Autonome Mobile Robots (5082AUMR6Y, Herfst 2012)
Tentame: Hoofdstuk 1 t/m 4

Week 44 t/m 46 (Donderdag 22 november, 15:00-17:00)

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Vraag 1

Consider an eight-legged walking robot. Consider gaits in the terms of lift/release events as in chapter 2.

(a) How many possible events exist for this eight-legged machine?

(b) Specify two different staticly stable walking gaits using the notation of figure 2.8.

Antwoord 1

(a) $k = 8 \Rightarrow N = (2k-1)! = (2*8-1)! = 15! = 1307674368000$ possible events exist.

(b) See figure 1.

Vraag 2

Consider a wheeled robot which moves over a flat surface. Each wheel has an y-axis. When a rolling motion occurs, all y-axis overlap at a point; the instantaneous Center of Curvature. Consider the case that the Instantaneous Center of Curvature is outside the robot, and the robot moves from point $(0,0)$ to (x,y) (see figure 2).

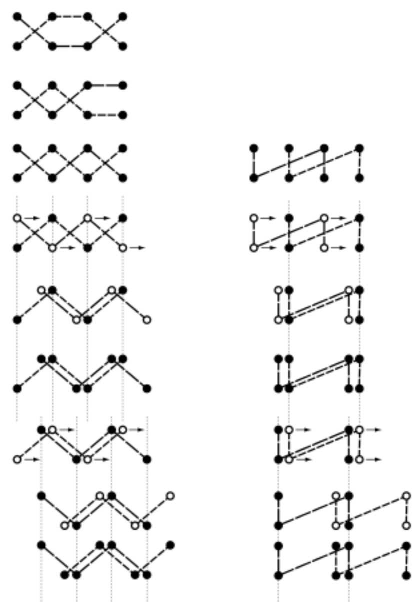

Figure 1: Two staticly stable walking gaits

A natural way to represent the movement as an circular movement with a radius R an the sector angle ϕ. (x, y) is a point on the circle, which means

$$\begin{cases} x = R\sin\phi \\ y = R(1-\cos\phi) \end{cases} \iff \begin{cases} R = \dfrac{x^2+y^2}{2y} \\ \phi = atan(\dfrac{2xy}{x^2-y^2}) \end{cases}$$

This representation has disadvantage that for small y (straight ahead!), a small change in (x, y) may cause a big change in parameter R. You can verify this with by computing the Jacobian; you should get:

$$\begin{pmatrix} dR \\ d\phi \end{pmatrix} = \begin{pmatrix} \frac{x}{y} & \frac{y^2-x^2}{2y^2} \\ \frac{-2y}{x^2+y^2} & \frac{2x}{x^2+y^2} \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix}$$

For this reason it is much better to characterize the path by the *curvature* $\kappa \equiv 1/R$, which changes smoothly around the forward direction.

Now, compute the Jacobian for the (κ, ϕ) representation and show that it changes more smoothly.

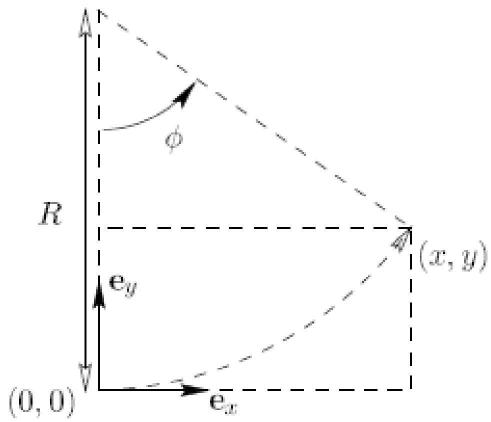

Figure 2: Turning to point (x, y)

Answer 2

The curvature is defined as $\kappa \equiv 1/R$, which means

$$\kappa = \frac{1}{R} = \frac{1}{\frac{x^2+y^2}{2y}} = \frac{2y}{x^2+y^2}. \tag{1}$$

The Jacobian with respect to κ and ϕ is defined as

$$J(\kappa, \phi) = \begin{pmatrix} \frac{\partial\kappa}{\partial x} & \frac{\partial\kappa}{\partial y} \\ \frac{\partial\phi}{\partial x} & \frac{\partial\phi}{\partial y} \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix} \tag{2}$$

