Portable calibration systems for robots[1]

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1.1. Abstract

In recent years both the attainable accuracy in terms of repeatability and the accuracy requirements for robots have increased. The more stringent requirements in terms of accuracy stem, e.g., from the advent of off-line programming techniques, which can only be used if the robot's positioning can be predicted with sufficient accuracy. It is, therefore, necessary to maintain this accuracy. This requires a measuring system that can easily be used on the work floor for incidental and periodic (partial) recalibration.

One of the goals of the CAR ESPRIT II project[3], is the development of a prototype for such a portable calibration system. The requirements study and a survey of possible techniques now have been completed and the calibration system is now in the design phase. Beside accuracy, the requirements also cover issues such as ease of use, portability, robustness, self-calibration and cost.

The measurements produced by the system will be used to compute improved parameters for the kinematical model for the robot. This approach allows the measurements to be used for a twofold purpose, viz. for improvements in the positioning accuracy and for diagnostic purposes, identifying any deteriorations in the system.

In order to obtain sufficient accuracy in the model's parameters, measurements of the tool center point's pose with a position accuracy of the order of 0.1 mm and an orientation accuracy of about 1' are required. The measurements should be well distributed over the joint coordinate space of the robot.

After an initial exploration several optical measuring techniques have been selected for further study. The advantage of such techniques is that they allow accurate measurements over a large range of distances. One such technique is based on the use of beams of light and position sensitive devices; another on the use of cameras and space resection.

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[3] The CAR project (ESPRIT II project 5220) is a collaboration between the Fraunhofer-Institut für Produktionsanlagen und Konstruktionstechnik, LEICA (UK) Ltd., the University of Amsterdam, TGT and KUKA Schweißanlagen und Roboter GmbH.

The performance of these systems has been analyzed using a first order analysis technique based on singular value decomposition. The technique has the advantage that it provides a clear indication of relative sensitivity to the various types of errors in the robot pose. It can also be used to analyze the systems capability for self-calibration.

1.2. Introduction

Robot calibration can serve various purposes. The most obvious of these is to improve the static and/or dynamic positioning accuracy of the robot, e.g. for off-line programming applications. A second important application is the use as a diagnostic tool. Accurate calibration measurements can be used to determine some or all parameters of a suitable kinematic or dynamic model of the robot. In this way inaccuracies and wear in specific components of the robot may be identified.

Depending on the aims of a calibration session and the circumstances under which it is performed, different requirements must be met by the measurement and data-analysis procedures. A moderate improvement in the positioning accuracy over a limited part of the reachable work space can be more easily attained than the accurate identification of all model parameters. Calibrating a robot at first installation is different from recalibrating a robot in a production line.

A large number of robot calibration tools in now available commercially, each with its own range of applicability and its own requirements. Prices and required levels of expertise vary over a very wide range.

In this chapter we shall specifically address the problem of calibration applied to parameter identification on the work floor, e.g. after repairs. The problem of parameter identification in general is treated in the chapter by [Albright and Schröer] in this book.

We shall begin by discussing the accuracy requirements for these types of measurement, as well as certain other important constraints. Next we will present a survey of possible measuring techniques and their limitations, with some examples of commercially available products. Finally, the expected performance of a few designs for measuring systems are analyzed in some detail. The analysis technique employed for this is discussed in an appendix.

1.3. Applications and requirements

1.3.1. Off-line programming and calibration

In recent years the accuracy requirements for robots have increased, mainly because of the broader use of off-line programming techniques, but also due to a wider range of applications.

The application of off-line robot programming techniques for tasks which require a high precision may be hampered by discrepancies between programmed position and the attained real position. It can only be successful if a good absolute positioning accuracy can be obtained, i.e. an accuracy of the same order as the repeatability. To minimize the positioning errors extensive calibration of the robot and its workcell is needed. In the course of such a calibration a full set of kinematical parameters of the robot can be obtained. However, currently extensive calibration still is an expensive procedure, which is unacceptable when it has to be applied repeatedly.

Errors in the positioning of a robot can be seen as a combination of two separate effects, viz. limited pose repeatability and model errors. Robot pose repeatability can range from 0.1 mm to 0.5 mm and up. Its effects cannot be removed in any simple way. Repeatability thus defines the limit of the accuracy attainable with a particular robot. In order to approach this limit, the contribution of the model errors should be small compared to those due to repeatability. The required model accuracy thus is about 0.05 mm to 0.25 mm at the tool center point. In order to construct a model with this kind of accuracy, pose measurements with at least this accuracy are required. Full pose information, including orientation information may be required e.g. when tools of various lengths will be employed. 0.05 mm at 30 cm corresponds to about 0.5 arc minute.

In many situations full accuracy calibration measurements covering only those parts of the workspace that are actually used will suffice. Almost any model fitting such measurements will provide the required positioning accuracy.

1.3.2. Parameter identification and calibration

When calibration measurements are used only to obtain a sufficiently good positioning accuracy, the accuracy requirements for the measurements will only be a few times better than the robot repeatability. When, however, the measurements should also serve diagnostic purposes, the requirements may be much more stringent. In order to successfully distinguish the error contributions of various components in the diagnostic model, a large number of highly accurate measurements, covering a large part of the robot joint space may be needed. Here it is of even more importance to obtain complete pose information.

Clearly, the pose information contained in each individual measurement cannot be better than the pose repeatability of the robot. However, as in any measurement procedure where the errors contain a stochastic component, the error in the final result can be reduced to below this limit by obtaining a sufficiently large sample. Any systematic contributions to the measurement errors must, of course, be much smaller than the required measurement accuracy.

A full calibration of a robot, involving all model parameters, requires a large number of highly accurate measurements, covering as large a part of the robot work-space as possible. Such a calibration must be performed at production and/or first installation of the robot.

However, it is known that during the lifetime of the robot only a limited and predictable subset of all the parameters needs to be recalibrated. Consequently, fewer measurements, covering a smaller fraction of the robot work-space will often suffice for a rapid, local recalibration of e.g. a newly installed or repaired robot.

1.3.3. Objectives of CAR

As argued above, highly accurate measurements of all six pose parameters, albeit over a limited part of the robot work space, will be needed at frequent intervals over the lifetime of a robot. Such measurements should be obtained with calibration system, which can be applied on-line for incidental and partial recalibration in between production cycles of the robot. It should meet the following requirements:

1) The system must be able to provide measurements of an accuracy similar to that provided by e.g. theodolite systems, but over a smaller part of the workspace of the robot.

2) The should be simple enough to be used by essentially untrained personnel and be usable in an industrial environment.

3) A derived requirement is that the measurement system should at least contain a number of consistency checks, so that damage to the system can be detected and reported - and if possible corrected for.

4) The system should be sufficiently inexpensive to produce that an economically viable product line can be established.

I.e., there is a need for reliable, low-cost measuring systems for partial calibration. Such systems should be portable, robust and easy to operate.

One of the goals of the CAR ESPRIT II project 5220: "Calibration Applied to Quality Control and Maintenance in Robot Production" is the development of a prototype for a portable calibration system such as described in the preceding section . The requirements study and a survey of possible techniques now have been completed and the low-cost calibration system is now in the design phase.

