Probabilistic Robotics
PRR06, Fall 2017
Exercise: Motion Model

Assigned: Tuesday September 19;
Due: Tuesday September 26; 13:00 in the afternoon

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Question 1

We have a robot equipped with wheel encoders and on-board software that combines the measurements from the multiple encoders in a tuple of time-discrete odometry measurements $(\delta_{\texttt{rot1}}\delta_{\texttt{trans}}\delta_{\texttt{rot2}})^T$.

(a) Let the robot start at pose $(xy\theta)^T = (0m, 0m, 0^o)^T$ and obtain the following subsequent odometry measurements: $(\delta^1_{\texttt{rot1}}\delta^1_{\texttt{trans}}\delta^1_{\texttt{rot2}})^T = (10^o, 3m, 10^0)^T$ and $(\delta^2_{\texttt{rot1}}\delta^2_{\texttt{trans}}\delta^2_{\texttt{rot2}})^T = (-20^o, 10m, -10^0)^T$.

Please assume perfect measurements and calculate the exact poses $(x^1y^1\theta^1)^T$ and $(x^2y^2\theta^2)^T$ with equation (5.40) from the book.

(b) How would your pose estimate look for first movement $(\delta^1_{\texttt{rot1}}\delta^1_{\texttt{trans}}\delta^1_{\texttt{rot2}})^T = (10^o, 3m, 10^0)^T$ under the following simple uniform error model? Please draw the extremes of the pose estimates into into a a diagram.

$$\begin{aligned}\hat{\delta}_{\texttt{rot1}} &= \delta_{\texttt{rot1}} \pm \epsilon_{\texttt{rot1}}\\ \hat{\delta}_{\texttt{trans}} &= \delta_{\texttt{trans}} \pm \epsilon_{\texttt{trans}}\\ \hat{\delta}_{\texttt{rot2}} &= \delta_{\texttt{rot2}} \pm \epsilon_{\texttt{rot2}}\end{aligned}$$

with $(\epsilon_{\texttt{rot1}}\epsilon_{\texttt{trans}}\epsilon_{\texttt{rot2}})^T = (5^o, 0.5m, 10^0)^T$.

(c) Apply the same error model again on the n extreme poses of the previous question. This would result in $n \times n$ poses. Please draw the $\frac{n\times n}{2}$ positions of the extremes of this motion estimate (i.e. without final orientation) after the second movement.

Question 2

Now consider a simple kinematic model of an idealized *bicycle*. Both tires are of a diameter d, and are mounted on a frame of length l. The font tire can swivel around a vertical axis, and its steering angle will be demoted α. The rear tire is always parallel to the bicycle frame and cannot swivel.

For the sake of this exercise, the pose of the bicycle shall be defined through three variables: the x-y location of the center of the front tire, and the angular orientation θ (yaw) of the bicycle frame relative to an external coordinate frame. The controls are the forward velocity v of the bicycle, and the steering angle α, which we will assume to be constant during each prediction cycle.

Provide the mathematical prediction model for a time interval Δt, assuming that it is subject to Gaussian noise in the steering angle α and the forward velocity v. The model will have to predict the posterior of the bicycle state after Δt, starting from a known state. If you cannot find an exact model, approximate it, and explain your approximations.

Question 3

Consider the kinematic bicycle model from Question 2. Implement a sampling function for posterior poses of the bicycles under the same noise assumptions.

For your simulation, you might assume $l = 100cm$, $d = 80cm$, $\Delta t = 1sec$. $|\alpha| \leq 80°$, $v \in [0; 100]cm/sec$. Assume further that the variance of the steering angle is $\sigma_\alpha^2 = 25^{\circ 2}$ and the variance of the velocity is $\sigma_v^2 = 50cm^2/sec^2 \cdot |v|$. Notice that the variance of the velocity depends on the commanded velocity.

For a bicycle starting at the origin, plot the resulting sample sets fro the following values of the control parameters:

problem number	α	v
1	$25°$	$20cm/sec$
2	$-25°$	$20cm/sec$
3	$25°$	$90cm/sec$
4	$80°$	$10cm/sec$
5	$85°$	$90cm/sec$

All your plots should show coordinate axes with units.

Hand-In

When you have completed the assignment, upload your solution to Blackboard. This should be a PDF, with your Matlab scripts as pseudo-code (for example with the matlab-prettifier package). If you have only partially solved the assignment, upload your partial solution.

Acknowledgements

This assignment is originally written by Wolfram Burgard, Jürgen Sturm and Boris Lau. The second question and third question are equavalent with question 5.8.4 and 5.8.5 from the book.