Agents express propositional goals over binary issues to reach a collective decision.

We study the strategy-proofness of three generalizations of the majority rule.

We also study the computational complexity of finding a successful manipulation.

### Framework

- A set $N$ of $n$ agents has to decide over a set $I$ of $m$ binary issues (no integrity constraint)
- Every agent $i$ has a propositional formula $y_i$ as her goal, whose models are in the set $\text{Mod}(y_i)$
- $m_i(j) = (m^0_j, m^1_j)$ indicates the number of 0s and 1s for issue $j$ in the models of $y_i$
- A goal profile $\Gamma = (y_1, \ldots, y_n)$ collects agents’ goals
- $L^*$ for $\star \in \{\land, \lor, \neg\}$ defined as $\varphi := p \mid \neg p \mid \varphi \star \varphi$ are language restrictions on the goals

Colleagues $\blacktriangle$, $\bullet$, and $\blacklozenge$ organize their next meeting. They have to decide whether to meet in the morning ($\bigstar$) or in the afternoon, to continue writing their paper ($\blacklozenge$) or to talk about practicalities, and whether they’ll meet at a local coffee shop ($\blacklozenge$) or in their office.

The following are their propositional goals:

- $\gamma_\blacktriangle : \bigstar \land \blacktriangle \land \blacklozenge$
- $\gamma_\bullet : \bigstar \land \neg \blacktriangle \land \neg \blacklozenge$
- $\gamma_\blacklozenge : (\bigstar \land \neg \blacktriangle \land \neg \blacklozenge) \lor (\neg \bigstar \land \neg \blacklozenge)$

- $\text{Mod}(\gamma_\blacktriangle) = \{(101), (010), (000)\}$
- $m_\blacktriangle(\blacktriangle) = (2, 1)$
- $\Gamma = (\gamma_\blacktriangle, \gamma_\bullet, \gamma_\blacklozenge)$

### Computational Complexity

How difficult it is to know if an agent can manipulate?

**Manipulation** profile $\Gamma$, agent $i$

- $\exists y'_i$ such that $F(\Gamma_{-i}, y'_i) \prec_i F(\Gamma)$?

**PP:** problems solvable by a probabilistic TM in poly time, where TM says yes $\iff$ a majority of computations accepts

$\text{MANIP(E Maj)}$ and $\text{MANIP(2s Maj)}$ are PP-hard.

### Summary of results

<table>
<thead>
<tr>
<th>$y_i$</th>
<th>$L$</th>
<th>$L^\land$</th>
<th>$L^\lor$</th>
<th>$L^\oplus$</th>
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</thead>
<tbody>
<tr>
<td>$\text{EMaj}$</td>
<td>M</td>
<td>M</td>
<td>SP</td>
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<tr>
<td>$\text{True Maj}$</td>
<td>M</td>
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<td>SP</td>
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<tr>
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\begin{align*}
\text{Erosion, Dilatation, Strategy-Proof, Manipulable}
\end{align*}

### Majoritarian Voting Rules

A goal-based voting rule is a collection of functions $F : (L_I)^n \to P(\{0, 1\}^m \setminus \emptyset)$ for all $n$ and $m$ and $L_I$ a propositional language over $I$.

- $\text{EMaj}(\Gamma) = 1 \iff \sum_{i \in N} \frac{m^i_j}{|\text{Mod}(y_i)|} \geq \left\lceil \frac{n+1}{2} \right\rceil$
- $\text{True Maj}(\Gamma) = \prod_{j \in I} M(\Gamma_j)$, where, for $j \in I$:

  $M(\Gamma_j) = \begin{cases} 
  \{x\} & \text{if } \sum_{i \in N} \frac{m^i_j}{|\text{Mod}(y_i)|} > \sum_{i \in N} \frac{m^{i-x}_j}{|\text{Mod}(y_i)|} \\
  \{0, 1\} & \text{otherwise}
  \end{cases}$

- $\text{2s Maj}(\Gamma) = \text{Max}(\text{Maj}(y_1), \ldots, \text{Maj}(y_n))$