1.3.4. Requirements for a portable measurement system

The most widely applied approach to calibration is the use of theodolite systems. Unfortunately theodolite systems impose some severe constraints on the calibration procedures. In the first place the theodolites (as well as most other global measuring systems) require a special set-up. This can limit their applicability on the workfloor. In the second place the cost of these calibration systems, which are in the order of $ 150 000, prohibits their application on a large scale. Thirdly, a full calibration is a time-consuming procedure, which requires the robot to be removed from the production process for a considerable amount of time.

A measuring system used for partial calibration needs to perform local measurements relative to some reference points or objects. Due to the robot accuracy the applied measuring system which has to be designed must be very precise to compete with robot repeatability. The measurements have to be performed within a volume of about 1 m^3.

The system should be mounted on the robot end-effector and identify the robot end-effector’s position and orientation when the robot takes one pose.

Measurements must be made with non-contact sensors to avoid external forces and the attendant deformations.

The measurements should be performed without manual interaction and in collaboration with an automatic system for robot calibration. The measuring system has to be easy to handle, portable and of low cost when concerning its utilization of partial calibration.

Accuracy

Calibration accuracy by itself depends on the accuracy of the measurement system and the inaccuracies due to robot parameters that are not modeled. Basically, the requirements of the measurement system are measurements with an accuracy in position of about 0.05 mm (systematic+3σ) and 0.01 degree in orientation can be performed for at least 40 significantly different poses.

As the positioning accuracy of the robot before calibration may be quite low, it is essential that the measuring device at least cannot be damaged by an error of about 40 mm, and preferably can start its measuring cycle given a positioning error of that magnitude.

1.4. Résumé of sensing techniques

The requirements state that both position and orientation must be measured with high accuracy. Position measurements throughout a volume can be obtained in a number of ways. Orientation measurements almost always are obtained as a tangential displacement. In order to obtain the required accuracy, a baseline of at least 0.3 m is implied by the above requirements if the same sensors are used for position and orientation measurements.

A large choice of sensing devices is available that can, in principle, be used as the basis of out low cost sensor design. Part of our effort was spent in obtaining information on the prices and capabilities of various devices.

The information obtained on commercially available devices will be reported separately.

1.4.1. Distance

One method of determining the position of the measuring head relative to the measurement station is to measure its distance to various reference objects in the measurement station. Optical, ultrasound and electrical techniques for distance measurements could be considered. Distances measured by the various devices can either be nearest distances, or distances in a specific direction.

Parallactic/triangulation

An optical technique of sufficient accuracy over ranges up to about 0.1 m. Devices based on a combination of a diode laser and a 1–D position-sensitive device are available commercially. They are moderately large and fairly expensive. As they provide only one data item per measurement, at least six such devices would be required in a single measuring head.

U.S. time of flight

An inexpensive technique. However, it is not sufficiently accurate at the frequencies usually employed and is sensitive to interference. It may be a good choice as an additional device to obtain an initial positioning and as a safety measure to prevent collisions.

Optical time of flight

Prohibitively expensive due to the technical complexity of the system.

Optical Interferometry, fringe counting

A very accurate distance measuring technique, retaining its full accuracy of the order of 0.1µ over its entire working range. Can, and actually must, be used dynamically, i.e. performing continuous measurements while the robot is moving. Is commonly used for partial calibration, but is then generally limited to 1-D position measurements along a single line.

Among others, the Leica "SMART" system and the laser tracking system developed by The University of Technology in Vienna (S. Decker et al., 1992) overcome this limitation.

Electric/electrostatic and inductive

Electric/electrostatic devices are based on the change in the capacitance of a condensor as the distance between the plates is varied; inductive devices make use of changes in the inductance of a coil. Neither has been studied in any great detail for this project as both can only provide a high measuring accuracy for small distances. Combination with bridge techniques should allow very accurate centering in null-type measurements.

1.4.2. Displacement

Displacement measurements as understood here are measurements in which the displacement of the measuring head parallel to the measuring device is to be measured.

Beams & knife edge

In this method a light-beam between to locations on the measurement station is cut by a knife edge in the measuring head. The requirement that the measurement volume be about 1 m^3 implies a beam length of about 1 m, which for optical wavelengths implies a beam-width of about 1 mm. I.e. the cut-off will not be sharp and complex photometric techniques may be required to obtain the required positioning accuracy. Furthermore, it is difficult to obtain all six position parameters in this manner.

Camera

A camera fixed in the MH may be used to observe reference points on the (entirely passive) MS. Using space resection techniques, the position of the MH may be reconstructed. The obtainable accuracy for normal video camera's appears to be marginal, especially as the solutions to the space resection problem are known to be somewhat unstable in many situations. The technique is, however, sufficiently promising still to be considered.

Position sensitive devices

Position sensitive devices are photo-electric devices that allow the intensity-weighted mean position of all the light incident on the device to be measured in either one or two dimensions with an accuracy up to one-thousandth of the device size. In combination with well-collimated (laser) beams and modulation techniques the intersection of a line fixed to the MH with the MS can be measured.

Rasnik

Rasnik is a position measuring device, developed at the Netherlands Institute for Nuclear and High Energy Physics by H. van der Graaf. The RASNIK device consists of three optical components and some specially designed electronics. The three optical components are a LED, a lens and a 4-quadrant detector (4QD), configured as shown in Fig.1.

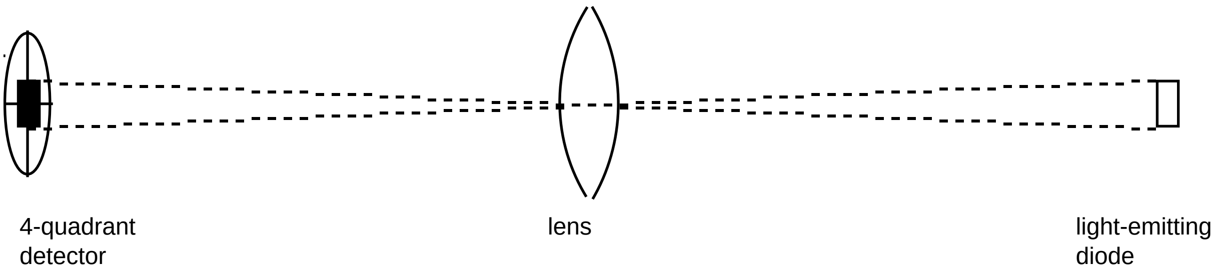

Figure 1. The three optical components of a Rasnik device.

The device measures the misalignment of the three components by comparing the 4 outputs of the 4-quadrant detector. It is made insensitive to ambient light by modulation of the LED output. The three optical components of a Rasnik device are selected and calibrated together and allow measurements with accuracies of the order of 1 micron over displacements of several millimeters. The system is not very sensitive to the accuracy with which the image of the LED is focussed on the 4QD.

1.4.3. Absolute Orientation

Various techniques are available to measure (components of) the orientation in space. We can distinguish two types of device, viz. devices that measure the inclination with respect to the vertical (two angles) and devices that measure the rotation around the vertical.

