Functionaal Analyse

Exercises 10

Thursday 4 May 2017

Exercise 10.1

Let $\mathscr{H}$ be the Hilbert space ℓ^2 of all square-summable sequences in $\mathbb{C}$. For every $n \geq 1$, define the operator $S_n : \mathscr{H} \to \mathscr{H}$,

$$S_n(x_1, x_2, x_3, \ldots) = (0, 0, \ldots, 0, x_1, x_2, x_3, \ldots)$$

that shifts x to the right n times.

a. Show that for every $x \in \mathscr{H}$,

$$\lim_{n\to\infty} S_n^* x = 0.$$

Exercise 10.2

Let $\mathscr{H}$ be a complex Hilbert space with $T \in B(\mathscr{H})$ self-adjoint. Let $S \subset \mathscr{H}$ be a linear subspace and let x_0 be a point in S. Show that the following are equivalent:

a. the map $S \to \mathbb{R} : x \mapsto \|Lx\|$ assumes its maximum in x_0;

b. the map $S \to \mathbb{R} : x \mapsto |(Lx, x)|$ assumes its maximum in x_0;

c. x_0 is eigenvector of L^2 with eigenvalue $\|L^2\|$;

d. x_0 is eigenvector of L with eigenvalue $\pm\|L\|$.

Exercise 10.3

Discuss the solutions of exercises 6.1, 6.5, 6.9–6.12, 6.14, 6.17, 6.21–6.23, 6.26 and 6.28 from Rynne and Youngson with your fellow students, until you understand all aspects in full detail.
