Probabilistic Robotics (BAIPR6, Fall 2009)
Examination: Basics & Markov localization

Assigned: Week 42, Due: Week 44 (Monday September 26th, 15:00)

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The solutions have to be mailed individually to Arnoud Visser <A.Visser@uva.nl>.

Question 1

A robot equipped with a sensor for detecting whether a door is open or closed obtains a reading $z = 42$ from the sensor. From previous experience you know that $P(z = 42|open) = 0.6$ (the conditional probability obtaining a reading $z = 42$ given that the door is open) and that $P(z = 42|\neg open) = 0.3$. The door can only be either completely open or completely closed and since you have no prior knowledge about the state of the door at the moment of the measurement, you assume that $P(open) = P(\neg open) = 0.5$.

Using the given information and Bayes rule compute the probability $p(open|z = 42)$ of the door being open given the reading $z = 42$.

Question 2

Assume the following 1-D lineair dynamic system, with a simple motion model:

$$x_t = x_{t-1} + u_t + \epsilon_t \tag{1}$$

and a simple measurement model:

$$y_t = x_t + \delta_t \tag{2}$$

The terms ϵ_t and δ_t represent respectively the control and measurement error, a random number from a Gaussian distribution $\mathcal{N}(x; 0, R_t)$ and $\mathcal{N}(y; 0, Q_t)$. For the moment you can assume that the variance $R_t = 0$ and $Q_t = 1$, which means that you have perfect control over the dynamic system (ϵ_t can be ignored). For all timesteps, the same input is given ($u_t = 0.5$). The initial estimate is represented with a Gaussian distribution $\mathcal{N}(x; \mu_0, \Sigma_0)$ with $\mu_0 = 5$ and $\Sigma_0 = 10$.

You receive the following measurements ($y_1 = 0.0, y_2 = 2.1, y_3 = 5.6$).

(a) Use the measurements (y_1, y_2, y_3) to estimate (μ_1, μ_2, μ_3). For this lineair system you can use a traditional Kalman Filter, as described in section 3.2 of the book. This will be a two step approach, a prediction and an update step. The result of the prediction step will be a Gaussian distribution $\mathcal{N}(x; \overline{\mu}_0, \overline{\Sigma}_t)$. In the update step you can shift and narrow this distribution to $\mathcal{N}(x; \mu_t, \Sigma_t)$ making use of the measurements and the following precalculated Kalman gain ($K_1 = \frac{10}{11}, K_2 = \frac{10}{21}, K_3 = \frac{10}{31}, K_4 = \frac{10}{41}, K_5 = \frac{10}{51}$).

(b) Explain why the Kalman Gain decreases for every time step.

(c) Lets drop the assumption of perfect control, and reintroduce the control noise ϵ_t modelled with a Gaussian distribution $\mathcal{N}(x; 0, 1)$. Recalculate $(K_1, K_2, K_3, K_4, K_5)$ for the given variance $Q_t = 1$. Explain the observed pattern in the Kalman Gain K_t.

(d) Make a new estimate of (μ_1, μ_2, μ_3) based on the recalculated Kalman Gain K_t.

Question 3

Solve exercise 6.10.1 from the Probabilistic Robotics book. [1]

[1] Question 1 originates from the course 'Introduction to Mobile Robotics' at the Albert-Ludwigs-Universität Freiburg. Question 2 originates from the course 'Design and Organization of Autonomous Systems' from the Universiteit van Amsterdam.