Inclination measurements can easily be performed with exceedingly high accuracy (less than an arc second, if desired) for angles near to the vertical, e.g. by using a pendulum or a mercury mirror. Both techniques do require the measured object to be at rest.

The design of a device providing a similar accuracy over a larger range of angles is much more difficult.

The measurement of rotations around the vertical is much more problematical, as no absolute reference is simply available (a magnetic compass is not particularly suitable for industrial environments). Such measurements, therefore, must be made using inertial systems, such as gyroscopes. A good accuracy can be attained, but e.g. the rotation of the earth must be taken into account.

1.4.4. Existing robot calibration systems

An extensive review of the available systems at that time is given by Schlüßler (1987, in German).

1.5. Measuring device designs considered for the CAR project

The measuring device is likely to consist of a "measurement station" (MS) fixed in the workspace and a "measuring head" (MH) or "measurement tool" fixed to the robot wrist. The MH can either replace the normal tool or be mounted separately on the wrist. The choice for the one or the other option is determined by the measuring principle and by weight and accuracy considerations.

1.5.1. Calibration procedure

From the point of view of the calibration procedure it would be attractive to control the robot directly from the calibration system, as such an approach would allow null-measurements to be performed, directing the robot precisely to a predetermined position and then obtaining the joint-encoder readings. However, such an approach would be technically complex and result in a system that is not easily adapted to different robots. Therefore, a procedure like the one described below must be followed.

In the calibration procedure a number of components must cooperate to obtain and process the required measurements. These components are:

1) The robot plus robot controller. Together, these must execute a program to move the robot to a specified position, provide the joint-encoder values to the measuring system, wait until a pose measurement has been obtained and move on to the next position.

2) The pose measuring system. This component includes the measuring sensor system, a control computer and the required software. The pose measuring system will wait for the robot to reach a specified pose, read the joint-encoder values, obtain the necessary data from the sensor system and signal the robot to move on to the next pose. When a measuring program has been completed, a file containing tuples of joint encoder values, pose information and error information can be generated[1].

3) The modelling program. This program will take the data generated by the pose measuring system and compute improved model parameters for the robot.

4) The robot program generator. This program will generate a program for the robot controller, taking into account the desired measuring poses generated by either the pose measuring system, or the modelling program. The output will be corrected using the information in the robot model.

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[1] For a measuring system incorporating self-calibration and consistency checks, the complete set of measurements must, in general, be available, before the poses can be reconstructed.

Depending on the type of sensors used by the pose measuring system, the calibration procedure may have to be executed more than once in order to obtain the desired accuracy. E.g. one sensor system may be used for an initial, rough calibration and a different one for a final, precise calibration.

1.5.2. FFC

The FFC has been described in the first interim report. Several variants have been studied. The original design where the four fingers are positioned near the centers of the sides of the reference cube proved to be unstable with respect to rotation around the vertical axis. A rotation of the MH by 30• around the vertical axis solves this problem.

The FFC as a measuring device still presents a large number of problems:

1. Measurements must be done in the immediate vicinity of a reference object. The design, construction and handling of a measurement station with reference objects covering a 1 m^3 3-D region in space is difficult.

2. A very large measuring head (40 to 50 cm!!) seems to be indicated, both to accommodate the six or more distance sensors, and to allow a good angular measuring accuracy.

3. Redundant measurements are difficult to obtain.

For these reasons the FFC probably will be dropped as a design option.

1.5.3. Three and four beam systems

In this and the following sections we shall consider a number of designs that make use of a number of well collimated light sources, e.g. generated by laser diodes in the MH and a grid of 2-D position sensitive devices in the MS. The collimated beams make fixed angles with each other.

Centering a light beam on a PSD fixes two degrees of freedom. With a suitable configuration of 3 beams and 3 PSD-s, all six degrees of freedom can be fixed; if more beams are used, a certain redundancy is introduced that can be used for e.g. self-calibration purposes.

For simplicity only symmetric configurations have been studied, where the beams are distributed evenly over the surface of a cone. Such configurations also have the advantage that the measuring head can simply be rotated by 90• (for a four beam system) to obtain a new measurement.

A problem that must be solved for all optical sensor systems is that of background light. As the output of a PSD is linear with the amount of light incident on the device, and as the response time of a PSD is short (in the order of 0.1 ms), modulation techniques and analog electronics can be used quite reliable to remove the effect of the background light. However, even so, the amount of background light must be kept reasonably small and care must be exercised to select a modulation frequency not present in the background.

Another problem specific to the three and four-beam systems described here, is that they require the use of light beams that make very considerable angles (e.g. 50•) to the normals of the PSD-s. This implies that any protective cover (glass) on the PSD-s may result in ghost images and thus in measurement errors.

Three beam system

The three beam system will consist of a measuring head to be mounted on the robot wrist or tool and a target plate MS to be mounted in the robot work area. The area of space where calibration measurements can be obtained is determined by the size of the MS target plate and the configuration of the PSD-s on this target plate as well as by the angle between the beams and the symmetry axis of the system.

The angle between the beams must be quite large to obtain a stable system. A good choice appears to be 90•, i.e. three mutually perpendicular beams. If we consider such a system, and assume that two beams have been aimed at selected PSD-s, we find that the third beam can be aimed at any PSD in the region illustrated in Fig. 2.

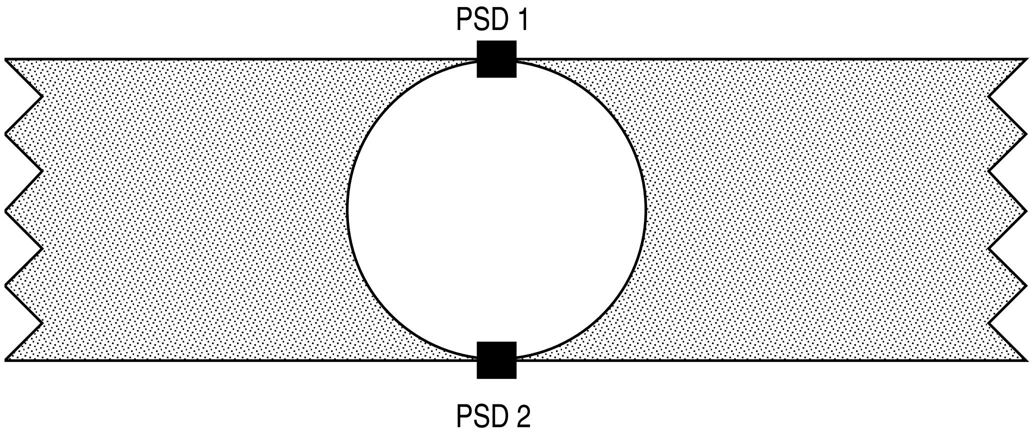

Figure 2. This figure shows the locations (in grey) where the third PSD may be located for a three beam system in which the beams are all perpendicular.

If we now consider the design of a target plate, we find that we can use a limited number of PSD-s to make a large number of measurements possible.

High accuracy measurements can be obtained if a null-technique is used to reposition the robot until the three beams are centered exactly on their target PSD-s.

