Binary Aggregation with Integrity Constraints

A set of \( n \) agents express their opinions on a set of \( m \) issues to reach a collective decision on them.

A profile \( B \) is a set of \( m \) ballots, one for each agent, where \( b_i \) \( \in \{0,1\}^m \) is the ballot of agent \( i \) on issues from \( \{1,\ldots,m\} \).

The integrity constraint \( IC \) is a propositional formula relating issues and valuation of each agent.

The aggregation rule is a function \( F : Mod(IC)^n \rightarrow P(\{0,1\}^m) \setminus \{\emptyset\} \)

Aggregation Rules

- Simple rules: dictatorship, majority, quota, ... 
- Max/Min rules: max: subagenda, min: number of atomic changes 
- Preference aggregation rules: Kemeny, Slater, ... 
- Representative voter rules: average voter, majority voter, ... 

Agenda Safety

IC properties \( \Rightarrow \) DL-PA formulas (ensuring correctness of translation)

- median property
- \( k \)-median property
- syntactic simplified median property

IC properties linked to classes of rules whose outcomes will always satisfy IC.

Dynamic Logic of Propositional Assignments

Atomic program \( +p (\lnot p) \) makes propositional variable \( p \) true (false).

Language formulas \( \varphi := p | T | \bot | \neg \varphi | \varphi \lor \varphi | (\pi) \varphi \)

programs \( \pi := +p | -p | \pi \lor \pi | \pi \land \pi | \pi^* | \varphi^? \)

for \( p \) in a countable set of propositional variables \( \mathcal{P} \)

+ some abbreviations for programs (if \( \varphi \) then \( p_1 \) else \( p_2 \), ...)

Translation of Judgment Aggregation into DL-PA

A graphical representation of the translation into DL-PA.

Axioms

- Single-profile: the outcome of \( F \) linked to structure of a single profile.
- unanimity, issue-neutrality, domain-neutrality, N-monotonicity
  \( \Rightarrow \) propositional logic

Unanimity

- BA | \( U = \text{For any } B, \text{ for all issues } j \text{ and for } x \in \{0,1\}, \text{ if } b_{ij} = x \text{ for all agents } i \text{ then } F(B)_j = x \)
- DL-PA | \( U = \bigwedge_{1 \leq i \leq m} (\neg (\bigwedge_{1 \leq i < s} p_{ij}) \rightarrow p_{ij}) \land (\bigwedge_{1 \leq i < s} (\neg p_{ij}) \rightarrow \neg p_{ij}) \)

Multi-profile: two outcomes of \( F \) linked to structures of multiple profiles.

- independence, 1-monotonicity, anonymity
  \( \Rightarrow \) DL-PA

Future Directions

- What about other rules, axioms, IC properties?
- What about the existing translation of DL-PA into propositional logic?
  \( \Rightarrow \) It could be used for automated reasoning with SAT-solvers.
- What about other areas of Judgment Aggregation?