Probabilistic Robotics
BAIPR6, Spring 2008
Examination: Basics & Localization
Assigned: Week 13, Due: Week 15
Friday April 11th, 24:00 in the night

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The solutions have to be mailed individually to Arnoud Visser `arnoud@science.uva.nl`.

Question 1

Consider a wheeled robot which moves over a flat surface. Each wheel has an y-axis. When a rolling motion occurs, all y-axis overlap at a point; the instantaneous Center of Curvature. Consider the case that the Instantaneous Center of Curvature is outside the robot, and the robot moves from point $(0,0)$ to (x,y) (see figure 2).

A natural way to represent the movement as an circular movement with a radius R an the sector angle ϕ. (x,y) is a point on the circle, which means

$$\begin{cases} x = R\sin\phi \\ y = R(1-\cos\phi) \end{cases} \quad \Longleftrightarrow \quad \begin{cases} R = \dfrac{x^2+y^2}{2y} \\ \phi = atan(\dfrac{2xy}{x^2-y^2}) \end{cases}$$

This representation has disadvantage that for small y (straight ahead!), a small change in (x,y) may cause a big change in parameter R. You can verify this with by computing the Jacobian; you should get:

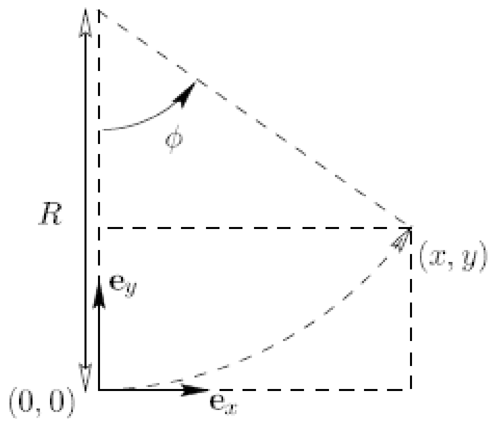

Figure 1: Turning to point (x, y)

$$\begin{pmatrix} dR \\ d\phi \end{pmatrix} = \begin{pmatrix} \frac{x}{y} & \frac{y^2-x^2}{2y^2} \\ \frac{-2y}{x^2+y^2} & \frac{2x}{x^2+y^2} \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix}$$

For this reason it is much better to characterize the path by the *curvature* $\kappa \equiv 1/R$, which changes smoothly around the forward direction.

Now, compute the Jacobian for the (κ, ϕ) representation and show that it changes more smoothly.

Question 2

A robot is moving along the middle of a corridor with a given accurate map, as depicted in figure 2.

At some of the given locations x_i the robot takes a measurement of the distance z_k, using a laser beam. Every measurement is corrupted only with additive Gaussian noise $\mathcal{N}(\mu, \sigma)$ with $\mu = 0m$ and $\sigma = 1m$. The scanner range is $80m$. The measured distances are $z_1 = 1.1m$, $z_2 = 2.1m$, $z_3 = 8.6m$, $z_4 = 9.4m$. The correspondence between z_k and x_i is unknown.

(a) For each measurement, determine the most likely robot pose by calculating the probabilities for each position given the measurement using Bayes'rule. Assume an uniform distributed *prior*. The *evidence* term (denominator) can be neglected, but the probabilities should be scaled such that $\sum_{i=1}^{4} P(x_i|z) = 1$.

(b) The robot believes that taking measurements at the positions x_2 and x_3 is in general three times as likely as doing so at x_1 and x_4. se this prior information to recalculate the probabilities of (a).

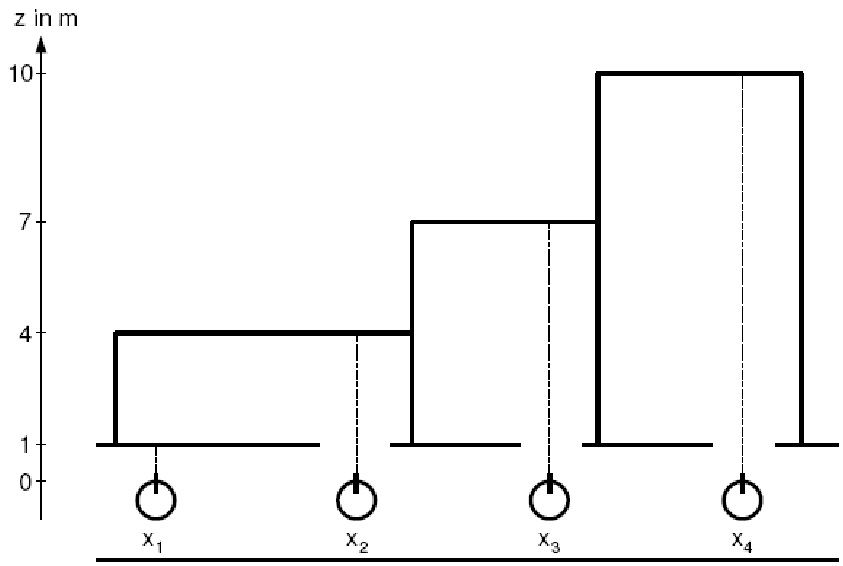

Figure 2: Accurate map of a corridor with three rooms

(c) Because people are present in the corridor, a faulty measurement of $z = 1m$ can occur in 33% of the cases, no matter the actual distance. How does this change the results of (a) and (b).

Question 3

Solve exercise 7.11.4 from the Probabilistic Robotics book.