A disadvantage inherent to the three-beam system is the lack of redundancy. Displacements in the PSD-s or the laser diodes cannot be easily detected in the course of the measuring process. For that reason four-beam-systems have been studied which do allow redundant measurements to be obtained.

Four beam systems

The four-beam-systems considered are entirely similar to the three beam system, except that four symmetrically distributed beams are used. The geometry of the four-beam-system is given in Fig. 3. It is clear that for any opening angle, the four beams can always be aimed at four PSD-s positioned at the corners of a square.

The redundancy in the system allows most of the system parameters to be verified and possibly even to be measured using a least-squares method. It is clear, however, that certain global characteristics cannot be found in this way, e.g. the position and orientation of the target plate (6 parameters), its scale (another two) and certain properties of the measuring head.

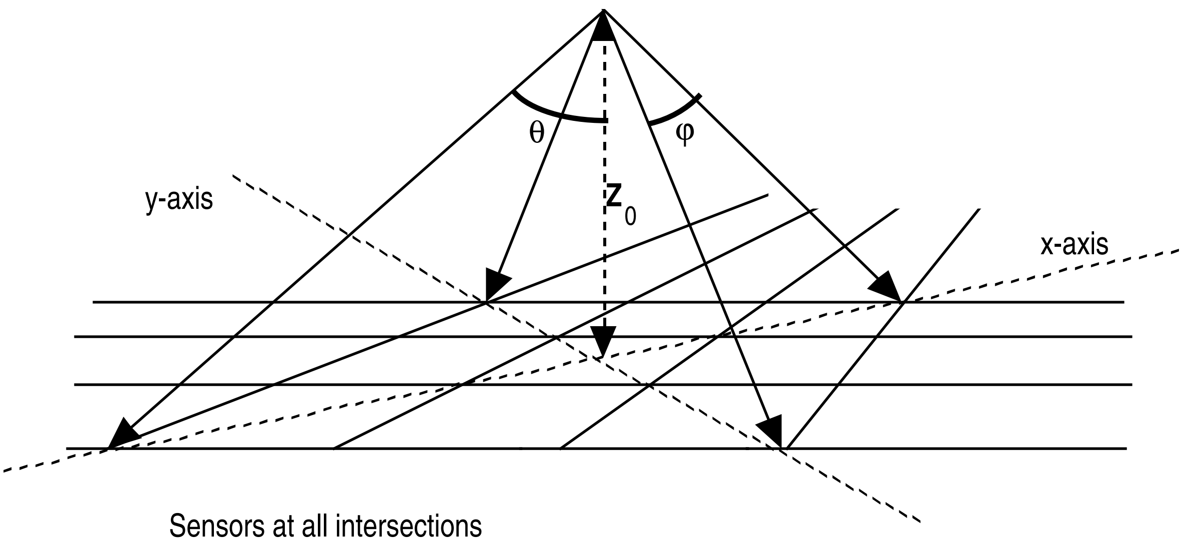

Figure 3. The 4-beam system. The full-drawn arrows represent the 4 beams, which are assumed to intersect at the TCP position. The height of the TCP above the measuring system is Z_0.

The minimal configuration of sensors is given in Fig. 4.

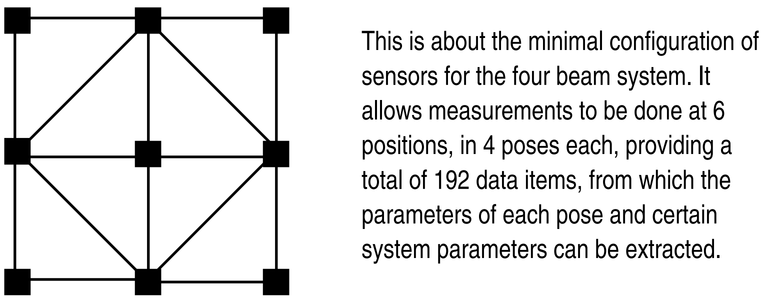

Figure 4. A target plate configuration with only 9 PSD-s.

1.6. Analyzing the design

1.6.1. Introduction

For many approaches to the calibration problem, it is not immediately clear what quantities can be measured to what accuracy. E.g. for the space resection problem it is clear that a displacement or a change in orientation can be measured with good accuracy - but a combination of the two can actually sometimes produce a null result and be undetectable. A sound design for a measuring system should not allow such undetectable motions.

A number of measuring systems considered for the CAR project was investigated with respect to the sensitivity of the measuring system for various deviations on basis of their geometric properties. The approach taken was to linearizing the problem in the neighborhood of an "ideal" measurement pose. I.e., the Jacobian of the measurements with respect to the pose was computed. To this Jacobian a singular-value decomposition was applied, which gives a clear indication of the relative sensitivity of the measuring system for various deviations from this ideal pose.

To demonstrate this analysis technique we apply this to a camera system and we will show that this measuring system has some "short-comings" under some circumstances. Then this technique is used to evaluate the Four Finger Calibration (FFC) sensor system, which will show that this system cannot detect rotations of the pose around the z-axis (vertical axis) if the four fingers are placed symmetrically around the cube. Also the new design for the sensor system (the "three beam" and "four beam" systems) was evaluated that appears promising.

1.6.2. Camera system

A basic problem in image analysis is establishing a relationship between a point in the world coordinate system (X, Y, Z) and the projection of this point onto the image plane (x, y). One variation of this problem is called the Location Determination Problem (LDP) which is formally defined as follows: Given a set of m control points, whose 3-dimensional coordinates are known in some coordinate frame, and given an image in which some subset of the m control points is visible, determine the location ($X_0, Y_0, Z_0, \alpha, \beta, \gamma$) (relative to the coordinate system of the control points) from which the image was obtained. (From Fischler, 1981.) The situation for m = 3 is depicted in Fig. 5.

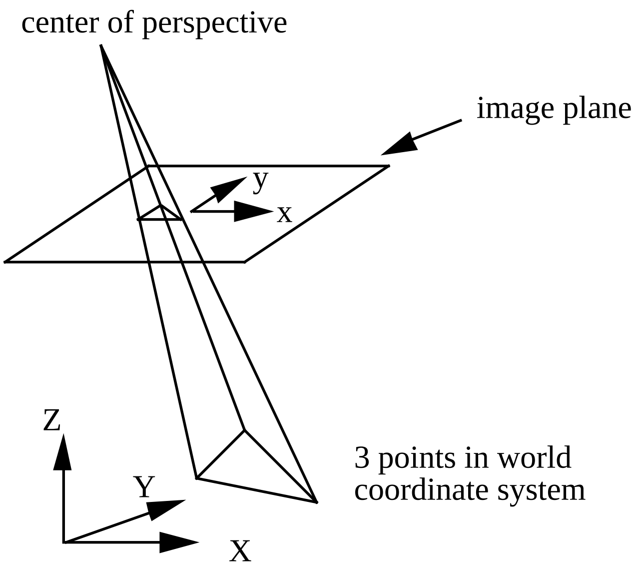

Figure 5. This figure shows the geometry of the Location Determination Problem (From Fischler, 1981).

