UNIVERSITEIT VAN AMSTERDAM

Faculteit der Natuurwetenschappen, Wiskunde en Informatica

Tentamen

Autonome Mobiele Robotica

Bachelor Kunstmatige Intelligentie jaar 3

2e Deeltentamen
Datum: 18 december 2014
Tijd: 9.00-11.00

Aantal pagina's: 6 (inclusief voorblad)
Aantal vragen: 5
Maximaal aantal te behalen punten: 10

VOORDAT U BEGINT

- **Wacht** tot u de instructie krijgt het tentamen te openen.
- Controleer of uw versie van het tentamen compleet is.
- Schrijf **uw naam en studentnummer en indien van toepassing versienummer** op **elk vel papier** dat u inlevert en **nummer de pagina's**.
- U dient uw **mobiele telefoon** uit te schakelen en te bewaren in uw jas of tas.
  Uw **jas en tas** moeten onder uw tafel liggen.
- **Toegestane hulpmiddelen**: boek & grafische rekenmachine.

HUISHOUDELIJKE MEDEDELINGEN

- De eerste 30 minuten en de laatste 15 minuten mag u de zaal niet verlaten, ook niet voor het bezoeken van het toilet.
- Op verzoek van de examinator (of diens vertegenwoordiger) moet u zich kunnen legitimeren met een bewijs van inschrijving of een geldig legitimatiebewijs.
- Tijdens het tentamen is toiletbezoek niet toegestaan, tenzij de surveillant hier toestemming voor geeft.
- 15 minuten voor het eind wordt u gewaarschuwd dat het inlevertijdstip nadert.
- Vul indien van toepassing na afloop van het tentamen alstublieft het evaluatieformulier in.

Succes!

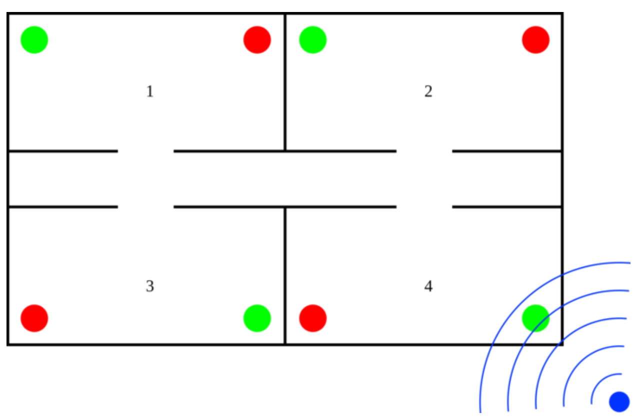
Autonomous Mobile Robots (AUMR6Y, Fall 2014)
Examination: Localization, Planning & Navigation

Thursday December December 18th, 09:00-11:00, G0.23

Arnoud Visser and Toto van Inge

Question 1

Consider the following discrete Markov localization problem. A robot needs to localize within the map, illustrated in Fig. 1. The red and green dots are for your convenience and not visible for the robot. You know already that the robot is in one of the four rooms (labeled $1 \ldots 4$). The robot is equipped with a laser scanner which enables the robot to localize and navigate through the room, but the map has not enough information to infer in which room it is. However, the robot can also measure the strength of the Wifi access point located at the blue dot in the image. It is your task to build a localization algorithm based on the Wifi strength.



Figuur 1: A map with four equivalent rooms

The robot first navigates to one corner of the room opposite to the door (indicated by a green dot), takes a first Wifi-strength measurement z_1 at that point. Then it navigates to the other corner of the room opposite the door (indicated with a red dot), and takes a second Wifi-strength z_2 measurement. Based on this two measurements, we like to know the probability that robot is respectively in room 1,2,3 or 4.

(a) What is the prior belief state **BelPrior** of the probability that robot is respectively in room 1,2,3 or 4? Specify this **BelPrior** as a $4x1$ vector.

The Wifi signal is modeled with a simple distance based model: $I(d) = \frac{1}{d^2}(1+\nu)$, where d is the Euclidean distance between both antennas and ν is white noise (with the Gaussian distribution $\mathcal{N}(0, 0.1)$). The first measurement z_1 on one of the four green dots gives a Wifi strength of 0.0042. This measurement will be compared with the measurement model $I(d)$.

(b) Give the formula to calculate the Euclidean distance, when the coordinates from the green, red and blue dots are known at the map.

The distance between the green dots in room 2 and 3 and the access point at the blue dot are nearly equivalent: $d_2 = 17.03m$ and $d_3 = 13.34m$. This corresponds an expected intensity of $I(d_2) = 0.0034$ and $I(d_3) = 0.0056$, close the actual observation $z_1 = 0.0042$. The likelihood of each of the observations could be represented with the formula:

$$\text{likelihood}(z, d) = \frac{1}{0.01\sqrt{2\pi}}\exp\Big(-\frac{(\frac{z}{I(d)}-1)^2}{0.2}\Big) \tag{1}$$

(c) Explain the origin of the term 0.01 and 0.2 in equation 1.

If one also calculates the expected intensity for observations at the green dots in room 1 and 4 ($d_1 = 24.69m$ and $d_4 = 4.24m$), the likelihood calculation with equation 1 gives the following result:

$$\text{likelihood}(z_1, d_i) = [0.0001 \quad 12.5494 \quad 11.5740 \quad 0.2220]$$

(d) If this likelihood represents the probability $p(z_t|x_t, M)$ of an observation z_t given a position $x_t = i$ and a map M, which two other terms are needed to calculate the posterior belief **BelPosterior** about the location of the robot (after the first Wifi-strength measurement)?

