Probabilistic Robotics (BAIPR6, Fall 2010)
Examination: Basics & Localization

Assigned: Week 42, Due: Week 44 (Wednesday November 3rd, 10:00)

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The solutions have to be mailed individually to Arnoud Visser `<A.Visser@uva.nl>`.

Question 1

Consider a robot that can be in any of 10 possible states ($x = 1, ..., x = 10$). The robot is equipped with a sensor that indicates the state x the robot is in. This sensor, however, is not perfect and indicates the correct state only 90% of the cases. In 10% of the cases, the sensor indicates an incorrect state with equal probability.

(a) Compute the probability $P(x = 5|z = 5)$ for the robot of being in state 5 given that the sensor indicates that the robot is in state 5, if the robot could be in any state before the first sensor indication. What is the probability $P(x = 4|z = 5)$ for the robot of being in state 4?

(b) Assuming that a second indication $z = 5$ is obtained, what would be the probability of being in state 5?

(c) What is the *belief state* (the probability $P(x)$ for all possible values of x) for the sequence of sensor indications: $z_1 = 4, z_2 = 5, and z_3 = 5$, if the robot could be in any state before the first sensor indication.

Question 2

Assume the following 1-D lineair dynamic system, with a simple motion model:

$$x_t = x_{t-1} + u_t + \epsilon_t \tag{1}$$

and a simple measurement model:

$$y_t = x_t + \delta_t \tag{2}$$

The terms ϵ_t and δ_t represent respectively the control and measurement error, a random number from a Gaussian distribution $\mathcal{N}(x; 0, R_t)$ and $\mathcal{N}(y; 0, Q_t)$. For the moment you can assume that the variance $R_t = 0$ and $Q_t = 1$, which means that you have perfect control over the dynamic system (ϵ_t can be ignored). For all timesteps, the same input is given ($u_t = 0.5$). The initial estimate is represented with a Gaussian distribution $\mathcal{N}(x; \mu_0, \Sigma_0)$ with $\mu_0 = 5$ and $\Sigma_0 = 10$.

You receive the following measurements ($y_1 = 0.0, y_2 = 2.1, y_3 = 5.6$).

(a) Use the measurements (y_1, y_2, y_3) to estimate (μ_1, μ_2, μ_3). For this lineair system you can use a traditional Kalman Filter, as described in section 3.2 of the book. This will be a two step approach, a prediction and an update step. The result of the prediction step will be a Gaussian distribution $\mathcal{N}(x; \overline{\mu}_0, \overline{\Sigma}_t)$. In the update step you can shift and narrow this distribution to $\mathcal{N}(x; \mu_t, \Sigma_t)$ making use of the measurements and the following precalculated Kalman gain ($K_1 = \frac{10}{11}, K_2 = \frac{10}{21}, K_3 = \frac{10}{31}, K_4 = \frac{10}{41}, K_5 = \frac{10}{51}$).

(b) Explain why the Kalman Gain decreases for every time step.

(c) Lets drop the assumption of perfect control, and reintroduce the control noise ϵ_t modelled with a Gaussian distribution $\mathcal{N}(x; 0, 1)$. Recalculate $(K_1, K_2, K_3, K_4, K_5)$ for the given variance $Q_t = 1$. Explain the observed pattern in the Kalman Gain K_t.

(d) Make a new estimate of (μ_1, μ_2, μ_3) based on the recalculated Kalman Gain K_t.

Question 3

Solve exercise 7.11.4 from the Probabilistic Robotics book. [1]

[1]Question 1 originates from the course 'Introduction to Mobile Robotics' at the Albert-Ludwigs-Universität Freiburg. Question 2 originates from the course 'Design and Organization of Autonomous Systems' from the Universiteit van Amsterdam.