In order to find the singularities of the transformation involved in the previous problem, we investigate the inverse transformation: Given the location ($X_0, Y_0, Z_0, \alpha, \beta, \gamma$) of the camera and given a set of m control points in the world coordinate system, determine the projection of the m control points onto the image plane. We investigate this problem for m = 3. The geometry of the camera is depicted in Fig. 6, which shows a world coordinate system (X, Y, Z) used to locate both the camera and 3D points (denoted by **w**), and the camera coordinate system (x, y, z) with image points **c**.

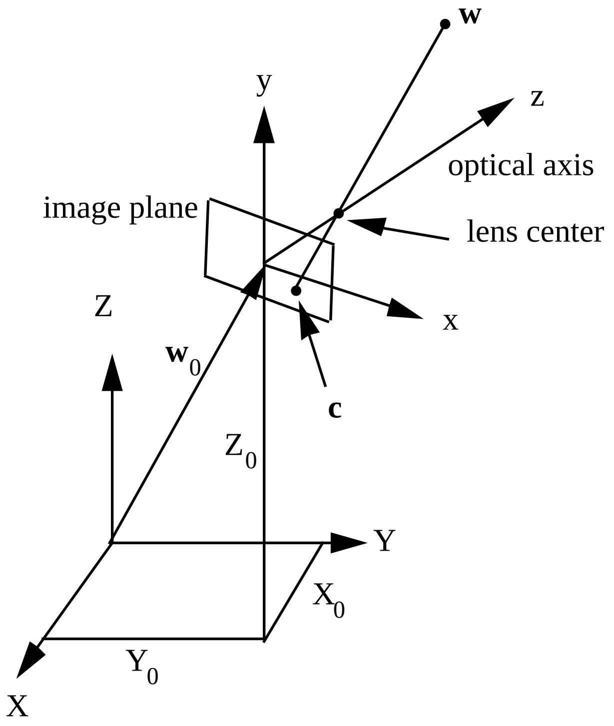

Figure 6. Imaging geometry with two coordinate systems (From Fu, 1987).

Fu (1987) gives the perspective transformation which maps a point in the world coordinate system $\mathbf{w}_h$ = (X, Y, Z, 1) (given in homogeneous coordinates) onto a point $\mathbf{c}_h$ in the image plane (also given in homogeneous coordinates):

$$\mathbf{c}_h = P\ R\ G\ \mathbf{w}_h,$$

with G the transformation matrix which translates the origin of the world coordinate system to the center of the camera system (= center of image plane), R the matrix which is a concatenation of the three basic rotation matrices (around the x axis, y axis and z axis) and P the perspective transformation matrix with focal length λ. The Cartesian coordinates (x, y) of the imaged point are obtained by dividing the first and second components of $\mathbf{c}_h$ by the fourth. This means we have the following functions which are dependent on the position and orientation of the camera:

$$f_x(X_0, Y_0, Z_0, \alpha, \beta, \gamma): \qquad X = \mathbf{w}_h(1) \quad -> \quad x = \mathbf{c}_h(1) / \mathbf{c}_h(4)$$

$$f_y(X_0, Y_0, Z_0, \alpha, \beta, \gamma): \qquad Y = \mathbf{w}_h(2) \quad -> \quad y = \mathbf{c}_h(2) / \mathbf{c}_h(4).$$

When we apply this functions to three known points in the world coordinate system we get six non-linear equations in six variables which give the coordinates of three points in the image plane. We denote these functions by f_{x1} , f_{y1} , f_{x2} , f_{y2} , f_{x3} and f_{y3} . Now we compute the Jacobian matrix with the partial differentials of these functions:

$$J = \begin{pmatrix} \frac{\bullet f_{x1}}{\bullet X_0} & \frac{\bullet f_{x1}}{\bullet Y_0} & \frac{\bullet f_{x1}}{\bullet Z_0} & \frac{\bullet f_{x1}}{\bullet \alpha} & \frac{\bullet f_{x1}}{\partial \beta} & \frac{\partial f_{x1}}{\partial \gamma} \\ \frac{\partial f_{y1}}{\partial X_0} & \frac{\partial f_{y1}}{\partial Y_0} & \frac{\partial f_{y1}}{\partial Z_0} & \frac{\partial f_{y1}}{\partial \alpha} & \frac{\partial f_{y1}}{\partial \beta} & \frac{\partial f_{y1}}{\partial \gamma} \\ & & \ldots & & & \\ & & \ldots & & & \\ & & \ldots & & & \\ \frac{\partial f_{y3}}{\partial X_0} & \frac{\partial f_{y3}}{\partial Y_0} & \frac{\partial f_{y3}}{\partial Z_0} & \frac{\partial f_{y3}}{\partial \alpha} & \frac{\partial f_{y3}}{\partial \beta} & \frac{\partial f_{y3}}{\partial \gamma} \end{pmatrix} .$$

This Jacobian matrix multiplied with the vector consisting of changes in the pose of the camera gives us the resulting changes of the 3 points in the image plane. If the Jacobian matrix is singular, some changes in the pose of the camera will not result in any changes of the three points in the image plane. This means that certain movements (a combination of translations and rotations) of the camera will not change the triangle (consisting of the 3 image points) in the image plane. In this case the pose of the camera cannot uniquely be determined from the three image points.

One way to investigate the Jacobian matrix is to apply a singular value decomposition (see Stewart, 1973) in which a general matrix A is reduced to a diagonal form by pre-multiplying and post-multiplying it by orthogonal matrices:

$$A = U^T M V,$$

with M a diagonal matrix, U and V orthogonal (unitary) matrices ($U^T U = I$ and $V^T V = I$). The diagonal elements of M are called the singular values of matrix A. The rank of the diagonal matrix M is equal to the rank of matrix A, which immediately indicates if matrix A is singular. If one of the singular values is small, matrix A is almost singular.

If we apply a singular value decomposition to the Jacobian matrix, the singular values will characterize the Jacobian matrix. If one of the singular values is small, certain movements in the camera pose will not result in changes in the triangle in the image plane. If one of the singular values is big, the resulting changes in the image will also be big.

In Fig. 7 the meaning of the matrices U^T, M, and V is given. The matrices U^T, M, and V identify separate, essential properties of the measuring process. The vectors comprising the V matrix represent distinct transformations of the state space to which the system has a well defined sensitivity. This sensitivity, or the measurement gains, are given by the associated number in the singular value matrix M. And the signature of this transformation in the measured values is given by the column vectors in U^T.

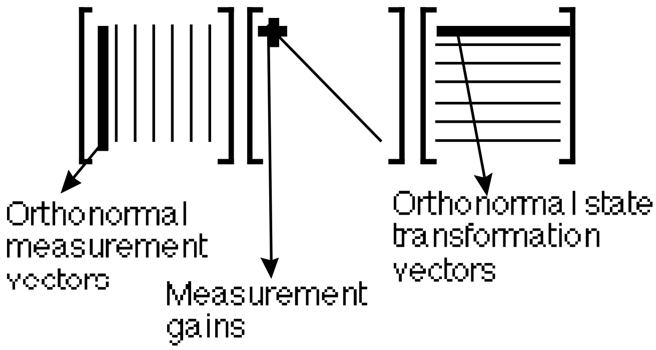

Figure 7. The matrices U^T, M, and V of the singular value decomposition.

