Category Theory 2017 - Exercise sheet 7

1. Show that the category of partial maps, **Par**, and the category of pointed sets, $\mathbf{Sets}_*$, are equivalent.

2. Let $F\colon \mathbb{C} \to \mathbb{D}$.

   We say F is *faithful* if for all objects X and Y of $\mathbb{C}$ and all morphisms $f, g\colon X \to Y$, if $F(f) = F(g)$ then $f = g$. (In other words, for all objects X and Y the function $F(-)\colon \mathrm{Hom}(X, Y) \to \mathrm{Hom}(FX, FY)$ is injective).

   We say F is *full* if for all objects X and Y of $\mathbb{C}$ and all morphisms $f\colon F(X) \to F(Y)$, there exists some $f'\colon X \to Y$ such that $F(f') = f$. (In other words, for all objects X and Y the function $F(-)\colon \mathrm{Hom}(X, Y) \to \mathrm{Hom}(FX, FY)$ is surjective.)

   We say F is *essentially surjective* if for every object Z of $\mathbb{D}$ there exists an object Z' in $\mathbb{C}$ together with an isomorphism $i\colon F(Z') \to Z$.

   Show that F is an equivalence of categories if and only if it is full, faithful and essentially surjective.

3. Show that equivalence of categories is an equivalence relation on the class of categories.

4. A category is *skeletal* if whenever two objects are isomorphic, they are in fact equal. Show that every category is equivalent to a skeletal category.