The partial derivatives of ϕ are already given, so only $\partial\kappa/\partial x$ and $\partial\kappa/\partial y$ have to be calculated with the quotient rule:

$$\frac{\partial\kappa}{\partial x} = -4yx\frac{1}{(x^2+y^2)^2} = \frac{-4yx}{(x^2+y^2)^2} \tag{3}$$

$$\frac{\partial\kappa}{\partial y} = \frac{2(x^2+y^2) - 2y2y}{(x^2+y^2)^2} = \frac{2(x^2-y^2)}{(x^2+y^2)^2} \tag{4}$$

Those two partial derivates can be written filled in the Jacobian matrix:

$$\begin{pmatrix} d\kappa \\ d\phi \end{pmatrix} = \begin{pmatrix} \frac{-4yx}{(x^2+y^2)^2} & \frac{2(x^2-y^2)}{(x^2+y^2)^2} \\ \frac{-2y}{x^2+y^2} & \frac{2x}{x^2+y^2} \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix} \tag{5}$$

The remaining question is too prove that the Jacobian $J(\kappa, \phi)$ behaves more smoothly for small y than $J(R, \phi)$. The second column of both Jacobian with the partial derivates of ϕ are equivalent, so only the behavior of $\partial\kappa/\partial x$ has to be compared with the behavior of $\partial R/\partial x$ (for the limit $y \to 0$) and equivallently $\partial\kappa/\partial y$ versus $\partial R/\partial y$.

$$\lim_{y\to 0} \frac{\partial \kappa}{\partial x} = \frac{-4yx}{(x^2+y^2)^2} = \frac{0}{x^4} = 0 \tag{6}$$

$$\lim_{y\to 0} \frac{\partial R}{\partial x} = \frac{x}{y} = \frac{x}{0} = \infty \tag{7}$$

$$\lim_{y\to 0} \frac{\partial \kappa}{\partial y} = \frac{2(x^2-y^2)}{(x^2+y^2)^2} = \frac{2x^2}{x^4} = \frac{2}{x^2} \tag{8}$$

$$\lim_{y\to 0} \frac{\partial R}{\partial y} = \frac{y^2-x^2}{2y} = \frac{-x^2}{0} = \infty \tag{9}$$

Both partial derivates of radius R go to infinity, while the partial derivates of curvature κ go respectively to zero and $2/x^2$.

Vraag 3

In the appendix there are three digital CMOS-based camera data sheets given. Select one of the product specifications, collect and compute the following values: (Show your derivations)

(a) Dynamic range

(b) Resolution (of a single pixel)

(c) Bandwith

Antwoord 3

	pixel depth	bandwidth	Dynamic range
NEO	30.000 e^-	560.200 MHz, 100 fps	
MT9V	10 bit	26.6 MHz, 60 fps	> 55 dB, $> 80 - 100$ dB
MT9M	10 bit	48 MPs, 30 fps	68.2 dB

Table 1: Selected camera data

The NEO camera falls outside the scope. So, the other two are chosen. From the table all answers are already there.

(a) Dynamic Range: MT9V, MT9M = > 55 dB, 68.2 dB

(b) Resolution MT9V, MT9M =

$$\frac{1}{2^{10}-1} = \frac{1}{2047} = 0.4 * 10^{-3} \tag{10}$$

(c) Bandwidth: MT9V, MT9M = 26.6 MPS/MHz, 48 MPS/MHz

Vraag 4

Determine the degrees of mobility, steerability, and maneuverability for each of the following:

(a) Bicycle

(b) Dynamically balanced robot with a single spherical wheel

(c) Automobile

Antwoord 4

	mobility δm	steerability δs	maneuverability δM
Bicycle	1	1	2
DBR	3	0	3
Auto	1	1	2

Table 2: Kinematics (see page 71 1^{st} edition)

	Bicycle	DBR	Auto
mobility δm	1	3	1
steerability δs	1	0	1
maneuverability δM	2	3	2

Table 3: Kinematics (see page 71 1^{st} edition)