Category Theory 2017 - Exercise sheet 3

Exercise 1. In this exercise $\mathbb{C}$ is a category with binary products and A, B and C are objects in $\mathbb{C}$.

(a) Show that $A \times (B \times C) \cong (A \times B) \times C$.

(b) Define the ternary product of objects A_0, A_1 and A_2 to be an object P equipped with morphisms $\pi_i : P \to A_i$ ($i \in \{0, 1, 2\}$) such that for any object Q and any triple of morphisms $f_i : Q \to A_i$ ($i \in \{0, 1, 2\}$) there exists a unique morphism $m : Q \to P$ such that $\pi_i m = f_i$ for every $i \in \{0, 1, 2\}$. Show that the ternary product of A_0, A_1, A_2 is unique up to isomorphism.

(c) Use (b) to give an alternative solution to (a).

Exercise 2. (a) Show that the category **Sets** has equalizers, coproducts and coequalizers.

(b) Do the same for the category **Top** of topological spaces and continuous maps.

Exercise 3. (a) Show that if 1 is a terminal object in a category $\mathbb{C}$, then $\mathbb{C}/1 \cong \mathbb{C}$.

(b) Show that if $A \times B$ is the product of two objects A and B in $\mathbb{C}$, then $\mathbb{C}/(A \times B) \cong \mathbb{C}/A \times \mathbb{C}/B$.

(c) Let X be an object in a category $\mathbb{C}$. Show that $\mathbb{C}/X$ has a terminal object. Show that if $\mathbb{C}$ has coproducts, then so does $\mathbb{C}/X$; and similarly for equalizers.

(d) Assume X is an object in a category $\mathbb{C}$ with products. Show that the functor $\Sigma_X : \mathbb{C}/X \to \mathbb{C}$ which sends $A \to X$ to A preserves epis.