(e) Show with a calculation that it is 4% more probable that the robot is room 2 than that it is in room 3.

At the red dot the robot does a second Wifi-strength measurement, with a value of $z_2 = 0.0026$. This measurement is close to the observations expected in room 1 and room 3 ($I(d_1) = 0.0030$ and $I(d_3) = 0.0022$).

(f) Which prior belief **BelPrior** should the robot use the calculate the posterior belief **BelPosterior** about the location of the robot after the second Wifi-strength measurement)?

(g) What will be the most probable location after the two Wifi-strength measurements $z_1 = 0.0042$ and $z_2 = 0.0026$ at respectively one of the green and one of the red dots?

Question 2

The full SLAM posterior can be written in the factored form:

$$p(x_{1:t}, m|z_{1:t}, u_{0:t-1})) = p(x_{1:t}|z_{1:t}, u_{0:t-1}) \prod_{n=1}^{N} p(m_n|x_{1:t}, z_{1:t}) \tag{2}$$

In the second factor of the factorization, the landmarks are supposed to be independent given the complete trajectory $x_{1:t}$ and the observations $z_{1:t}$. Is it possible to condition the map on the most recent pose x_t only? That is:

$$p(x_{1:t}, m|z_{1:t}, u_{0:t-1})) = p(x_{1:t}|z_{1:t}, u_{0:t-1}) \prod_{n=1}^{N} p(m_n|x_t, z_{1:t}) \tag{3}$$

Explain your answer.

Question 3

Name three key, distinct advantages for each of the following SLAM algorithms: EKF SLAM, GraphSLAM, and FastSLAM.

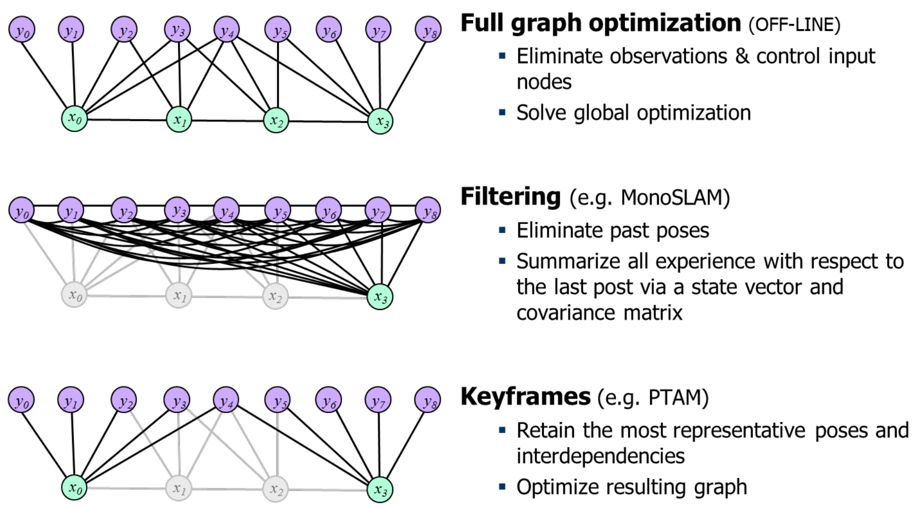
Question 4

In Chapter 5 is explained that the SLAM problem could be expressed as finding correlations between the robot position and the locations of the robot. The full SLAM problem could be approximated with the filtering or keyframes approach, as illustrated in Fig. 2. In this figure x_i denotes the robot poses and y_j the landmark positions.

(a) Suppose we have 8 landmarks in the SLAM map, as is the case in the figure above. If we would choose the keyframes approach, no links would be introduced among the landmarks. If we would follow the filtering approach, how many new links between the landmarks would have to be introduced? Note: do not consider any new links introduced between the current pose and the landmarks.

(b) If instead the number of landmarks in the SLAM map is now 200, how many new links between the landmarks would have to be introduced?

(c) Consider now the case, where we are conducting SLAM using the Extended Kalman Filter using a wheeled robot that moves in a plane measuring point features on this plane. So the



Figuur 2: The correlation between landmarks and robot positions for different SLAM-approaches

state of this robot is characterized by its location (x, y) and its direction (θ), while the state of each point feature (landmark) is characterized by its estimated location (in x and y). If we have constructed a SLAM map with 200 features as in question **4b** above, how many elements does the EKF covariance matrix contain?

(d) A covariance matrix is actually symmetric (and positive semi-definite). As a result, there is a lot of repetition in the elements stored, with the unique elements contained in the upper (or lower) triangular space of this matrix (i.e. containing also the elements along the diagonal). Considering the links introduced in the graphical representation of the filtering approach, how many elements does one link *between two point features* (each characterized in x and y) correspond to in the unique elements of the covariance matrix?

(e) How many elements does one link *between a point feature and the robot state* correspond to in the unique elements of the covariance matrix?

Question 5[1]

Does the wave-front planner in a discrete grid yield the shortest distance? (If so, in what metric?)

[1] Question 1 and 4 originate from the ETX Autonomous Mobile Robots course. Question 3 orginates from the book 'Probabilistic Robotics' by Sebastian Thrun *et al*. Question 5 originates from 'Principles of Robot Motion' by Howie Choset *et al*.