Agents ▲, ●, and ■ want to visit a city together. There are three points of interest: an ancient belfry (▲), a music museum (●), and the beach (■). ▲ wants to visit everything, ● wants to go only to the museum, and ■ wants to visit a single place...

Agents express goals with propositional formulas:

\[ y_▲ : ▲ \land ▲ \land ▲ \quad y_● : ▲ \land ▲ \land ▲ \land ▲ \quad y_■ : (▲ \land ▲ \land ▲ \land ▲) \lor (▲ \land ▲ \land ▲ \land ▲) \lor (▲ \land ▲ \land ▲ \land ▲) \]

A goal profile \( \Gamma = (y_1, \ldots, y_n) \) collects agents’ goals.

A goal-based voting rule is a collection of functions \( F : (\mathcal{L}_I)^n \to \mathcal{P}((0, 1)^m) \setminus \emptyset \) for all \( n \) and \( m \), where \( \mathcal{L}_I \) is a propositional language over \( I \).

F is resolute if it always returns a singleton (irresolute otherwise)

Goal-based Voting Rules

\[ Con_j(\Gamma) = \begin{cases} \text{Mod}(y_1 \land \cdots \land y_n) & \text{if non-empty} \\ \{v\} & \text{for } v \in \{0, 1\}^m \text{ otherwise} \end{cases} \]

\[ Approval(\Gamma) = \text{arg max}_{v \in \text{Mod}(\forall, \exists \land y)} |\{i \in N \mid v \in \text{Mod}(y_i)\}| \]

1. \( EMaj(\Gamma)_j = 1 \iff \sum_{i \in N} m_{y_i}^j \geq \frac{n}{2} \)
2. \( TrueMaj(\Gamma) = \Pi_{j \in I} M(\Gamma)_j \) where, for \( j \in I \):

\[ M(\Gamma)_j = \begin{cases} \{x\} & \text{if } \sum_{i \in N} m_{y_i}^j \geq \sum_{i \in N} m_{y_i}^j \text{ for } (0, 1) \text{ otherwise} \\ 2sMaj(\Gamma) = Maj(Maj(y_1), \ldots, Maj(y_n)) \]

\[ \begin{array}{c|c|c} \text{Agents} & \text{Goal profile} & \text{Output} \\ \hline ▲ & (▲ \land ▲ \land ▲) & (111) \\ ● & (▲ \land ▲ \land ▲) & (000) \\ ■ & (▲ \land ▲ \land ▲) & (010) \\ (▲ \land ▲ \land ▲) & (101) \\ (▲ \land ▲ \land ▲) & (001) \\ (▲ \land ▲ \land ▲) & (100) \end{array} \]

What is the output of the different rules?

Axiomatics - Characterization

Anonymity (A): Agents’ (goals) are equally important
Neutrality (N): Issues are equally important
Independence (I): Each issue \( j \) is decided by a function \( f_j \)
Unanimity (U): Result follows agents’ unanimous choice
Positive responsiveness (PR): Adding (deleting) support for an issue when the result is equally irresolute or favoring acceptance (rejection), gives a result strictly favoring acceptance (rejection)
Egalitarianism (E): Every model of a goal has a weight proportional to the total number of models of the goal
Duality (D): Rule isn’t biased for acceptance/rejection

Similar to results for majority in Judgment Aggregation.

Computational Complexity

We study the complexity to compute the result of rules.

\[ \text{WinDet}(F) \text{ profile } \Gamma, \text{ issue } j \]
\[ \text{WinDet}^*(F) \text{ profile } \Gamma, \text{ set } S \subseteq I, \rho : S \to \{0, 1\} \]
\[ \Theta^p_2 \text{ problems solvable in poly time with } O(\log n) \text{ queries to an NP oracle} \]

PP: problems solvable by a probabilistic TM in poly time, with error probability < \( 1/2 \)

\[ \text{WinDet}^*(\text{Conj}) \text{ is NP-hard.} \]
\[ \text{WinDet}^*(\text{Approval}) \text{ is } \Theta^p_2 \text{-complete.} \]
\[ \text{WinDet of majorities is PP-hard.} \]