Question 1

Solve exercise 5.9.2 from the Autonomous Mobile Robots book\(^1\).

Question 2

Assume the following 1-D lineair dynamic system, with a simple probabilistic motion model:

\[
\hat{x}_t = x_{t-1} + u_t + \epsilon_t
\]  \hfill (1)

and a simple probabilistic measurement model:

\[
\hat{z}_t = \hat{x}_t + \delta_t
\]  \hfill (2)

The terms \(\epsilon_t\) and \(\delta_t\) represent respectively the control and measurement error, a random number from a Gaussian distribution \(\mathcal{N}(x; 0, R_t)\) and \(\mathcal{N}(z; 0, Q_t)\). For the moment you can assume that the variance \(R_t = 0\) and \(Q_t = 1\), which means that you have perfect control over the dynamic system (\(\epsilon_t\) can be ignored). For all timesteps, the same input is given \((u_t = 0.5)\). The initial estimate is represented with a Gaussian distribution \(\mathcal{N}(x; \mu_0, \Sigma_0)\) with \(\mu_0 = 5\) and \(\Sigma_0 = 10\).

You receive the following measurements \((z_1 = 0.0, z_2 = 2.1, z_3 = 5.6)\).

(a) Is in this case the assumption of white noise made? Explain your answer.

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\(^1\)Question 2 originates from the course 'Design and Organization of Autonomous Systems' from the Universiteit van Amsterdam. Question 3 originates from the book 'Principles of Robot Motion - Theory, Algorithms and Implementations'. Question 4 originates from the book 'Artificial Intelligence - A Modern Approach'.
(b) Use the measurements \((z_1, z_2, z_3)\) to estimate \((\mu_1, \mu_2, \mu_3)\). For this linear system you can use a traditional Kalman Filter, as described in section 5.6.8 of the book. This will be a two step approach, a prediction and an update step. The result of the prediction step will be a Gaussian distribution \(N(x; \hat{\mu}_0, \hat{\Sigma}_t)\). In the update step you can shift and narrow this distribution to \(N(x; \mu_t, \Sigma_t)\) making use of the measurements and the following precalculated Kalman gain \(K_1 = \frac{10}{11}, K_2 = \frac{10}{21}, K_3 = \frac{10}{31}, K_4 = \frac{10}{41}, K_5 = \frac{10}{51}\).

(c) Explain why the Kalman Gain decreases for every time step.

(d) Lets drop the assumption of perfect control, and reintroduce the control noise \(\epsilon_t\) modelled with a Gaussian distribution \(N(x; 0, 1)\). Recalculate \((K_1, K_2, K_3, K_4, K_5)\) for the given variance \(Q_t = 1\). Explain the observed pattern in the Kalman Gain \(K_t\).

(e) Make a new estimate of \((\mu_1, \mu_2, \mu_3)\) based on the recalculated Kalman Gain \(K_t\).

**Question 3**

What happens if you apply the particle filter SLAM algorithm to a robot whose sensor is almost perfect? For example, what happens when the robot uses (almost) noise-free range sensors? Hint: For near-perfect sensors, the likelihood-function \(P(z|x)\) will be extremely peaked, i.e., it will be almost zero for all measurements that are slightly off the correct noise-free value. How does the accuracy of the sensor affect the number of particles needed?

**Question 4**

Which of the following statements are true and which are false? Explain your answers.

(a) Depth-first search always expands at least as many nodes as A* with an admissible heuristic\(^2\).

(b) A* is of no use in robotics because observations, states and movements are continuous.

(c) Breath-first search is complete\(^3\) even if zero step costs are allowed.

(d) Assume a rook on a chessboard; the piece can move any number of squares in a straight line, horizontally or vertically, but cannot jump over other pieces. Manhattan distance\(^4\) is an admissible heuristic for moving the rook from square A to square B in the smallest number of moves.

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\(^2\)An **admissible heuristic** is a measure that *never overestimates* the cost to reach a goal.

\(^3\)Completeness indicates that the algorithm is guaranteed to find a solution when there is one.

\(^4\)City block or **Manhattan distance** is the sum of horizontal and vertical distances between grid cells.