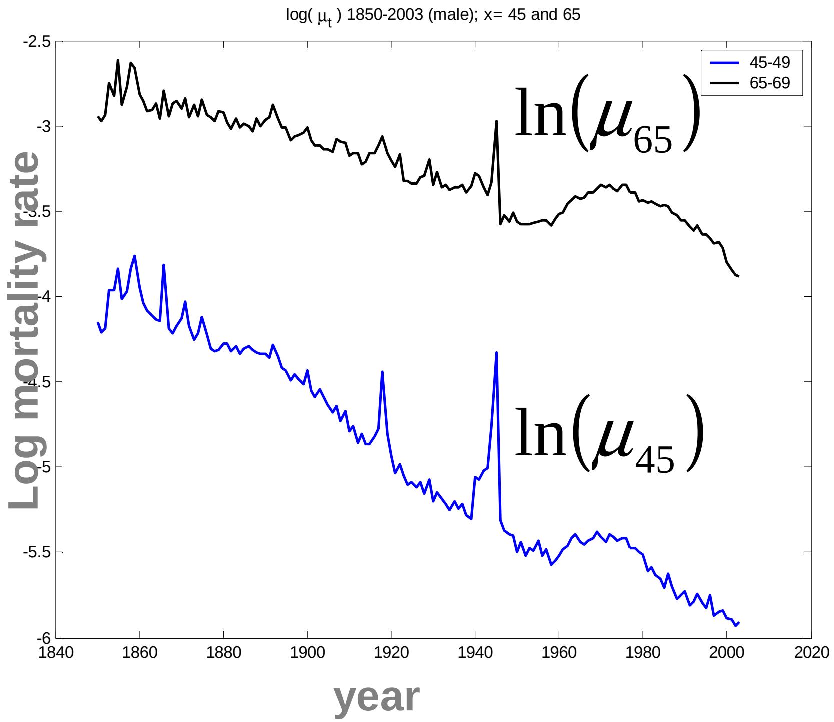
Blake, Cairns and Dowd (2006), '*Living with Mortality: Longevity bonds and other mortality-linked securities*', British Actuarial Journal, Vol. 12, No. 1, 2006 , pp. 153-197

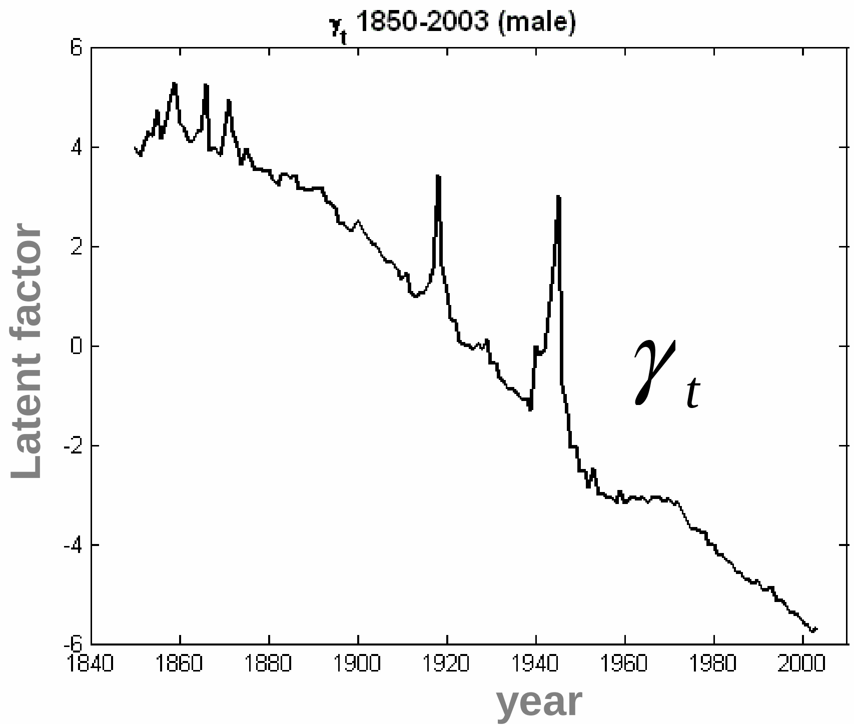
Stochastic mortality models

- Lee-Carter (1992)

$$\ln(\mu_{x,t}) = \alpha_x + \beta_x \gamma_t + \delta_{x,t}$$

$$\gamma_t = c + \gamma_{t-1} + \varepsilon_t$$





Equivalent Utility Pricing

- Compensate the longevity bond seller (e.g. EIB/BNP), such that, he is indifferent between bearing risk after compensation and not bearing risk.
  - Seller's minimum price
- The longevity bond buyer (e.g. annuity providers) pays, such that, she is indifferent between bearing risk and not bearing risk after the payment.
  - Buyer's maximum price
- Negotiation range: [min, max]

