3rd Homework sheet Category Theory

- Deadline: 2 May, 13:00 sharp.
- Submit your solutions by handing them to the lecturer at the *beginning of the lecture*.
- Good luck!

Exercise 1 A poset P is *complete* if every subset $X \subseteq P$ has a least upper bound and a greatest lower bound (equivalently, it is a poset which is both complete and cocomplete as a category). This means in particular that it has a least and greatest element. A *complete Heyting algebra* is a complete poset which is also cartesian closed as a category. Throughout this exercise H will be a fixed complete Heyting algebra.

An *H-set* is a pair (A, α) where A is a set and α is a function $A \to H$. A morphism of H-sets $f \colon (B, \beta) \to (A, \alpha)$ is a function $f \colon B \to A$ such that

$$\beta(b) \leq \alpha(f(b))$$

for every $b \in B$. Since such functions are closed under composition and contain all identities, this defines a category which we will denote by $H\mathbf{Sets}$.

(a) *(2 points)* Show that the category $H\mathbf{Sets}$ has all small limits.

(b) *(4 points)* Show that the category $H\mathbf{Sets}$ is cartesian closed.

(c) *(4 points)* Show that category $H\mathbf{Sets}$ has a natural numbers object.