KdV institute	Nonparametric Bayesian Statistics, assignment set 3
UvA	December 2, 2014

- **Deadline: December 15, 2014.**
- **Send a pdf file with your answers to hvzanten@uva.nl.**
- **Your name and student number should be on the answer sheet!**

1. Make the following exercises from the lecture notes: 10.5, 10.6, 11.4, 11.5, 12.1.
2. Note that Theorems 12.1 and 12.3 remain true if the Gaussian process defining the prior depends on the sample size n. For a sequence of positive constants c_n, consider the setting of Section 12.2 with as prior Π_n the law of p_{W_n}, where $W_n = c_n W$ and W is a Brownian motion with standard normal initial distribution (as in Section 12.3.1).
   (a) For a given sequence c_n, determine a (good) upper bound for $-\log Pr(\|W_n\|_\infty < \varepsilon)$ (with $\|\cdot\|_\infty$ the supremum-norm on $[0, 1]$).
   (b) For a given sequence c_n, determine the RKHS $\mathbb{H}_n$ of the process W_n and the corresponding RKHS-norm.
   (c) For a given sequence c_n and a $w_0 \in C^\beta[0, 1]$ for $\beta \in (0, 1]$, determine a (good) upper bound for
   $$\inf_{h \in \mathbb{H}_n : \|h - w_0\|_\infty \leq \varepsilon} \|h\|^2_{\mathbb{H}_n}.$$
   (d) For the density estimation problem, show that there exists a choice for the rescaling sequence c_n such that if $\log p_0 \in C^\beta[0, 1]$ for $\beta \in (0, 1]$, then the posterior contracts around p_0 at the rate $n^{-\beta/(1+2\beta)}$.