The mod $p$ local Langlands program has the ambition to study the cohomology of arithmetic varieties, with coefficients in local systems over fields of characteristic $p > 0$, in terms of Galois parameters. The behavior of the Galois isotypical components of such spaces become more and more complicated as the level at $p$ deepens and there are no geometric tools to control the cohomology of such spaces.

In particular, the local representations of the $p$-adic groups obtained as Galois component with “infinite level at $p$” are extremely complicated objects and work of Breuil and Paskunas show that there are a priori infinitely many local representations of the $p$-adic group which could be attached to a single local parameter (even by taking into account the constraints coming from Serre’s conjectures).

In this talk I will present a recent study where we show that the local representation in cohomology behaves as expected by Breuil and Paskunas when the level at $p$ is the principal congruence subgroup. In other words, the space of $K_1$-invariant of these local representations is “as small as possible” and depends only on the local Galois parameter at $p$ in an explicit way.

This ongoing joint work with Benjamin Schraen.

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