Seller's minimum price

- **Without** longevity risk

$$V_0 = \max_{\{D_t, x_t\}} E\left[\int_0^T \beta^{-t} u(D_t)dt + \beta^{-T} u(W_T)\right]$$

$$s.t. \qquad E\left[\int_0^T M_t D_t dt + M_T W_T\right] = W_0$$

- **With** longevity risk

$$V_0^{\pi} = \max_{\{D^{\pi}{}_t, x_t\}} E\left[\int_0^T \beta^{-t} u\left(D_t^{\pi} + E(S_t) - S_t\right)dt + \beta^{-T} u\left(W_T^{\pi}\right)\right]$$

$$s.t. \qquad E\left[\int_0^T M_t D_t^{\pi} dt + M_T W_T^{\pi}\right] = W_0 + \pi$$

- Indifferent

$$V_0^{\pi} = V_0$$

Buyer's maximum price

- **Without** longevity risk

$$V_t^{\pi} = \max_{\{D_t^{\pi}, x_t\}} E\left[\int_0^T \beta^{-t} u\left(D_t^{\pi}\right) dt + \beta^{-T} u\left(W_T^{\pi}\right)\right]$$

$$s.t. \qquad E\left[\int_0^T M_t D_t^{\pi} dt + M_T W_T^{\pi}\right] = W_0 - \pi$$

- **With** longevity risk

$$V_0 = \max_{\{D_t, x_t\}} E\left[\int_0^T \beta^{-t} u\left(D_t + E\left(S_t\right) - S_t\right) dt + \beta^{-T} u\left(W_T\right)\right]$$

$$s.t. \qquad E\left[\int_0^T M_t D_t dt + M_T W_T\right] = W_0$$

- Indifferent

$$V_0^{\pi} = V_0$$

Utility function assumption

- Negative exponential utility with wealth-dependent risk aversion
  - Risk aversion decreases as wealth increases
  - b = 0, …, 1 (from CARA to CRRA)

$$u(X) = -\frac{1}{\alpha(W_0)} \exp(-\alpha(W_0)X)$$

$$where \qquad \alpha(W_0) = \overline{\alpha}(W_0)^{-b}$$

Results

- Risk loading ($) π

$$\pi = \frac{1}{\alpha} E\left[\int_0^T e^{-rt} \ln G_t dt\right]$$

$$where \qquad G \equiv E\left[\exp\left(-\alpha\left(E[S_t] - S_t\right)\right)\right]$$

- Risk premium (bp) R_p

$$\int_0^T e^{-rt} E[S_t] dt + \pi = \int_0^T e^{-\left(r+R_p\right)t} E[S_t] dt$$

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Longevity Bond: Risk premia in basis points

maturity	equity capital = 10000				equity capital = 100			
(year)	b=1	b=1/4	b=1/8	b=0	b=1	b=1/4	b=1/8	b=0
5	0	0	0	-1	0	0	-1	-1
10	0	0	-1	-3	0	-1	-2	-3
15	0	-1	-2	-7	0	-2	-4	-7
20	0	-1	-4	-11	0	-4	-7	-11
25	0	-2	-5	-15	0	-5	-8	-15
30	0	-2	-5	-16	0	-5	-9	-16
35	0	-2	-5	-16	0	-5	-9	-16

- Financially stronger seller requires lower risk premium
- Smaller amount of principal requires lower risk premium
- Implication: More participants. Every one issues moderate amount of longevity bonds, which are linked to the same survivor index.