The ideal pose of the camera is equal to $(X_0, Y_0, Z_0, \alpha, \beta, \gamma) = (0, 0, 3, 0, 0, 0)$. This means the camera is placed on the z axis at height 3 looking downwards. The focal length $\lambda = 1$. The situation is depicted in Fig. 8.

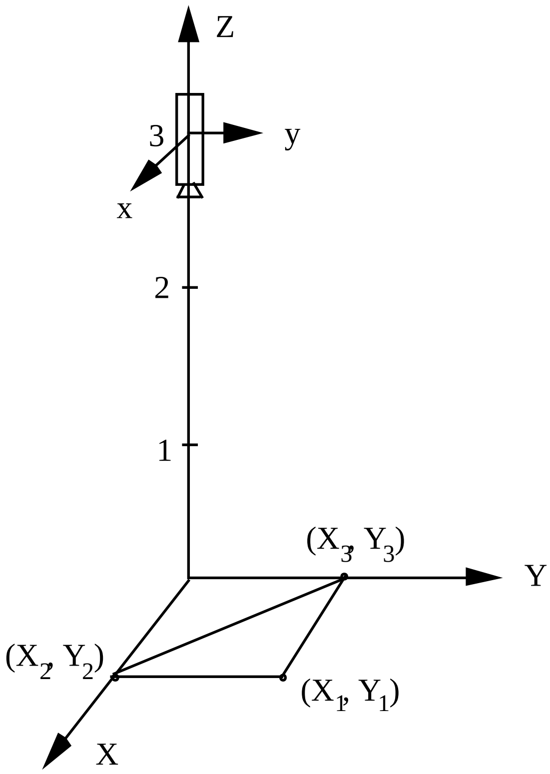

Figure 8. Camera viewing a 3-D scene (From Fu, 1987).

This resulted in the following Jacobian matrix:

$$J = \begin{pmatrix} 0.5 & 0 & 0.25 & 0.25 & -1.75 & -0.5 \\ 0 & 0.5 & 0.25 & 1.75 & -0.25 & 0.5 \\ 0.5 & 0 & 0.25 & 0. & -1.75 & 0 \\ 0 & 0.5 & 0 & 1.5 & 0 & 0.5 \\ 0.5 & 0 & 0 & 0 & -1.5 & -0.5 \\ 0 & 0.5 & 0.25 & 1.75 & 0 & 0 \end{pmatrix}$$

The Jacobian matrix shows for example that when the camera is lifted up, i.e. Z_0 is changed with ΔZ_0, the resulting changes of the images of (X_1, Y_1), (X_2, Y_2) and (X_3, Y_3) are resp.

$$(0.25\ \Delta Z_0,\ 0.25\ \Delta Z_0),\ (0.25\ \Delta Z_0,\ 0) \text{ and } (0,\ 0.25\ \Delta Z_0).$$

This result is obtained by multiplying J with vector $(0, 0, \Delta Z_0, 0, 0, 0)^T$. This means that the y-coordinate of the image of the second world point and the x-coordinate of the image of the third world point do not change as result of this movement of the camera. The singular value decomposition of matrix J gives the following diagonal matrix M:

$$M = \begin{pmatrix} 3.2 & 0 & 0 & 0 & 0 \\ 0 & 2.99 & 0 & 0 & 0 \\ 0 & 0 & 0.61 & 0 & 0 \\ 0 & 0 & 0 & 1.26 & 0 \\ 0 & 0 & 0 & 0 & 0.05 \end{pmatrix}$$

This shows that the Jacobian is singular and that its rank equals 5. It means that a certain movement of the camera will not result in a change of the image coordinates, and is therefore not noticed. Because the camera is placed in vertically above the circumscribed circle of the three world points, this is exactly what we expected. If the Z = 0 plane is tilted a little bit (i.e. the camera is rotated around the x and/or y axis over a small angle), the image is not changed. The fifth singular value is very small, which means that the rank of the Jacobian matrix is "almost 4". The meaning of this small singular value can be explained as follows. The Jacobian matrix gives the change of the measurements as a result of the change in the pose of the system. The singular value decomposition $J = U^T MV$ leads us to the remark that matrix V consists of rows containing a combination of translations and rotations of the camera position. From this decomposition of the Jacobian matrix it is easy to deduct what the result will be of a movement defined by for example the fifth row v_5 in matrix V. Because V is an orthogonal matrix

$$V.\ v_5^T = (0, 0, 0, 0, 1)^T\ .$$

From this we get:

$$J.v_5^T = U^T\ .M.(0, 0, 0, 0, 1)^T = U^T\ .(0, 0, 0, 0, 0.05)^T = 0.05\ .\ u_5$$

with u_5 the fifth column of matrix U^T. This means that the singular values on the diagonal of matrix M act as weighting factors of the movements given by the rows of matrix V. If a weighting factor is small, the effect of the movement given by the corresponding row in matrix V is small. From this we deduce that small singular values lead to insensitivities of the measuring system.

This previous results show that the camera should be placed in a more horizontal direction and should have the image of the triangle centered around the origin of the image plane. The Jacobian computed for such a situation appears to be non singular, which means that this measuring system is sensitive to all possible deviations from the original pose.

1.6.3. The Four Finger Calibration (FFC) sensor system

Using the previous procedure, we first investigate the Four Finger Calibration sensor system with the four finger frame placed symmetrically around the cube. Instead of determining the position and orientation of the TCP from the 8 measured distances, we look again at the inverse transformation. Suppose the pose (X_0,Y_0, Z_0, α, β, γ) of the TCP is given, what are the 8 distances to the cube as a function of the pose. This gives us 8 equations in six variables for which the Jacobian matrix is computed, which is in this case an 8-6 matrix. The singular value decomposition for the case with (α, β, γ) = (0, 0, 0) gives a diagonal matrix M of 5-5, and

$$M = \begin{pmatrix} 10.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 10.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 2.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.4 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 1.4 \end{pmatrix}$$

which means that the Jacobian matrix is singular. This is exactly what we expected, because it is clear from the geometry of the FFC that rotations around the z axis can not be measured by this system when it is placed in the symmetric position. Better results are gained when the system is placed in a position rotated 30^0 around the z axis (see Fig. 9).

The singular values now are: 10.0, 10.0, 6.67, 2.0, 1.63, 1.63.

The singular value decomposition of the Jacobian matrix gives 6 singular values between 1.63 and 10.0, which means that in this case all deviations can be measured.

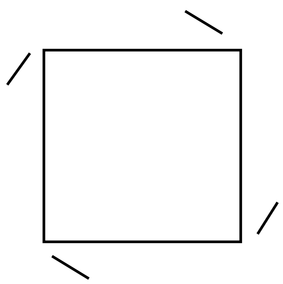

Figure 9. Top view of cube with four fingers rotated around the z axis by 30 degrees.

1.6.4. Three beam sensor system

The same mathematical approach was derived for the 3 beam system, consisting of 3 laser beams mounted on the TCP of the arm, which should each be centered on one of three 2-D position sensitive devices placed on the working plane in an equilateral triangle. The geometry of this system is depicted in Fig. 10.