Longevity Bond: When to deal in

Sell side

$$\overline{\alpha} = 3$$

Minimum Ask price

Buy side

$$\overline{\alpha} = 5$$

Maximum Bid price

equity	w0 = 10000			
maturity	b=1	b=1/4	b=1/8	b=0
5	0	0	0	-1
10	0	0	-1	-3
15	0	-1	-2	-7
20	0	-1	-4	-11
25	0	-2	-5	-15
30	0	-2	-5	-16
35	0	-2	-5	-16

equity capital = 100			
b=1	b=1/4	b=1/8	b=0
0	0	-1	-2
0	-2	-3	-5
0	-4	-7	-11
0	-6	-11	-16
0	-8	-14	-21
0	-8	-15	-23
0	-8	-15	-23

– A simple table to facilitate deal making

Other Securities: Deferred

- Deferred starting longevity bonds

# year	equity capital = 10000				equity capital = 100			
defer	b=1	b=1/4	b=1/8	b=0	b=1	b=1/4	b=1/8	b=0
0	0	-2	-5	-16	0	-5	-9	-16
5	0	-2	-6	-17	0	-6	-10	-17
10	0	-2	-7	-21	0	-7	-12	-21
15	0	-3	-9	-26	0	-9	-15	-26
20	0	-3	-10	-30	0	-10	-18	-30

- Skip inefficient coupons
- Higher risk premium than longevity bond
- **Different payoff, different risk premium in incomplete market**

Other Securities: Swaps, floors & Caps

- Longevity swaps, floors & Caps

	equity capital = 10000			equity capital = 100		
maturity	swap	floor	cap	swap	floor	cap
5	0	-3	3	-1	-3	3
10	-1	-5	4	-2	-5	4
15	-2	-7	5	-4	-8	5
20	-4	-9	7	-7	-11	6
25	-5	-11	8	-8	-14	7
30	-5	-12	8	-9	-15	8
35	-5	-12	9	-9	-15	8

- More efficient way of using capital
- **Different payoff leads to different risk premium in incomplete market**

The impact of natural hedging

- Term insurance is a natural hedge for annuity
- Suppose a life insurer has both annuity and term insurance business units. Is EIB/BNP survivor bond a good deal for the life insurer?
- Buyer's view

maturity	equity capital = 10000				equity capital = 100			
(year)	b=1	b=1/4	b=1/8	b=0	b=1	b=1/4	b=1/8	b=0
5	0	0	0	0	0	0	0	0
10	0	0	0	-1	0	0	0	-1
15	0	0	-1	-2	0	-1	-1	-2
20	0	0	-1	-5	0	-1	-3	-5
25	0	-1	-2	-6	0	-2	-3	-6
30	0	-1	-2	-7	0	-2	-4	-7

Conclusion: Natural hedging may significantly reduce the risk premium

The impact of basis risk

- Basis risk:
  - a discrepancy between the reference population and the annuitant population
- EIB/BNP survivor bond is linked British survivor index.
  - Is it a good deal for Dutch pension fund?
- Buyer’s view
  - Dutch pension fund

Dutch buyer's price to EIB/BNP bonds

- Without basis risk (if bond links to Dutch mortality)

	equity capital = 10000				equity capital = 100			
maturity	b=1	b=1/4	b=1/8	b=0	b=1	b=1/4	b=1/8	b=0
5	0	0	-1	-2	0	-1	-1	-2
10	0	-1	-2	-7	0	-2	-4	-7
15	0	-2	-5	-15	0	-5	-9	-15
20	0	-3	-8	-23	0	-8	-14	-23
25	0	-3	-10	-28	0	-10	-17	-28
30	0	-3	-10	-30	0	-10	-18	-30

- With basis risk (if bond links to British mortality)

equity	equity capital = 10000				equity capital = 100			
maturity	b=1	b=1/4	b=1/8	b=0	b=1	b=1/4	b=1/8	b=0
5	0	0	0	-1	0	0	0	-1
10	0	0	-1	-2	0	-1	-1	-2
15	0	-1	-2	-6	0	-2	-4	-6
20	0	-1	-4	-11	0	-4	-7	-11
25	0	-2	-6	-16	0	-6	-10	-16
30	0	-2	-7	-18	0	-7	-12	-18

- **Conclusion: Basis risk matters.**

Conclusions (1)

- Longevity risk imposes severe solvency issue.
- Longevity linked securities offer a solution
- Advantages of our pricing method:
  - Pricing in incomplete market
  - Different payoff structures require different risk premia (bonds, deferred, swaps, floors, caps)
  - Consistent with observed equity premium
  - Able to tell when to deal in
  - Narrow price range
  - Capable to handle realities: natural hedging and basis risk. These matter!

Conclusion (2)

Our results also imply:

- Natural hedging and basis risk may have significant impacts on pricing.
- Financially stronger sellers require lower risk premiums
- Smaller amount of principal requires lower risk premium
- Distributing instead of accumulating. Market calls for more sellers. Every one issues a moderate amount of longevity bonds linked to same survivor index.