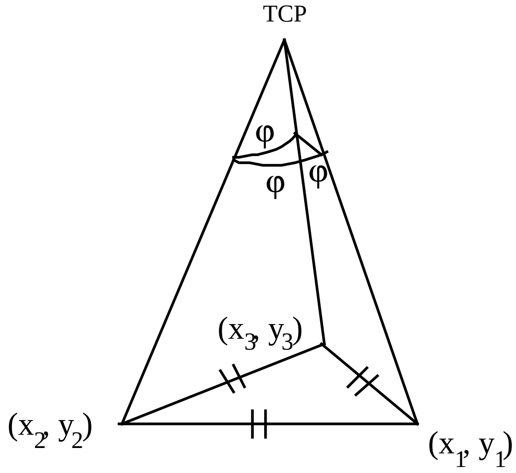

Figure 10. The geometry of the 3 beam system.

We consider the situation in which the angles between the beams (denoted by φ) are the same. The problem is the following: Given the three locations of the PSD-s with the light beams centered on the PSD-s, find the position and orientation ($X_0, Y_0, Z_0, \alpha, \beta, \gamma$) of the TCP. We again study the inverse problem: Given the position and orientation of the TCP, find the intersection of the 3 beams with the working plane. We define the following functions which are dependent on the position and orientation of the TCP:

$$f_{xi}(X_0, Y_0, Z_0, \alpha, \beta, \gamma) : \quad X_i \quad -> \quad x_i$$

$$f_{yi}(X_0, Y_0, Z_0, \alpha, \beta, \gamma) : \quad Y_i \quad -> \quad y_i \qquad i = 1, 2, 3,$$

with (X_i , Y_i) the coordinates of the unit vector along the i-th beam in the local coordinate system (with the TCP as origin) and (x_i , y_i) the intersection points of the 3 beams with the working plane. We form the Jacobian matrix with the partial differentials of the functions f_{x1} , f_{y1} , f_{x2} , f_{y2} , f_{x3} and f_{y3} . We computed the Jacobian matrix for the following case. Pose of the TCP: ($X_0, Y_0, Z_0, \alpha, \beta, \gamma$) = (0, 0, 1, 0, 0, 0) and angle $\varphi = 90^0$. This resulted in the following Jacobian matrix J:

$$J = \begin{pmatrix} 1 & 0 & 1.22 & -0.86 & -2.5 & 0.71 \\ 0 & 1 & -0.71 & 1.5 & 0.86 & 1.22 \\ 1 & 0 & -1.22 & 0.86 & -2.5 & 0.71 \\ 0 & 1 & -0.71 & 1.5 & -0.86 & -1.22 \\ 1 & 0 & 0 & 0 & -1 & -1.41 \\ 0 & 1 & 1.41 & 3 & 0 & 0 \end{pmatrix} .$$

As an example we make the following remarks about this matrix. Translations of the TCP in the direction of the x- or y axis result in the same translations of the intersection points. A rotation of the TCP around the X axis results in no change of the x coordinate of the third point of the triangle and the change in the y coordinate is weighted with factor 3. The singular value decomposition of J gives us the following singular values:

(4.18, 4.18, 0.71, 0.71, 2.45, 2.45).

These are all close to one, which means the measuring system is sensitive to all kinds of movements of the TCP. These values are pairwise equal because of the symmetry of the measuring system. It is worthwhile to show matrix V, because it contains mainly rows having only two components.

$$V = \begin{pmatrix} -0.38 & 0 & 0 & 0 & 0.92 & 0 \\ 0 & 0.38 & 0 & 0.92 & 0 & 0 \\ -0.92 & -0.02 & 0 & 0 & -0.38 & 0 \\ 0.02 & -0.92 & 0 & 0.38 & 0 & 0 \\ 0 & 0 & 0.46 & 0 & 0 & -0.88 \\ 0 & 0 & 0.88 & 0 & 0 & 0.46 \end{pmatrix}.$$

It consists of combinations of translations in the direction of the x or y axis and rotations around the y resp. x axis. In the last two rows a translation along the z axis and a rotation around the z axis are combined. The first two rows have the biggest weighting factor (4.18), which means that the movements represented by the first two rows cause a relative big change of the 3 intersection points in the plane. As mentioned before matrix U^T contains in the columns the results of the translations specified in the rows of matrix V. Matrix U^T looks like this:

$$U^T = \begin{pmatrix} -0.64 & -0.19 & 0.04 & -0.46 & -0.02 & 0.58 \\ 0.19 & 0.42 & -0.47 & -0.48 & -0.58 & -0.02 \\ -0.64 & 0.19 & 0.06 & 0.46 & -0.49 & -0.31 \\ -0.19 & 0.42 & 0.45 & -0.50 & 0.31 & -0.49 \\ -0.31 & 0 & -0.75 & 0.02 & 0.51 & -0.27 \\ 0 & 0.75 & 0.01 & 0.31 & 0.27 & 0.51 \end{pmatrix}.$$

The singular value decomposition has another advantage which we didn't mention until now. For matrix A with singular value decomposition $A = U^T MV$, we can write: $A^{-1} = V^T M^{-1} U$ with M^{-1} the diagonal matrix with the inverse singular values. Thus the singular value decomposition clearly shows how the inverse transformation looks like. For the 3 beam system the singular values of the inverse transformation show that the system is sensitive for deviations in the measuring points as well.

1.6.5. Four beam sensor system

We now investigate the 4-beam system with the four collimated beams making fixed angles with each other. Centering the beams on the PSD-s fixes two degrees of freedom. With 3 beams and 3 PSD-s all six degrees of freedom (position of the TCP) can be fixed. A system of 4 beams centered on 4 PSD-s is used to introduce redundancy which can be used for self-calibration purposes.

It is clear that for any opening angle, the four beams can always be aimed at four PSD-s positioned at the corners of a square. This configuration has the advantage that the measuring head can simply be rotated by 90• to obtain a new measurement.

A minimal configuration of the PSD-s has been studied, which consists of 9 PSD-s in a 3-3 array. It allows measurements to be done at 6 positions, in 4 poses each, providing a total of 192 data items, from which the parameters of each pose and certain system parameters (4 parameters of each beam) can be extracted. It is clear, however, that certain global characteristics cannot be found in this way, e.g. the position and orientation of the target plate (6 parameters), its scale (another 2) and certain properties of the measuring head.

We first analyze the properties of the set of equations resulting from one measurement, which reveals a number of geometric properties of the system. These equations are formed by the Jacobian matrix (J) which gives a relation between errors in the pose of the TCP (X, Y, Z, α, β, γ) and the resulting deviations from the intersection points of the 4 beams ((x_i, y_i), i = 1, 2, 3, 4) with the working plane:

$$J \cdot \begin{pmatrix} \Delta X \\ \Delta Y \\ \Delta Z \\ \Delta\alpha \\ \Delta\beta \\ \Delta\gamma \end{pmatrix} = \begin{pmatrix} \Delta x_1 \\ \Delta y_1 \\ \Delta x_2 \\ \Delta y_2 \\ \Delta x_3 \\ \Delta y_3 \\ \Delta x_4 \\ \Delta y_4 \end{pmatrix} .$$

From the Jacobian matrix we can deduct the effect of errors in the position and orientation of the TCP. We first study the effect of varying heights and varying opening angles on the singular values of the Jacobian matrix. The results of the analysis are given in Table 1, where the angle θ (= angle between beams and axis of symmetry, see Fig. 3) ranges from 30• to 70• and the height Z_0 of the TCP is equal to 1 or 2. The table shows that the Jacobian matrix is non-singular in all cases, which means that the system is sensitive to all kinds of transformations of the TCP. It shows that the optimal opening angle φ (= the angle between the beams) for the system is equal to about 70•. This corresponds to an angle of about 45• with the axis of symmetry of the system. The measuring system is more sensitive to orientation errors (the singular values are bigger), when the TCP is placed in a higher position.

θ	φ	Z_0	Singular values	$\frac{S_{max}}{S_{min}}$
30^0	41.4^0	1	3.08 3.08 1.15 1.15 0.22 0.22	14.3
		2	5.11 5.11 2.31 1.15 0.26 0.26	19.6
40^0	54.1^0	1	3.41 3.41 1.68 1.68 0.41 0.41	8.3
		2	5.92 5.92 3.36 1.68 0.48 0.48	12.4
45^0	60.0^0	1	3.70 3.70 2.00 2.00 0.54 0.54	6.9
		2	6.60 6.60 4.00 2.00 0.61 0.61	10.9
50^0	65.6^0	1	4.15 4.15 2.38 2.38 0.68 0.68	6.1
		2	7.64 7.64 4.77 2.38 0.74 0.74	10.3
55^0	70.8^0	1	4.88 4.88 2.86 2.86 0.84 0.84	5.8
		2	9.23 9.23 5.71 2.86 0.88 0.88	10.4
60^0	75.5^0	1	6.09 6.09 3.46 3.46 0.99 0.99	6.2
		2	11.8 11.8 6.93 3.46 1.02 1.02	11.6
70^0	83.3^0	1	12.3 12.3 5.49 5.49 1.23 1.23	10.0
		2	24.4 24. 4,11.0 5.49 1.24 1.24	19.7

Table 1: Singular values for Z_0 = 1 and 2.

The condition number of the Jacobian (ratio of largest and smallest eigenvalue), gives a clear indication of the sensitivity of the system. If this ratio becomes too large, a small offset in a mode to which the system is very sensitive can easily drown out a larger offset in a mode to which the system is less sensitive. If the ratio is small, the Jacobian matrix is well-conditioned.

From Table 1 we can see that the singular values increase as the opening angle φ increases, but the condition number is the smallest when the opening angle is close to 70•. This means the optimal opening angle of the 4-beam system is about 70•. The singular values for Z_0 = 2 are bigger than the values for Z_0 = 1. This means that the measuring system is more sensitive to orientation errors (e.g. the singular values are bigger), when the TCP is placed in a higher position. All in all, initial analysis of the 4-beam system indicate that it leads to a viable design.

As a next step, we investigated the self-checking properties of the system. I.e. we included various relevant system parameters with the unknowns. This requires a complete series of measurements in various poses to be treated as a unit. As expected, when all system parameters were included in this way, a singular system was obtained, i.e. not all parameters can be measured. Yet, a significant amount of information regarding the system can be extracted.

Next, we assumed the target plate, i.e. the PSD positions to be fully calibrated, but include the errors in the beam parameters (4 each) into the system. This means we have to compute the Jacobian of the measurements with respect to the pose and the beam parameters. This adds 16 unknowns to system, which means the Jacobian is 8 by 22.

To investigate the system as a whole, we have to study the complete set of equations obtained from all possible measurement poses for an array of 9 PSD-s. It allows measurements to be done at 6 positions, in 4 poses each, which provides 6*4*8 = 192 measurement data. The number of unknowns are the 6*4*6 pose parameters and the 4*4 beam parameters. This results in a Jacobian matrix of 192 by 160. The singular value decomposition of the Jacobian matrix of the complete measuring system shows that its rank is equal to 152, which means the equations cannot be solved. The situation is even worse, because the 4 smallest singular values are very small (less than 0.005). This means in fact that 12 unknowns still cannot be determined.

The above implies that, used in this way, the system is self-checking, i.e. when the system is damaged, this will show up as inconsistencies in the measurements. The system is not self-calibrating, however, as the cause of the error cannot be precisely identified. More information has to be added to the system.

This can be done by performing some calibration measurements on the measuring system itself. A suitable procedure for this is to use a calibration stand in which the position of the measuring head relative to the target plate can be precisely fixed. By rotating the target plate, four poses can be measured.

This adds 4*8 equations to the system, which give a relation between errors in the beam parameters and the resulting errors in the intersection points. The resulting Jacobian is a matrix of 224 by 160. From the singular value decomposition we deduct that the rank of the Jacobian is equal to 160. The condition number of the Jacobian (ratio of largest and smallest eigenvalue), gives a clear indication of the sensitivity of the system. This number would exceed 23 000 in this case, which means the Jacobian matrix is ill-conditioned. I.e. matrix J is nearly singular and J^{-1} is very large. From the 6 smallest singular values (less than 0.006) we deduct that 6 unknowns still cannot be determined. Examination of the corresponding row vectors of the V matrix shows, that the system cannot make a difference between increasing the angle between the beams and lifting the system as a whole. This is the reason why we have to measure at known positions from different heights.

So we assume that the parameters of the position at another height are also known,which adds again 32 equations to the system. The resulting Jacobian of 256 by 160 has rank 160. A histogram of the singular values is given in Fig. 11. The condition number is about 79 which means that this Jacobian is sufficiently well-conditioned and that all the parameters can be deduced from this system. In fact the solution is given by the pseudo-inverse of J defined by $J^{-1} = V^T D^{-1} U$. This matrix gives the optimal solution of the system:

$$J \, . \, \Delta(\text{pose parameters and beam parameters}) = \Delta(\text{measurements})$$

in the least square sense.

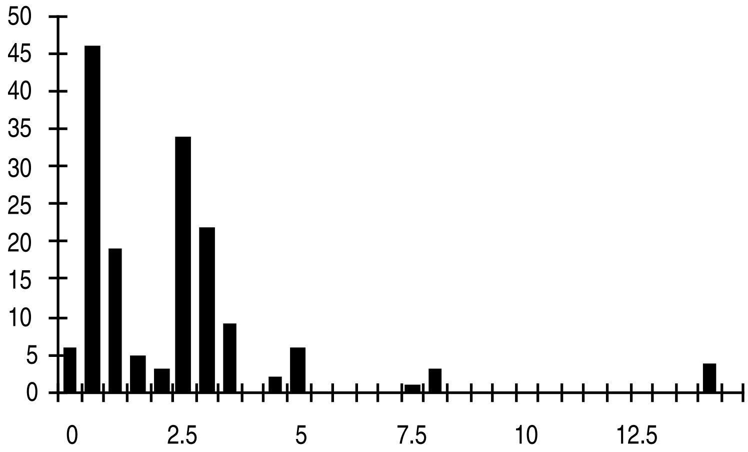

Figure 11. Histogram of the singular values.

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